An Analytic Solution of Free Convective MHD Flow of Radiating and Reacting Fluid Past an Infinite Vertical Plate With Constant Suction

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Abstract

An analytic solution of unsteady flow of an incompressible and electrically conducting fluid past an infinite vertical plate, under the action of uniformly applied magnetic field has been presented. A uniform magnetic field is applied along an axis perpendicular to plane of the plate. The plate temperature is raised with time ($t^{'} > 0$). The dimensionless governing equations are solved in closed form by using Laplace transform tecnique. The solution are expressed in terms of exponential and complimentary error function. The effect of flow parameters on velocity, temperature, concentration, the rate of heat and mass transfer and sherwood number have been discussed in detail with the help of graphs. It is found that velocity profile increases with Grashoff number whereas it adversely effect the skin friction coefficient.

Subject Classification (2010) **:**76D05, 76D10.
Keywords: MHD, porous medium, radiation, chemical reaction, accelerated plate.

INTRODUCTION

The study of MHD free convective flow with heat and mass transfer have attaracted the attention of a number of scholars due to diverse applications. In astrophysics and geophysics it is applied to study the steller and solar structures, radio propagation through the ionosphere etc. In engineering we find its applications in MHD pumps, MHD bearings and MHD converter etc.From tecnological point of view, MHD convection flow problem are also very significant in the fields of stellar and planetary magnetosphere, aeronautics, chemical engineering and electronics. The study of radiative heat transfer flow is very important in manufacturing industry for the design of reliable equipments, nuclear power plants, gas turbines and various propulsion devices for air craft, missiles, satellites and space vehicles.

In view of the importance of the thermal radiation effect along with, chemical reaction and heat source several authors have carried out their researches to investigate the effect of these on heat and mass transfer problems. Recently Garg [1] has analysed the magnetohydrodynamics and rdiation effects on the flow due to moving vertical porous plate with variable temperature. Effects of radiation on MHD flow past an accelerated isothrmal vertical plate with heat and mass diffusion were studied by Muthucumaraswamy et al.[2]. Prakash et al.[3] studied diffusion-thermo and radiation effects on unsteady MHD flow through porous medium past an impulsively started infinite vertical plate with variable temperature and mass diffusion. Singh and Garg [4] studied exact solution of an oscillatory free convective MHD flow in a rotating porous channel with radiative heat effects . Hall current on free convective flow past an accelerated vertical porous plate in a rotating system with heat source/sink have been investigated by Singh and Garg [5]. Rajput and Kumar [6] have presented radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer. Prasad et al. [7] have presented combined effects of thermal radiation and hall current on vertical moving porous plate in a rotating system with variable temperature.Combined heat and mass transfer by mixed convection MHD flow along a porous plate with chemical reaction in the presence of heat source investigated by Zueco and Ahmed [8] .Poonia and Chaudhary [9] studied the effect of heat transfer on MHD free convective flow through porous medium with viscous dissipation. Effects of heat and mass transfer on MHD unsteady convective flow along a vertical porous plate with constant suction studied by Chand et al. [10].Ahmed and Kalita [11] peresented analytical and numerical study for MHD radiating flow over an infinite vertical surface bounded by a porous medium in presence of chemical reaction. Effects of chemical reactions and radiation on an unsteady MHD flow past an accelerated infinite vertical plate with variable temperature and mass transfer investigated by Ahmed et al.[12]. Rana [13] presented free convection effects on the oscillatory flow past a vertical porous plate in the presence of radiation for an optically thin fluid.

The object of the present paper is to study the effect of magnetic field on unsteady free convective flow past along an accelerated infinite vertical porous plate in the presence of variable temperature, thermal radiation, heat source and constant suction.The dimensionless governing equations are solved using Laplace transform tecnique.The expressions for velocity, temperature and concentration have been obtained in terms of exponential and complementary error function.

NOTATION

$A = \left(\frac{a^2}{\nu}\right)^{\frac{1}{3}}$ =constant

a :acceleration

B_0 :the magnetic induction

c_p :specific heat of the fluid at constant pressure

G_r :Grashoff number

G_m :modified Grashoff number

g : acceleration due to gravity

k_p :permeability of porous medium

$K^{'}$:porosity parameter

M :magnetic field parameter

N :radiation parameter

P_r : Prandtl number

S_c :Schmidt number

t :time
u :fluid velocity along x-axis
v :fluid velocity along y-axis
w :suction parameter
κ :coefficient of thermal conductivity
μ :coefficient of viscosity
ν :kinematic viscosity
σ :electrical conductivity
θ :Temperature
ρ :density of the fluid

GOVERNING EQUATIONS

Consider an unsteady, free convective flow of an incompressible, electrically conducting viscous fluid past an accelerated infinite vertical plate with variable temperature and mass transfer under the influence of a uniform transverse magnetic field B_0. We introduce a co-ordinate system $\left(x', y', z'\right)$ with origin at the accelerated plate which is subjected to constant suction velocity V_0, the $x' - axis$ is taken along the plate in the upward vertical direction, $y' - axis$ is taken along normal to the plate directed in to the fluid region. Let $\left(u', 0, 0\right)$ be the fluid velocity at the $\left(x', y', z'\right)$ when time $t' > 0$. Initially $\left(i.e\ when\ t' \leq 0\right)$, the plate is at rest relative to the fluid $\left(i.e\ u' = 0\ \right)$ and the fluid at the plate's surface has the same temperature and concentration as those at the edge of the boundary layer i.e T'_∞ and C'_∞respectively. Initially when $\left(t' \leq 0\right)$ the fluid temperature and concentration are assumed to remain constant through out the fluid region. At time $t' > 0$ the plate is accelerated with a velocity $u' = at'$ in its own plane and the temperature at the plate is raised linearly with respect to time and concentration near the plate is assumed C'_w.

Under Boussinesq approximation the equations governing the flow are:

Equation of continuity

$$\frac{\partial v'}{\partial y'} = 0. \tag{1}$$

Equation of motion

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta_c\left(C' - C'_\infty\right) - \frac{\sigma B_0^2}{\rho}u' - \frac{\nu}{K'}u'. \tag{2}$$

Equation of energy

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q'}{\partial y'} - \frac{Q'_0}{\rho c_p}\left(T' - T'_\infty\right) \tag{3}$$

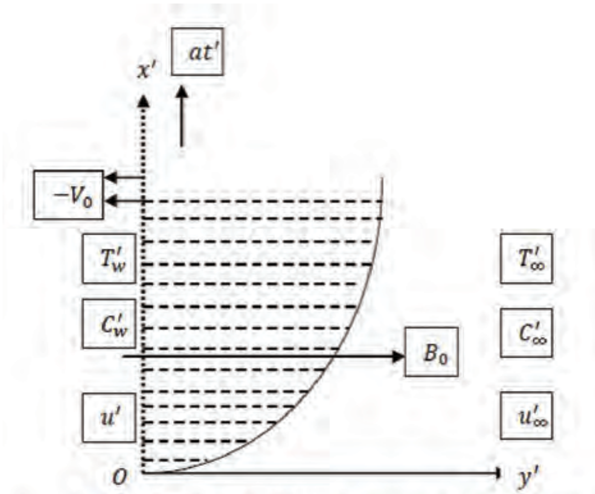

Fig.1.-Geometrical Configuration of Problem.

Equation of mass transfer

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - \left(C' - C'_\infty\right)K'_1 \tag{4}$$

where $u'$and v'- denotes the velocity component in the boundary layer in direction $x' - axis$ and $y' - axis$ respectively; T'- the temperature inside the boundary layer; T'_∞- the temperature of the free stream; t'- the time; β and β_c- the volumetric coefficient of thermal and concentration expansion respectively; C'-the concentration in the boundary layer; C'_∞- the concentration in the fluid far away from the plate; Q'_0-heat source parameter; D-thermal diffusivity; K'_1-coefficient of first order chemical reaction parameter. Here

$$\frac{\partial q'}{\partial y'} = 4\alpha^2\left(T' - T'_\infty\right)$$

is a radiative heat flux Cogely et al.[14] where α is radiation absorption cofficient.
The flow is governed by the following initial and boundary conditions:

$$u' = 0, T' = T'_\infty, C' = C'_\infty \quad \text{for all } y', \quad t' \leq 0 \tag{5}$$

$$\left\{\begin{array}{c} \text{at } y' = 0, u' = at', T' = T'_\infty + \left(T'_w - T'_\infty\right)At', C' = C'_w \\ \text{at } y' \to \infty, u' \to 0, T' \to T'_\infty, C' \to C'_\infty \end{array}\right\} \text{for all } t' > 0 \tag{6}$$

From equation of continuity (1), it is clear that suction velocity normal to the plate is constant. Hence from the equation of the continuity we obtained:

$$v' = -V_0 \tag{7}$$

V_0 is a non zero positive constant suction velocity, the negative sign indicates that the suction is towards the plate.

Governing equations in non-dimensional form are,

$$\frac{\partial u}{\partial t} - w\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m C - \left(M + \frac{1}{k_p}\right)u \tag{8}$$

$$P_r\frac{\partial \theta}{\partial t} - wP_r\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} - \left(N^2 + QP_r\right)\theta \tag{9}$$

$$S_c\frac{\partial C}{\partial t} - wS_c\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} - S_c K_1 C \tag{10}$$

where the non-dimensional quantities used above are

$$\left\{\begin{array}{c} u = \frac{u'}{(\nu a)^{\frac{1}{3}}}, y = y'\left(\frac{a}{\nu^2}\right)^{\frac{1}{3}}, t = t'\left(\frac{a^2}{\nu}\right)^{\frac{1}{3}}, w = \frac{V_0}{(\nu a)^{\frac{1}{3}}}, N = \frac{2\alpha}{\sqrt{k}}\left(\frac{\nu^2}{a}\right)^{\frac{1}{3}} \\ G_r = \frac{g\beta\left(T'_w - T'_\infty\right)}{a}, G_m = \frac{g\beta_c\left(C'_w - C'_\infty\right)}{a}, k_p = K'\left(\frac{a}{\nu^2}\right)^{\frac{2}{3}}, P_r = \frac{\mu C_p}{\kappa}, S_c = \frac{\nu}{D} \\ M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{a^2}\right)^{\frac{1}{3}}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Q = \frac{Q'_0}{\rho C_p}\left(\frac{\nu}{a^2}\right)^{\frac{1}{3}}, K_1 = K'_1\left(\frac{\nu}{a^2}\right)^{\frac{1}{3}} \end{array}\right\} \tag{11}$$

The initial and boundary conditions in dimensionless form are as follows:

$$u = 0, \theta = 0, C = 0 \quad \text{for all } y,\ t \le 0 \tag{12}$$

$$\left\{\begin{array}{c} \text{at } y = 0, u = t, \theta = t, C = 1 \\ \text{at } y \to \infty, u \to 0, \theta \to 0, C \to 0 \end{array}\right\} \text{ for all } t > 0 \tag{13}$$

The dimensionless governing equations (8)to (10), subject to boundary conditions (12) and (13) are solved by using Laplace transform tecnique and it transform to following set of equations.

$$\frac{d^2\overline{u}}{dy^2} + w\frac{d\overline{u}}{dy} - \left(S + M + \frac{1}{k_p}\right)\overline{u} = -G_r\overline{\theta} - G_m\overline{C} \tag{14}$$

$$\frac{d^2\overline{\theta}}{dy^2} + wP_r\frac{d\overline{\theta}}{dy} - \left(P_r S + N^2 + QP_r\right)\overline{\theta} = 0 \tag{15}$$

$$\frac{d^2\overline{C}}{dy^2} + wS_c\frac{d\overline{C}}{dy} - S_c\left(S + K_1\right)\overline{C} = 0 \tag{16}$$

subject to the boundary conditions:

$$\left\{\begin{array}{l} \text{at } y=0, \overline{u}=\frac{1}{S^2}, \overline{\theta}=\frac{1}{S^2}, \overline{C}=\frac{1}{S} \\ \text{at } \ y\to\infty, \overline{u}\to 0, \overline{\theta}\to 0, \overline{C}\to 0 \end{array}\right\} \tag{17}$$

The solution of the equations (14) to (16) under the boundary condition (17) are given by following expressions.

$$\overline{u}=\left[\begin{array}{c} \left\{\frac{1}{S^2}+\begin{array}{c}\frac{G_r}{(P_r-1)}\left(\frac{M_1}{S}+\frac{M_2}{S^2}+\frac{M_3}{(S-\alpha_1)}+\frac{M_4}{(S-\beta_1)}\right)\\ +\frac{G_m}{(S_c-1)}\left(\frac{M_5}{S}+\frac{M_6}{(S-\alpha_2)}+\frac{M_7}{(S-\beta_2)}\right)\end{array}\right\}e^{-A_6 y} \\ -\frac{G_r}{(P_r-1)}\left(\frac{M_1}{S}+\frac{M_2}{S^2}+\frac{M_3}{(S-\alpha_1)}+\frac{M_4}{(S-\beta_1)}\right)e^{-A_4 y} \\ -\frac{G_m}{(S_c-1)}\left(\frac{M_5}{S}+\frac{M_6}{(S-\alpha_2)}+\frac{M_7}{(S-\beta_2)}\right)e^{-A_2 y} \end{array}\right] \tag{18}$$

$$\overline{\theta}=\frac{1}{S^2}e^{-A_4 y} \tag{19}$$

$$\overline{C}=\frac{1}{S^2}e^{-A_2 y} \tag{20}$$

Taking inverse Laplace transforms of the equations(18) to (20) ,we obtained the following expression for the velocity, temperature and concentration profile,

$$u=\left[\begin{array}{c} -A_3\left(\exp X_1 erfc\eta_1+\exp X_2 erfc\eta_2\right)-A_4\left(\eta_3\exp X_3 erfc\eta_3-\eta_4\exp X_4 erfc\eta_4\right) \\ -A_5\left(\exp X_3 erfc\eta_3+\exp X_4 erfc\eta_4\right)+A_6\left(\eta_5\exp X_5 erfc\eta_5-\eta_6\exp X_6 erfc\eta_6\right) \\ +A_7\left(\exp X_5 erfc\eta_5+\exp X_6 erfc\eta_6\right)+A_8\left(\exp X_7 erfc\eta_7+\exp X_8 erfc\eta_8\right) \\ +A_9\left(\exp X_9 erfc\eta_9+\exp X_{10} erfc\eta_{10}\right)+A_{10}\left(\exp X_{11} erfc\eta_{11}+\exp X_{12} erfc\eta_{12}\right) \\ +A_{11}\left(\exp X_{13} erfc\eta_{13}+\exp X_{14} erfc\eta_{14}\right)-A_{12}\left(\exp X_{15} erfc\eta_{15}+\exp X_{16} erfc\eta_{16}\right) \\ -A_{13}\left(\exp X_{17} erfc\eta_{17}+\exp X_{18} erfc\eta_{18}\right)-A_{14}\left(\exp X_{19} erfc\eta_{19}+\exp X_{20} erfc\eta_{20}\right) \\ -A_{15}\left(\exp X_{21} erfc\eta_{21}+\exp X_{22} erfc\eta_{22}\right) \end{array}\right] \tag{21}$$

$$\theta=\left[\ A_1\left(\eta_3\exp X_3 erfc\eta_3-\eta_4\exp X_4 erfc\eta_4\right)\ \right] \tag{22}$$

$$C=\frac{1}{2}\left(\exp X_1 erfc\eta_1+\exp X_2 erfc\eta_2\right) \tag{23}$$

SOME IMPORTANT CHARACTERSTICS OF FLOW FIELD

The skin friction coefficient at the plate in non – dimensional form is given by

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$(24) \qquad \tau = \begin{bmatrix} A_3\left(m_1 erfc\eta_1 - m_2 erfc\eta_2\right) + A_4\left(\eta_3 m_3 erfc\eta_3 + \eta_4 m_4 erfc\eta_4\right) \\ +A_5\left(m_3 erfc\eta_3 - m_4 erfc\eta_4\right) - A_6\left(\eta_5 m_5 erfc\eta_5 + \eta_6 m_6 erfc\eta_6\right) \\ -A_7\left(m_5 erfc\eta_5 - m_6 erfc\eta_6\right) - A_8\left(m_7 erfc\eta_7 - m_8 erfc\eta_8\right) \\ -A_9\left(m_9 erfc\eta_9 - m_{10} erfc\eta_{10}\right) - A_{10}\left(m_{11} erfc\eta_{11} - m_{12} erfc\eta_{12}\right) \\ -A_{11}\left(m_{13} erfc\eta_{13} - m_{14} erfc\eta_{14}\right) + A_{12}\left(m_{15} erfc\eta_{15} - m_{16} erfc\eta_{16}\right) \\ +A_{13}\left(m_{17} erfc\eta_{17} - m_{18} erfc\eta_{18}\right) + A_{14}\left(m_{19} erfc\eta_{19} - m_{20} erfc\eta_{20}\right) \\ +A_{15}\left(m_{21} erfc\eta_{21} - m_{22} erfc\eta_{22}\right) \end{bmatrix}$$

From the temperature profile the rate of heat transfer (Nu), in non - dimensional form is given by,

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$(25) \qquad Nu = \left[-A_1\left(\eta_3 m_3 erfc\eta_3 + \eta_4 m_4 erfc\eta_4\right)\right]$$

The mass transfer coefficient(Sh), at the plate in non – dimensional form is given by,

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$(26) \qquad Sh = -\frac{1}{2}\left(m_1 erfc\eta_1 - m_2 erfc\eta_2\right)$$

The constants used above have been listed in the appendix.

RESULTS AND DISCUSSION

In order to illustrate the physical significance of the results the numerical calculations have been carried out. The value of Schmidt number is taken 0.22 and 0.66 which corresponds to Hydrogen and Oxygen respectively 20^0C. The value of Prandtl number is taken 0.71, 3 and 7 which corresponds to air, Freon and water. Freon represents several different chlorofluorocarbons which are used in commerce and industries. The values of all the other parameters are taken arbitrarily. Only real part of the result has been considered. The effects of various parameters on the velocity profile are shown graphically in Fig.2. It is observed from this figure, that the velocity increases with the increase in the Grashoff number G_r, the modified Grashoff number G_m,the porosity parameter k_p whereas it decreases with the increase in Magnetic field parameter M, the Schmidt number S_c, radiation parameter N, heat source parameter Q, chemical reaction parameter K_1 and suction parameter w.

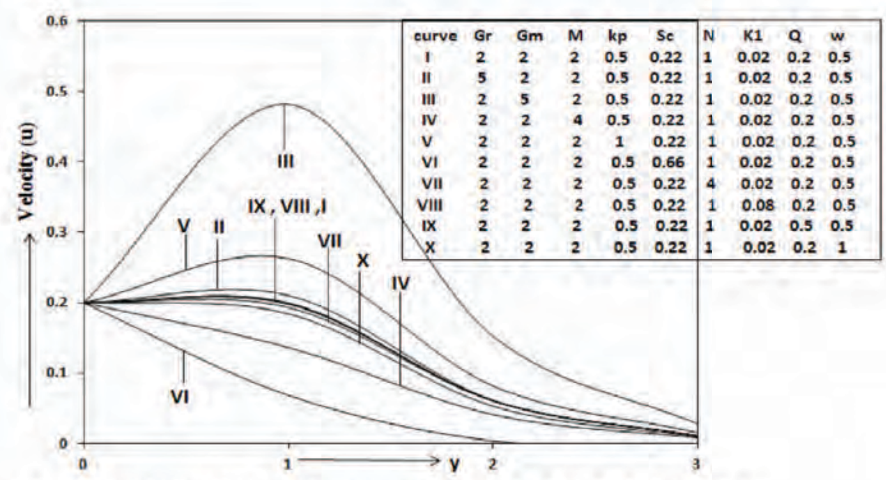

Fig.2. -Variation in velocity with various parameters, Pr=0.71, Gr=2, Gm=2 ,M=2 kp=0.5, Sc=0.22, K₁=0.02, N=1, Q=0.2, w=0.5, t=0.2

The variation in temperature profile with radiation parameter is shown in Fig.-3. This figure clearly depicts that the increasing radiation parameter has adverse effect on the temperature.The

variation in temperature profile with time is shown in Fig. - 4 and we observed that temperature increases with increase in time. From the Fig. - 5 it is observed that the concentration increases with the increase of time whereas it decreases with the increase in Schmidt number and chemical reaction parameter. From the Fig.-6 it is clear that coefficient of skin friction increases with the increase in Magnetic field parameter M, Schmidt number S_c, chemical reaction parameter K_1, heat source parameter Q, suction parameter w and it decreases with the increase in Grashoff number G_r, the modified Grashoff number G_m, porosity parameter k_p. From the Figs. -7& 8 it can be interpreted that the Nusselt number increases with increase in Prandtl number P_r and radiation parameter N. From the Fig.-9 it is evident that Sherwood number increases with the increase in Schmidt number S_c and chemical reaction parameter K_1.

PHYSICAL SIGNIFICANCE OF RESULTS

Physically if $(G_r > 0)$, it means cooling of the plate (or heating of the fluid) i.e. in free convection current which transfer heat away from the plate into the boundary layer region, therefore increasing value of Grashof number accelerates the flow.With the increase of magnetic field parameter, Lorentz forc e increases opposite to the direction of flow of fluid. This force has tendency to slow down the motion of the fluid therefore velocity decrease. With the increase of porosity parameter, resistance offered by porous medium decreases and consequently fluid velocity increases. Prandtl number is defined as the ratio of kinematic viscosity to the thermal diffusivity, therefore with the increase in Prandtl number viscosity increases which retard the fluid. As Schmidt number increases, the concentration decreases. This is attributed to the fact that higher value of Schmidt number amount to fall in the

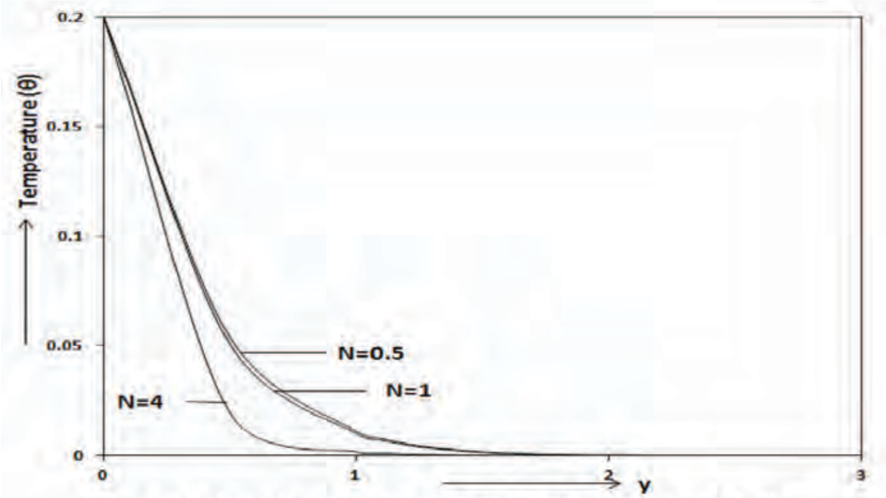

Fig.3.- Variation in temperature with various value of radiation parameter , for Pr=0.71, Q=0.2, w=0.5 , t=0.2.

chemical molecular diffusivity therefore less diffusion takes place by species transfer causing a reduction in concentration.

CONCLUSION

In this paper chemical reaction ,heat source and radiation effects on unsteady MHD heat and mass transfer flow of a viscous, incompressible, electrically conducting fluid past an accelerated infinite vertical plate with variable temperature in the presence of applied magnetic field through porous medium has been studied. The dimensionless governing equations are solved using Laplace transform technique. The main finding can be summarized as:

1. The velocity increases with the increase of the Grashoff number and porosity parameter, whereas it decreases as the magnetic field parameter , radiation parameter and heat source parameter increases.
2. Velocity decreases with the increase of suction parameter indicating the usual fact that suction stabilize the boundary layer . Sucking decelerate fluid particle through the porous wall reduce the growth of fluid boundary layer and hence velocity decreases.
3. Radiation parameter has adverse effect on temperature profile.
4. The skin friction coefficient decreases with the Grashoff number and porosity parameter whereas it increases with magnetic field parameter.
5. Concentration decreases with the increase in Schmidt number and chemical reaction parameter.

References

[1] Garg B.P. (2013), Megnetohydrodynamics and Radiation effects on the flow due to moving porous plate with variable temperature. Proceeding of the National Academy

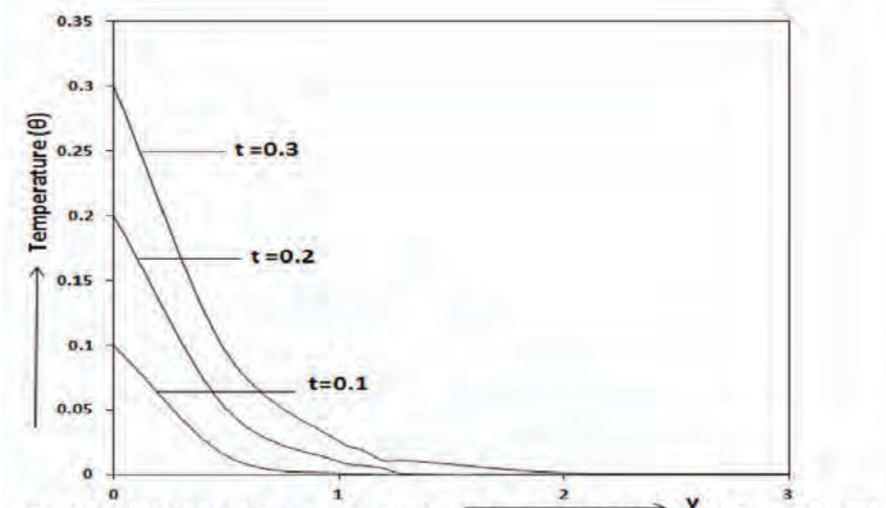

Fig.4.- Variation in temperature with various value of time ,for Pr=0.71, N=1, Q=0.2, w=0.5.

of sciences, 83, 327-331.

[2] Muthucumaraswamy R. , Lal Tina and Ranganayakulu, (2010), Effects of radiation on MHD flow past an accelerated isothermal vertical plate with heat and mass diffusion. Journal of Theortical and Appled Mechanics, 37, 189-202.

[3] Prakash J., Bhanumati and Kumar Vijaya (2013), Diffusion thermo and radiation effects on unsteady MHD flow through porous medium past an impulsively started infinite vertical plate with variable temperature and mass diffusion.Transport in Porous Media, 96, 135-151

[4] Singh K.D., Garg B.P. (2010), Exact solution of an oscillatory free convective MHD flow in a rotating porous channel with Radiative Heat. Proc. Natl. Acad., 80A, 81-89.

[5] Singh K.D., Garg B.P. (2009), Effects of Hall current on free convective flow past an accelerated vertical porous plate in a rotating system with Heat source/sink. J. Rajasthan Acad. Phy. Sci., 8, 191-202.

[6] Rajput U.S. and Kumar S. (2012), Radiation effects on MHD flow past an impulsively started vertical plate with variable Heat and Mass Transfer. Int. J. of Appl. Math and Mech., 8, 66-85.

[7] Prasad R.V. and Bhaskar R. N. (2008),Combined effects of thermal radiation and Hall current on vertical moving porous plate in a rotating system with variable temperature. Journals of Energy Heat and Mass Transfer, 30, 57-68.

[8] Zueco J. and Ahmed S.(2010), Combined heat and mass transfer by mixed convection MHD flow along a porous plate with chemical reaction in the presence of heat source.Appl. Math. Mech. Eng.Ed.,31,1217-1230.

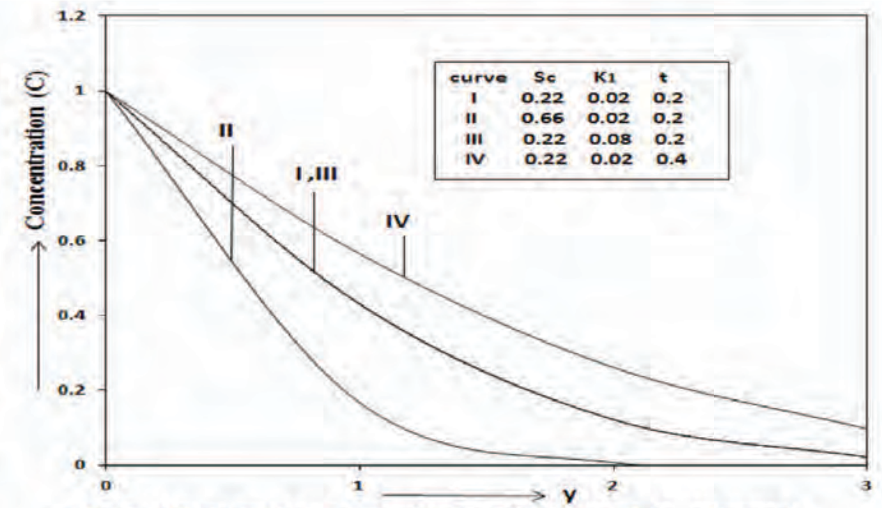

Fig.5.-Variation in concentration with Sc number, chemical reaction parameter and time.

[9] Poonia H. and R.C. Chaudhary (2012), Effect of heat transfer on MHD free convective flow through porous medium with viscous dissipation. Journal of Energy, Heat and Mass transfer, 33,103-120.

[10] Chand K., Singh K.D. and Sharma B.(2013),Effects of heat and mass transfer on MHD unsteady convective flow along a vertical porous plate with constant suction. Bull. Cal. Math. Soc.,105,411-428.

[11] Ahmed S. and K. Kalita (2013), Analytical and numerical study for MHD radiating flow over an infinite vertical surface bounded by a porous medium in presence of chemical reaction. 6, 597-607.

[12] Ahmed N., Goswami J.K. and Barua D.P.(2013), Effects of chemical reactions and radiation on an unsteady MHD flow past an accelerated infinite vertical plate with variable temperature and mass transfer.Indian J. Pure Appl. Math., 44,443-466.

[13] Rana S.(2013), Free convection effects on the oscillatory flow past a vertical porous plate in the presence of radiation for an optically thin fluid. Indian J. Pure Appl. Math., 44,757-770.

[14] Cogley A.C. ,Vincentine W.G. & Gilles S.E. (1986). Differential approximation for radiative transfer in a non- grey fluid near equilibrium. American Institute of Aeronautics and Astronautics, 6, 551-555.

APPENDIX

$X_1 = \left(b_1 - \frac{S_cW}{2}\right) y$ $\quad X_2 = -\left(b_1 + \frac{S_cW}{2}\right) y$ $\quad X_3 = \left(b_2 - \frac{P_rW}{2}\right) y$

$X_4 = -\left(b_2 + \frac{P_rW}{2}\right) y$ $\quad X_5 = \left(b_3 - \frac{W}{2}\right) y$ $\quad X_6 = -\left(b_3 + \frac{W}{2}\right) y$

$X_7 = \left(b_4 - \frac{W}{2}\right) y$ $\quad X_8 = -\left(b_4 + \frac{W}{2}\right) y$ $\quad X_9 = \left(b_5 - \frac{W}{2}\right) y$

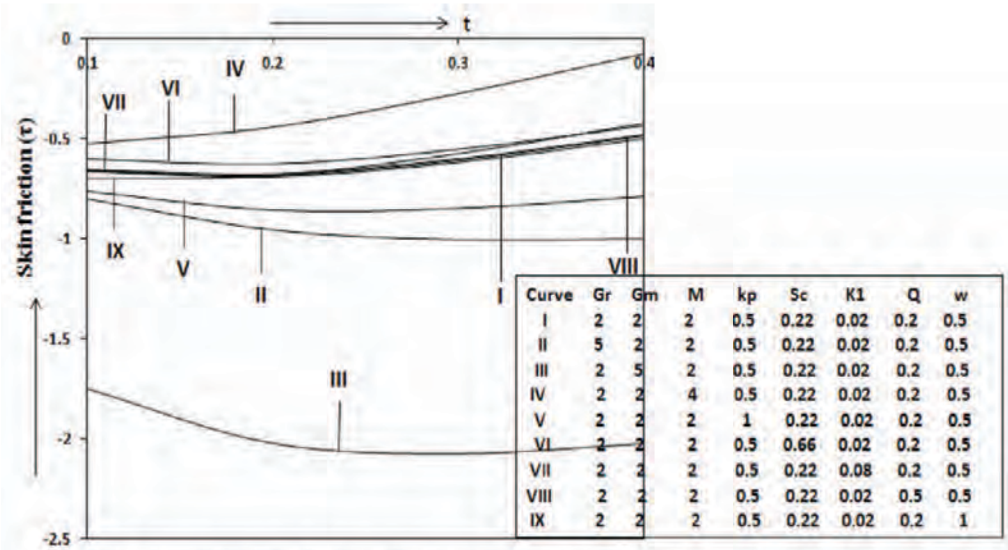

Fig.6.-Variation in skin friction with various parameters, Pr=0.71, Gr=2, Gm=2, N=1, Q=0.2, w=0.5, Kp=0.5, K₁=0.02, M=2.

$$X_{10} = -\left(b_5 + \tfrac{W}{2}\right) y \qquad X_{11} = \left(b_6 - \tfrac{W}{2}\right) y \qquad X_{12} = -\left(b_6 + \tfrac{W}{2}\right) y$$
$$X_{13} = \left(b_7 - \tfrac{W}{2}\right) y \qquad X_{14} = -\left(b_7 + \tfrac{W}{2}\right) y \qquad X_{15} = \left(b_8 - \tfrac{P_r W}{2}\right) y$$
$$X_{16} = -\left(b_8 + \tfrac{P_r W}{2}\right) y \qquad X_{17} = \left(b_9 - \tfrac{P_r W}{2}\right) y \qquad X_{18} = -\left(b_9 + \tfrac{P_r W}{2}\right) y$$
$$X_{19} = \left(b_{10} - \tfrac{S_c W}{2}\right) y \qquad X_{20} = -\left(b_{10} + \tfrac{S_c W}{2}\right) y \qquad X_{21} = \left(b_{11} - \tfrac{S_c W}{2}\right) y$$
$$X_{22} = -\left(b_{11} + \tfrac{S_c W}{2}\right) y \qquad m_1 = \left(b_1 - \tfrac{S_c W}{2}\right) \qquad m_2 = \left(b_1 + \tfrac{S_c W}{2}\right)$$
$$m_3 = \left(b_2 - \tfrac{P_r W}{2}\right) \qquad m_4 = \left(b_2 + \tfrac{P_r W}{2}\right) \qquad m_5 = \left(b_3 - \tfrac{W}{2}\right)$$
$$m_6 = \left(b_3 + \tfrac{W}{2}\right) \qquad m_7 = \left(b_4 - \tfrac{W}{2}\right) \qquad m_8 = \left(b_4 + \tfrac{W}{2}\right)$$
$$m_9 = \left(b_5 - \tfrac{W}{2}\right) \qquad m_{10} = \left(b_5 + \tfrac{W}{2}\right) \qquad m_{11} = \left(b_6 - \tfrac{W}{2}\right)$$
$$m_{12} = \left(b_6 + \tfrac{W}{2}\right) \qquad m_{13} = \left(b_7 - \tfrac{W}{2}\right) \qquad m_{14} = \left(b_7 + \tfrac{W}{2}\right)$$
$$m_{15} = \left(b_8 - \tfrac{P_r W}{2}\right) \qquad m_{16} = \left(b_8 + \tfrac{P_r W}{2}\right) \qquad m_{17} = \left(b_9 - \tfrac{P_r W}{2}\right)$$
$$m_{18} = \left(b_9 + \tfrac{P_r W}{2}\right) \qquad m_{19} = \left(b_{10} - \tfrac{S_c W}{2}\right) \qquad m_{20} = \left(b_{10} + \tfrac{S_c W}{2}\right)$$
$$\eta_1 = \frac{S_c y + 2b_1 t}{2\sqrt{t}\sqrt{S_c}} \qquad \eta_2 = \frac{S_c y - 2b_1 t}{2\sqrt{t}\sqrt{S_c}} \qquad \eta_3 = \frac{P_r y + 2b_2 t}{2\sqrt{t}\sqrt{P_r}}$$
$$\eta_4 = \frac{P_r y - 2b_2 t}{2\sqrt{t}\sqrt{P_r}} \qquad \eta_5 = \frac{y + 2b_3 t}{2\sqrt{t}} \qquad \eta_6 = \frac{y - 2b_3 t}{2\sqrt{t}}$$
$$\eta_7 = \frac{y + 2b_4 t}{2\sqrt{t}} \qquad \eta_8 = \frac{y - 2b_4 t}{2\sqrt{t}} \qquad \eta_9 = \frac{y + 2b_5 t}{2\sqrt{t}}$$
$$\eta_{10} = \frac{y - 2b_5 t}{2\sqrt{t}} \qquad \eta_{11} = \frac{y + 2b_6 t}{2\sqrt{t}} \qquad \eta_{12} = \frac{y - 2b_6 t}{2\sqrt{t}}$$
$$\eta_{13} = \frac{y + 2b_7 t}{2\sqrt{t}} \qquad \eta_{14} = \frac{y - 2b_7 t}{2\sqrt{t}} \qquad \eta_{15} = \frac{P_r y + 2b_8 t}{2\sqrt{t}\sqrt{P_r}}$$
$$\eta_{16} = \frac{P_r y - 2b_8 t}{2\sqrt{t}\sqrt{P_r}} \qquad \eta_{17} = \frac{P_r y + 2b_9 t}{2\sqrt{t}\sqrt{P_r}} \qquad \eta_{18} = \frac{P_r y - 2b_9 t}{2\sqrt{t}\sqrt{P_r}}$$
$$\eta_{19} = \frac{S_c y + 2b_{10} t}{2\sqrt{t}\sqrt{S_c}} \qquad \eta_{20} = \frac{S_c y - 2b_{10} t}{2\sqrt{t}\sqrt{S_c}} \qquad \eta_{21} = \frac{S_c y + 2b_{11} t}{2\sqrt{t}\sqrt{S_c}}$$

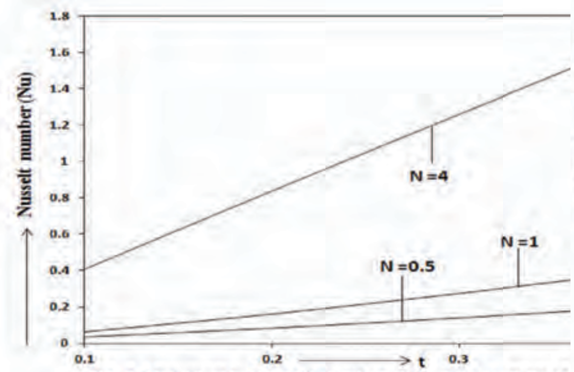

Fig. 7.-Variation in Nusselt number with various value of Radiation parameter, Pr=0.71, Q=0.2, w=0.5.

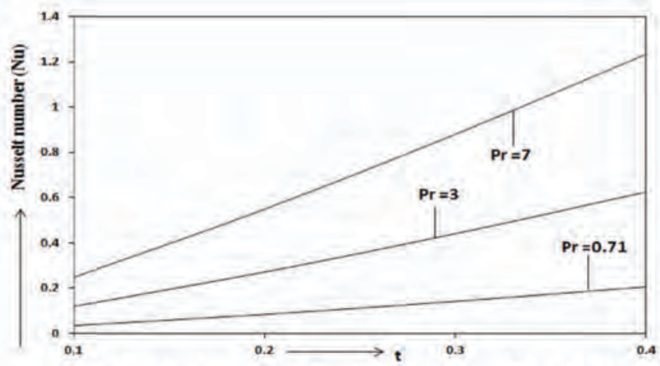

Fig. 8.-Variation in Nusselt number with various value of Pr number, for N=1, Q=0.2, w=0.5

$$\eta_{22} = \frac{S_c y - 2b_{11}t}{2\sqrt{t}\sqrt{S_c}}$$

$$a_1 = \frac{-B_1 + W\sqrt{P_r}\sqrt{(P_r-1)(r-\xi)}}{(P_r-1)} \quad a_2 = \frac{-B_1 - W\sqrt{P_r}\sqrt{(P_r-1)(r-\xi)}}{(P_r-1)} \quad a_3 = \frac{-B_2 + W\sqrt{S_c}\sqrt{(S_c-1)(r-K_1)}}{(S_c-1)}$$

$$a_4 = \frac{-B_2 - W\sqrt{S_c}\sqrt{(S_c-1)(r-K_1)}}{(S_c-1)} \quad b_1 = \sqrt{\frac{S_c^2 W^2}{4} + K_1 S_c} \quad b_2 = \sqrt{\frac{P_r^2 W^2}{4} + N^2 + QP_r}$$

$$b_3 = \sqrt{\frac{W^2}{4} + r} \quad b_4 = \sqrt{a_1 + \frac{W^2}{4} + r} \quad b_5 = \sqrt{a_2 + \frac{W^2}{4} + r}$$

$$b_6 = \sqrt{a_3 + \frac{W^2}{4} + r} \quad b_7 = \sqrt{a_4 + \frac{W^2}{4} + r} \quad b_8 = \sqrt{P_r}\sqrt{a_1 + \frac{W^2 P_r}{4} + \xi}$$

$$b_9 = \sqrt{P_r}\sqrt{a_2 + \frac{W^2 P_r}{4} + \xi} \quad b_{10} = \sqrt{S_c}\sqrt{a_3 + \frac{W^2 S_c}{4} + K_1}$$

$$b_{11} = \sqrt{S_c}\sqrt{a_4 + \frac{W^2 S_c}{4} + K_1}$$

$$A_1 = \frac{\sqrt{t}\sqrt{P_r}}{2b_2} \quad A_2 = \frac{\sqrt{t}}{2b_3} \quad A_3 = \frac{G_m}{S_c-1}\frac{M_5}{2}$$

$$A_4 = \frac{G_r}{P_r-1}M_2 A_1 \quad A_5 = \frac{G_r}{P_r-1}\frac{M_1}{2} \quad A_6 = \left(1 + \frac{G_r}{P_r-1}M_2\right)A_2$$

$$A_7 = \left(\frac{G_r}{P_r-1}\frac{M_1}{2} + \frac{G_m}{S_c-1}\frac{M_5}{2}\right) \quad A_8 = \left(\frac{G_r}{P_r-1}\frac{M_3 e^{a_1 t}}{2}\right) \quad A_9 = \left(\frac{G_r}{P_r-1}\frac{M_4 e^{a_2 t}}{2}\right)$$

$$A_{10} = \left(\frac{G_m}{S_c-1}\frac{M_6 e^{a_3 t}}{2}\right) \quad A_{11} = \left(\frac{G_m}{S_c-1}\frac{M_7 e^{a_4 t}}{2}\right) \quad A_{12} = \left(\frac{G_r}{P_r-1}\frac{M_3 e^{a_1 t}}{2}\right)$$

$$A_{13} = \left(\frac{G_r}{P_r-1}\frac{M_4 e^{a_2 t}}{2}\right) \quad A_{14} = \frac{G_m}{S_c-1}\frac{M_6 e^{a_3 t}}{2} \quad A_{15} = \frac{G_m}{S_c-1}\frac{M_7 e^{a_4 t}}{2}$$

$$\xi = \frac{N^2}{P_r} + Q \quad r = M + \frac{1}{K_P}$$

$$B_1 = N^2 + QP_r - M - \frac{1}{K_P} \quad B_2 = S_c K_1 - M - \frac{1}{K_P}$$

$$M_1 = \frac{2 - W\sqrt{P_r}\sqrt{\frac{W^2 P_r}{4} + \xi + 1} - \sqrt{\frac{W^2 P_r}{4} + \xi - 1} - 2M_3\left(1 + \frac{B_1^2}{(P_r-1)^2} - \frac{W^2 P_r (P_r-1)(r-\xi)}{(P_r-1)^2}\right) - 2M_3 - 2M_4}{2\left(1 + \frac{B_1^2}{(P_r-1)^2} - \frac{W^2 P_r (P_r-1)(r-\xi)}{(P_r-1)^2}\right)}$$

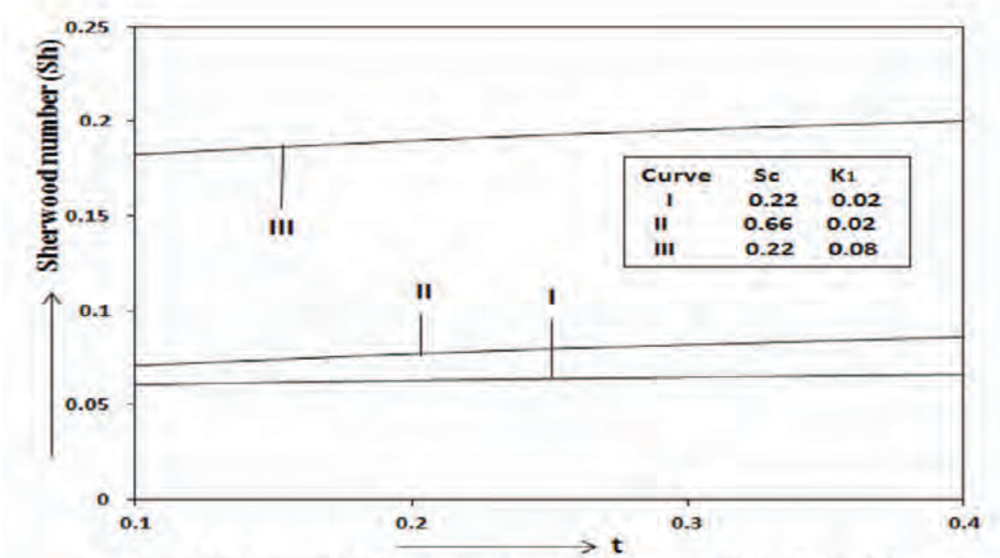

Fig. 9.-Variation in Sherwood number with Sc number and Chemical reaction parameter.

$$M_2 = \frac{\frac{W^2P_r}{2}+\frac{B_1}{(P_r-1)}-W\sqrt{P_r}\sqrt{\frac{W^2P_r}{4}+\xi}}{\frac{B_1^2}{(P_r-1)^2}-\frac{W^2P_r(P_r-1)(r-\xi)}{(P_r-1)^2}}$$

$$M_3 = \frac{\frac{W^2P_r}{2}+W\sqrt{P_r}\left(\frac{\sqrt{(P_r-1)(r-\xi)}}{(P_r-1)}-\sqrt{\frac{W^2P_r}{4}+\xi-\frac{B_1}{(P_r-1)}+\frac{W\sqrt{P_r}\sqrt{(P_r-1)(r-\xi)}}{(P_r-1)}}\right)}{\frac{1}{(P_r-1)^3}\left(2B_1^2W\sqrt{P_r}\sqrt{(P_r-1)(r-\xi)}+2W^3P_r(P_r-1)(r-\xi)\sqrt{P_r}\sqrt{(P_r-1)(r-\xi)}-4B_1W^2P_r(P_r-1)(r-\xi)\right)}$$

$$M_4 = \frac{\frac{W^2P_r}{2}-W\sqrt{P_r}\left(\frac{\sqrt{(P_r-1)(r-\xi)}}{(P_r-1)}+\sqrt{\frac{W^2P_r}{4}+\xi-\frac{B_1}{(P_r-1)}+\frac{W\sqrt{P_r}\sqrt{(P_r-1)(r-\xi)}}{(P_r-1)}}\right)}{\frac{1}{(P_r-1)^3}\left(-2B_1^2W\sqrt{P_r}\sqrt{(P_r-1)(r-\xi)}-2W^3P_r(P_r-1)(r-\xi)\sqrt{P_r}\sqrt{(P_r-1)(r-\xi)}-4B_1W^2P_r(P_r-1)(r-\xi)\right)}$$

$$M_5 = \frac{\frac{W^2S_c}{2}+\frac{B_2}{(S_c-1)}-W\sqrt{S_c}\sqrt{\frac{W^2S_c}{4}+K_1}}{\frac{B_2^2}{(S_c-1)^2}-\frac{W^2S_c(S_c-1)(r-K_1)}{(S_c-1)^2}}$$

$$M_6 = \frac{\frac{W^2S_c}{2}+\frac{W\sqrt{S_c}\sqrt{(S_c-1)(r-K_1)}}{(S_c-1)}-W\sqrt{S_c}\sqrt{\frac{W^2S_c}{4}+K_1-\frac{B_2}{(S_c-1)}+\frac{W\sqrt{S_c}\sqrt{(S_c-1)(r-K_1)}}{(S_c-1)}}}{-2\frac{B_2}{(S_c-1)^2}W\sqrt{S_c}\sqrt{(S_c-1)(r-K_1)}+2\frac{W^2S_c(S_c-1)(r-K_1)}{(S_c-1)^2}}$$

$$M_7 = \frac{\frac{W^2S_c}{2}-\frac{W\sqrt{S_c}\sqrt{(S_c-1)(r-K_1)}}{(S_c-1)}-W\sqrt{S_c}\sqrt{\frac{W^2S_c}{4}+K_1-\frac{B_2}{(S_c-1)}-\frac{W\sqrt{S_c}\sqrt{(S_c-1)(r-K_1)}}{(S_c-1)}}}{2\frac{B_2}{(S_c-1)^2}W\sqrt{S_c}\sqrt{(S_c-1)(r-K_1)}+2\frac{W^2S_c(S_c-1)(r-K_1)}{(S_c-1)^2}}$$