On An $(\epsilon, \delta)-$ Trans-Sasakian Manifold With Semi-Symmetric Metric Connecton

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Abstract

The objective of the present paper is to study of $(\epsilon, \delta)-$trans-Sasakian manifold with a semi-symmetric metric connection. We have found the relations between curvature tensors, Ricci tensors and scalar curvature of $(\epsilon, \delta)-$trans-Sasakian manifolds with semi-symmetric metric connection and with metric connection. Also, we have proved some results on quasi-projectively flat, $\phi-$projectively flat, conformally flat and $\xi-$conformally flat manifolds with respect to semi-symmetric metric connection. Finally, it is shown that the manifold satisfying $\bar{R}.\bar{S} = 0$ or $\bar{P}.\bar{S} = 0$ is an $\eta-$Einstein manifold if $\alpha = 0$ and $\beta = constant$.

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1 Introduction

The study of manifolds with indefinite metrics is of interest from the standpoint of physics and relativity. Manifolds with indefinite metrics have been studied by several authors. Bejancu and Duggal [3] introduced the concept of $(\epsilon)-$ Sasakian manifolds and Xufeng and Xiaoli [15] established that these manifolds are real hypersurfaces of indefinite Kahlerian manifolds. Kumar et al. [10] studied the curvature conditions of these manifolds.

The notion of semi-symmetric linear connection on a differentiable manifold was introduced by Friedman and Schouten [6] and metric connection with a torsion on a Riemannain manifold was studied by Hayden [7]. Semi-symmetric metric connection on a Riemannain manifold had been given by Yano [14] and later studied by Bagewadi [1], Amur and Pujar [9], Sharafuddin and Hussain [8], De et al. [5] and Bagewadi *et al.* [2] previously provided some results on the conservativeness of Projective, Pseudo projective, Conformal, Concircular, Quasi conformal curvature tensors on K-contact, Kenmotsu and trans-Sasakian manifolds.

As a natural generalization of both Sasakian and Kenmotsu manifolds, the notion of trans-Sasakian manifold was introduced by $Oubi\widetilde{n}a$ [13]. Trans-Sasskian manifold are closely related to the locally conformal Kahler manifold. Further, study about the local structures of trans-Sasakian manifolds was carried by Marrero [11]. Trans-Sasskian manifold of type $(\alpha, 0)$, $(0, \beta)$ and $(0, 0)$ are respectively, called the $\alpha-$ Sasakian, β-Kenmotsu manifolds and Cosymplectic with α, β being scalar functions. In particular, if $\alpha = 1$, $\beta = 0$

; $\alpha = 0$, $\beta = 1$; then a trans-Sasakian manifold becomes Sasakian and Kenmotsu manifolds respectively. Hence trans-Saskian structures give a large class of generalized Quasi-Sasakian structures.

In present paper we shall study some results on curvature tensors, Ricci tensor, scalar curvature, quasi projectively flat, $\phi-$projectively flat, Weyl conformally flat, Weyl $\xi-$conformally flat, $\bar{R}.\bar{S} = 0$, and $\bar{P}.\bar{S} = 0$, in $n-$dimensional $(\epsilon, \delta)-$ trans-Sasakian manifold M with a semi-symmetric metric connection.

2 PRELIMINARIES

An $n-$dimensional smooth manifold (M, g) is said to be an $(\epsilon)-$ almost contact metric manifold [4], if it admits a $(1,1)$ tensor field ϕ, a structure vector field ξ, a $1-$form η and an $(\epsilon)-$metric g such that,

$$\phi^2 X = -X + \eta(X)\xi, \ \phi\xi = 0, \ \eta(\phi X) = 0, \tag{2.1}$$

$$\eta(\xi) = 1, \tag{2.2}$$

$$g(\xi, \xi) = \epsilon, \tag{2.3}$$

$$\eta(X) = \epsilon g(X, \xi), \tag{2.4}$$

$$g(\phi X, \phi Y) = g(X, Y) - \epsilon\eta(X)\eta(Y), \tag{2.5}$$

for all vector fields X, Y on M, where ϵ is 1 or -1 according as ξ is space like or time like vector field and rank ϕ is $n-1$. If

$$d\eta(X, Y) = g(X, \phi Y), \tag{2.6}$$

then $M(\phi, \xi, \eta, g)$ is called an (ϵ)-contact metric manifold. An (ϵ)-almost contact metric manifold is called an (ϵ, δ) trans-Sasakian manifold if

$$(\nabla_X \phi)Y = \alpha\{g(X, Y)\xi - \epsilon\eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \delta\eta(Y)\phi X\}, \tag{2.7}$$

holds for some smooth functions α and β on M and $\epsilon = \pm 1$, $\delta = \pm 1$. For $\alpha = 0$, $\beta = 1$, an (ϵ, δ)-trans-Sasakian manifold reduces to an $(\delta)-$Kenmotsu manifold and for $\alpha = 1$, $\beta = 0$ it reduces to a $(\epsilon)-$Sasakian manifold.
From equations (2.1), (2.2), (2.3), (2.4), (2.5) and (2.7), we have

$$\nabla_X \xi = -\epsilon\alpha\phi X + \delta\beta(X - \eta(X)\xi)\}, \tag{2.8}$$

$$(\nabla_X \eta)Y = -\alpha g(\phi X, Y) + \delta\beta\{\epsilon g(X, Y) - \eta(X)\eta(Y)\}. \tag{2.9}$$

Morever, on such a $(\epsilon, \delta)-$trans-Sasakian manifold M of dimension n with structure $M(\phi, \xi, \eta, g)$ the following relations holds [12]:

$$\begin{aligned} R(X,Y)\xi &= (\alpha^2-\beta^2)\{\eta(Y)X-\eta(X)Y\}+2\alpha\beta(\delta-\epsilon)g(\phi X,Y)\xi \\ &+2\epsilon\delta\alpha\beta\{\eta(Y)\phi X-\eta(X)\phi Y\}+\epsilon\{(Y\alpha)\phi X \\ &-(X\alpha)\phi Y\}+\delta\{(Y\beta)\phi^2X-(X\beta)\phi^2Y\}, \end{aligned} \tag{2.10}$$

$$\begin{aligned} R(\xi,Y)Z &= (\alpha^2-\beta^2)\{g(Y,Z)\xi-\eta(Z)Y\}+2\epsilon\delta\alpha\beta\{g(\phi Z,Y)\xi \\ &+\eta(Z)\phi Y\}+\epsilon[(grad\alpha)g(\phi Z,Y)+(Z\beta)\phi Y] \\ &+\delta[(grad\beta)g(\phi^2Z,Y)-(Z\beta)\phi^2Y] \\ &+2(\delta-\epsilon)\alpha\beta g(Y,\xi)\phi Z, \end{aligned} \tag{2.11}$$

$$S(Y,\xi)=\{(n-1)(\epsilon\alpha^2-\delta\beta^2)-(\xi\beta)\}\eta(Y)-(n-2)(Y\beta)-(\phi Y)\alpha, \tag{2.12}$$

$$S(\xi,\xi)=(n-1)(\epsilon\alpha^2-\delta\beta^2)-(n-1)(\xi\beta), \tag{2.13}$$

$$S(\phi Y,\xi)=(Y\alpha)-\eta(Y)(\xi\alpha)-(n-2)(\phi Y)\beta, \tag{2.14}$$

$$\begin{aligned} C(X,Y)Z &= R(X,Y)Z-\frac{1}{(n-2)}[S(Y,Z)X-S(X,Z)Y \\ &+g(Y,Z)QX-g(X,Z)QY]+\frac{r}{(n-1)(n-2)} \\ &[g(Y,Z)X-g(X,Z)Y], \end{aligned} \tag{2.15}$$

$$r=S(e_i,e_i)=\sum_{i=1}^{n}\epsilon_i\prime R(e_i,e_i,e_i,e_i), \tag{2.16}$$

$$\begin{aligned} g(R(\xi,Y)Z,\xi) &= [\epsilon(\alpha^2-\beta^2)-\delta(\xi\beta)]g(Y,Z)+[\epsilon\delta(\xi\beta) \\ &-\epsilon(\alpha^2-\beta^2)]\eta(Y)\eta(Z)+[\alpha(\xi\alpha)+2\delta\alpha\beta] \\ &g(\phi Z,Y)-\delta(Z\beta)\eta(Y)\xi+\delta(Z\beta)Y. \end{aligned} \tag{2.17}$$

Let M be an $n-$dimensional (ϵ, δ)-trans-Sasakian manifold and ∇ be the metric connection on M. The relation between the semi-symmetric metric connection $\bar{\nabla}$ and the metric connection ∇ on M is given by

$$\bar{\nabla}_XY=\nabla_XY+\eta(Y)X-\epsilon g(X,Y)\xi. \tag{2.18}$$

3 Curvature tensor on (ϵ,δ) -trans-Sasakian manifold with semi-symmetric metric connection

Let M be an $n-$dimensional (ϵ,δ)-trans-Sasakian manifold. The curvature tensor $\bar{R}$ of M with respect to the semi-symmetric metric connection $\bar{\nabla}$ is defined by

$$\bar{R}(X,Y)Z=\bar{\nabla}_X\bar{\nabla}_YZ-\bar{\nabla}_Y\bar{\nabla}_XZ-\bar{\nabla}_{[X,Y]}Z. \tag{3.1}$$

By using equations (2.2), (2.4), (2.18) and (3.1), we get

$$\begin{aligned}\bar{R}(X,Y)Z &= R(X,Y)Z+\epsilon(1+2\delta\beta)[g(X,Z)Y-g(Y,Z)X\ ]\\ &+\epsilon(1+\delta\beta)[\eta(X)g(Y,Z)-\eta(Y)g(X,Z)]\xi\\ &+(1+\delta\beta)\eta(Z)[\eta(Y)X-\eta(X)Y\ ]\\ &-\alpha[g(\phi X,Z)Y-g(\phi Y,Z)X+g(X,Z)\phi Y-g(Y,Z)\phi X],\end{aligned} \tag{3.2}$$

where

$$R(X,Y)Z=\nabla_X\nabla_YZ-\nabla_Y\nabla_XZ-\nabla_{[X,Y]}Z$$

is the Riemannian curvature tensor of metric connection ∇.

Taking inner product of equation (3.2) with U and using equation (2.4), we have

$$\begin{aligned}&{}^{|}\bar{R}(X,Y,Z,U)\\ =\ &{}^{|}R(X,Y,Z,U)+\epsilon(1+2\delta\beta)[g(X,Z)g(Y,U)-g(Y,Z)g(X,U)]\\ &+(1+\delta\beta)[\eta(X)g(Y,Z)-\eta(Y)g(X,Z)]\eta(U)\\ &+(1+\delta\beta)[\eta(Y)g(X,U)-\eta(X)g(Y,U,)]\eta(Z)\\ &-\alpha[g(\phi X,Z)g(Y,U)-g(\phi Y,Z)g(X,U)-g(Y,Z)g(\phi X,U)\\ &+g(X,Z)g(\phi Y,U)].\end{aligned} \tag{3.3}$$

Where${}^{|}\bar{R}(X,Y,Z,U)=g(\bar{R}(X,Y)Z,U)$ and ${}^{|}R(X,Y,Z,U)=g(R(X,Y)Z,U)$.

Let $\{e_1,e_{2,}............,e_{n-1,}\xi\}$ be a local orthonormal basis of vector fields on (ϵ,δ)-trans-Sasakian manifold M and $\{\phi e_1,\phi e_{2,}............,\phi e_{n-1},\ \xi\}$ is a also local orthonormal basis of vector fields on $(\epsilon,\delta)-$trans-Sasakian manifold M. We putting value $X=U=e_{\rm i}$ in the equation (3.3) and taking summation over $i, 1 \leq i \leq n-1$, we get

$$\begin{aligned}
(3.4) \quad & \sum_{i=1}^{n-1} {}^{|}\bar{R}(e_i, Y, Z, e_i) \\
= & \sum_{i=1}^{n-1} {}^{|}R(e_i, Y, Z, e_i) + \epsilon(1+2\delta\beta)\sum_{i=1}^{n-1}[g(e_i, Z)g(Y, e_i \ ) - g(Y, Z)g(e_i, e)] \\
& +(1+\delta\beta)\sum_{i=1}^{n-1}[g(Y, Z)\eta(e_i) - \eta(Y)g(e_i, Z)]\eta(e_i) \\
& +(1+\delta\beta)\sum_{i=1}^{n-1}[\eta(Z)\eta(Y)g(e_i, e_i \ ) - \eta(Z)\eta(e_i)g(Y, e_i) \ ] \\
& -\alpha\sum_{i=1}^{n-1}[g(e_i\phi, Z)g(Y, e_i) - g(\phi Y, Z)g(e_i, e_i) - g(Y, Z)g(\phi e_i, e_i) \\
& +g(e_i, Z)g(\phi Y, e_i)].
\end{aligned}$$

Then, we have

$$(3.5) \qquad S(Y, Z) = \sum_{i=1}^{n} \epsilon_i \ {}^{|}R(e_i, Y, Z, e_i), \ \ g(Y, Z) = \sum_{i=1}^{n} \epsilon_i g(Y, e_i)g(e_i \ , Z).$$

$$(3.6) \qquad \bar{S}(Y, Z) = \sum_{i=1}^{n} \epsilon_i \ {}^{|}\bar{R}(e_i, Y, Z, e).$$

Also

$$(3.7) \qquad \sum_{i=1}^{n-1} {}^{|}\bar{R}(e_i, Y, Z, e) = \bar{S}(Y, Z) - \epsilon \ \ {}^{|}\bar{R}(\xi, Y, Z, \xi),$$

$$(3.8) \qquad \sum_{i=1}^{n-1} {}^{|}R(e_i, Y, Z, e_i) = S(Y, Z) - \epsilon \ \ {}^{|}R(\xi, Y, Z, \xi),$$

$$(3.9) \qquad \sum_{i=1}^{n-1} g(e, Z)g(Y, e_i) = g(Y, Z) - \epsilon g(\xi, Z)g(Y, \xi \ ),$$

$$(3.10) \qquad \sum_{i=1}^{n-1} g(e_i \ , e_i \ \ ) = n-1,$$

$$\sum_{i=1}^{n-1} g(e_i\ ,\xi)g(\xi,e_i) = 0, \tag{3.11}$$

$$\sum_{i=1}^{n-1} g(e_i,Z)g(\xi,e_i) = 0, \tag{3.12}$$

$$\sum_{i=1}^{n-1} g(e_i\phi,Z)g(Y,e_i) = g(\phi Y,Z), \tag{3.13}$$

$$\sum_{i=1}^{n-1} g(\phi e_i,e_i) = 0, \tag{3.14}$$

$$\sum_{i=1}^{n-1} g(\phi e_i,\phi e_i) = n-1, \tag{3.15}$$

$$\sum_{i=1}^{n-1} g(\phi e_i,Z)g(Y,\phi e_i\ ) = g(Y,Z) - \epsilon g(\xi,Z)g(Y,\xi). \tag{3.16}$$

Hence, using equations (3.5) – (3.16) in equation (3.4), we get

$$\begin{aligned} \bar{S}(Y,Z) \ &= \ S(Y,Z) - \epsilon[(n-2)(1+2\delta\beta)+\delta\beta]g(Y,Z) \\ &\quad +(n-2)(1+\delta\beta)\eta(Y)\eta(Z) + \alpha(n-2)g(\phi Y,Z). \end{aligned} \tag{3.17}$$

Where $\bar{S}$ and S are the Ricci tensor of connection $\bar{\nabla}$ and ∇, respectively in M .

$$\bar{Q}Y = QY - \epsilon[(n-2)(1+2\delta\beta)+\delta\beta]Y + (n-2)\epsilon(1+\delta\beta)\eta(Y)\xi + \alpha(n-2)\phi Y, \tag{3.18}$$

where $\bar{Q}$ and Q are Ricci operator with respect to the semi-symmetric metric connection and metric connection respectively and are define as $g(\bar{Q}Y,Z) = \bar{S}(Y,Z)$ and $g(QY,Z) = S(Y,Z)$ respectively.

Replace $Y = \xi$ in equation (3.18) and using equation (2.12), we get

$$\bar{Q}\xi = \epsilon(n-1)(\epsilon\alpha^2 - \delta\beta^2)\xi - \epsilon(\xi\beta)\xi - \epsilon\delta\beta(n-1)\xi. \tag{3.19}$$

Putting the value $Y = Z = e_i$, and taking summation over $1 \le i \le n-1$ in equation (3.17) and using equations (2.16), (3.10), (3.11), (3.14), we get

$$\bar{r}\ = r - \epsilon(n-1)[(1+2\delta\beta)(n-2)+2\delta\beta], \tag{3.20}$$

where $\bar{r}$ and r are the scalar curvatures of the connection $\bar{\nabla}$ and ∇, respectively. Replace the value $Y \to \phi Y$ in equation (3.17), we get

$$\begin{aligned} \bar{S}(\phi Y, Z) &= S(\phi Y, Z) - \epsilon[(1+2\delta\beta)(n-2)+\delta\beta]g(\phi Y, Z) \\ &\quad -\alpha(n-2)g(Y,Z) + \epsilon\alpha(n-2)\eta(Z)\eta(Y). \end{aligned} \tag{3.21}$$

Using equations (2.1), (2.2), (2.4) and (2.12) in eqution (3.21), we get

$$\bar{S}(\phi Y, \xi) = S(\phi Y, \xi) = -\epsilon(\phi^2 Y)\alpha - \epsilon(n-2)(\phi Y)\beta. \tag{3.22}$$

Using equations (2.1), (2.2), (2.4), (2.12) and (2.13) in equation (3.17), we get

$$\begin{aligned} \bar{S}(Y, \xi) &= [(n-1)(\epsilon\alpha^2 - \delta\beta^2) - \delta\beta(n-1) - (\xi\beta)]\eta(Y) \\ &\quad -(n-2)(Y\beta) - (\phi Y)\alpha, \end{aligned} \tag{3.23}$$

$$\bar{S}(\xi, \xi) = (n-1)(\epsilon\alpha^2 - \delta\beta^2) - \delta\beta(n-1) - (n-1)(\xi\beta), \tag{3.24}$$

$$\begin{aligned} \bar{S}(grad\alpha, \xi) &= \epsilon(n-1)(\epsilon\alpha^2 - \delta\beta^2)(\xi\alpha) - \delta\beta(n-1)(\xi\alpha) - \epsilon(\xi\alpha)(\xi\beta) \\ &\quad -(n-2)g(grad\alpha, grad\beta) - (\phi grad\alpha)\alpha, \end{aligned} \tag{3.25}$$

$$\begin{aligned} \bar{S}(grad\beta, \xi) &= \epsilon(n-1)(\epsilon\alpha^2 - \delta\beta^2)(\xi\beta) - \delta\beta(n-1)(\xi\beta) - \epsilon(\xi\beta)^2 \\ &\quad -(n-2)(grad\beta)^2 - (\phi grad\beta)\alpha, \end{aligned} \tag{3.26}$$

From equation (3.2) by the cyclic permutations of X, Y and Z, we get

$$\begin{aligned} &\bar{R}(Y,Z)X \\ =\ &R(Y,Z)X + \epsilon(1+2\delta\beta)[g(Y,X)Z - g(Z,X)Y\ ] \\ &+\epsilon(1+\delta\beta)[\eta(Y)g(Z,X) - \eta(Z)g(Y,X)]\xi \\ &+(1+\delta\beta)\eta(X)[\eta(Z)Y - \eta(Y)Z\ ] \\ &-\alpha[g(\phi Y,X)Z - g(\phi Z,X)Y - g(Z,X)\phi Y + g(Y,X)\phi Z], \end{aligned} \tag{3.27}$$

$$\begin{aligned} &\bar{R}(Z,X)Y \\ =\ &R(Z,X)Y + \epsilon(1+2\delta\beta)[g(Z,Y)X - g(X,Y)Z\ ] \\ &+\epsilon(1+\delta\beta)[\eta(Z)g(X,Y) - \eta(X)g(Y,Z)]\xi \\ &+(1+\delta\beta)\eta(Y)[\eta(X)Z - \eta(Z)X\ ] \\ &-\alpha[g(\phi Z,Y)X - g(\phi X,Y)Z - g(X,Y)\phi Z + g(Z,Y)\phi X], \end{aligned} \tag{3.28}$$

Adding equations (3.2), (3.27) and (3.28) and using the Bianchi' first identity (with respect to metric connection ∇), we get

$$\begin{aligned} &\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y \\ =\ &2\alpha[g(\phi Y,Z)X + g(\phi Z,X)Y + g(\phi X,Y)Z], \end{aligned} \tag{3.29}$$

If $\alpha = 0$ in equation (4.29), we get

$$\bar{R}(X,Y)Z + \bar{R}(Y,Z)X + \bar{R}(Z,X)Y = 0$$

Theorem 3.1. *Let M be an $n-$dimensional $(\epsilon, \delta)-$trans-Sasakian manifold with semi-symmetric metric connection $\bar{\nabla}$ and curvature tensor $\bar{R}$ satisfies* (3.29) *Bianchi first identity (with respect to semi-symmetric metric connection $\bar{\nabla}$) if and only if $\alpha = 0$.*

Now, from equation (3.3) interchanging X and Y, we get

$$\begin{aligned} & {}'\bar{R}(Y,X,Z,U) \\ = \; & {}'R(Y,X,Z,U) + \epsilon(1+2\delta\beta)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)] \\ & +(1+\delta\beta)[\eta(Y)g(X,Z) - \eta(X)g(Y,Z)]\eta(U) \\ & +(1+\delta\beta)[\eta(X)g(Y,U) - \eta(Y)g(X,U,)]\eta(Z) \\ & -\alpha[g(\phi Y,Z)g(X,U) - g(\phi X,Z)g(Y,U) - g(X,Z)g(\phi Y,U) \\ & +g(Y,Z)g(\phi X,U)]. \end{aligned} \tag{3.30}$$

From equations (3.3) and (3.30), we get

$$ {}'\bar{R}(X,Y,Z,U) = -{}'\bar{R}(Y,X,Z,U), \tag{3.31}$$

where

$$ {}'R(X,Y,Z,U) = -{}'R(Y,X,Z,U).$$

Again from equation (3.3) interchanging Z and U, we get

$$\begin{aligned} & {}'\bar{R}(X,Y,U,Z) \\ = \; & {}'R(X,Y,U,Z) + \epsilon(1+2\delta\beta)[g(X,U)g(Y,Z) - g(Y,U)g(X,Z)] \\ & +(1+\delta\beta)[\eta(X)g(Y,U) - \eta(Y)g(X,U)]\eta(Z) \\ & +(1+\delta\beta)[\eta(Y)g(X,Z) - \eta(X)g(Y,Z)]\eta(U) \\ & -\alpha[g(\phi X,U)g(Y,Z) - g(\phi Y,U)g(X,Z) - g(Y,U)g(\phi X,Z) \\ & +g(X,U)g(\phi Y,Z)]. \end{aligned} \tag{3.32}$$

From equations (3.3) and (3.32), we get

$$ {}'\bar{R}(X,Y,Z,U) = -{}'\bar{R}(X,Y,U,Z) \tag{3.33}$$

where

$$ {}'R(X,Y,Z,U) = -{}'R(X,Y,U,Z).$$

Again from equation (3.3) interchanging pair of slots, we get

$$
\begin{aligned}
(3.34)\qquad & {}'\bar{R}(Z,U,X,Y) \\
= & {}'R(Z,U,X,Y)+\epsilon(1+2\delta\beta)[g(Z,X)g(U,Y)-g(U,X)g(Z,Y)] \\
& +(1+\delta\beta)[\eta(Z)g(U,X)-\eta(U)g(Z,X)]\eta(Y) \\
& +(1+\delta\beta)[\eta(U)g(Z,Y)-\eta(Z)g(U,Y)]\eta(X) \\
& -\alpha[g(\phi Z,X)g(U,Y)-g(\phi U,X)g(Z,Y)-g(\phi Z,Y)g(U,X) \\
& +g(\phi U,Y)g(Z,X)].
\end{aligned}
$$

From equations (3.3) and (3.34), we get

$$
\begin{aligned}
(3.35)\qquad & {}'\bar{R}(X,Y,Z,U) \\
= & {}'\bar{R}(Z,U,X,Y)+2\alpha[g(\phi Z,X)g(U,Y) \\
& +g(\phi Y,Z)g(X,U)+g(\phi X,U)g(Y,Z) \\
& +g(\phi U,Y)g(Z,X)].
\end{aligned}
$$

If $\alpha=0$ in equation (3.35), we get

$$
{}'\bar{R}(X,Y,Z,U)={}'\bar{R}(Z,U,X,Y).
$$

Theorem 3.2. *The curvature tensor* $\bar{R}$ *of type* $(0,4)$ *of semi-symmetric metric connection* $\bar{\nabla}$ *is an* $(\epsilon,\delta)-$*trans-Sasakian manifolds is*

(i) Skew symmetric in first two slots.
(ii) Skew symmetric in last two slots.
(iii) Symmetric in pair of slots if and only if $\alpha=0$.

Now, let $\bar{R}(X,Y)Z=0$ in equation (3.2), we get

$$
\begin{aligned}
(3.36)\qquad R(X,Y)Z = & \epsilon(1+2\delta\beta)[g(Y,Z)X- g(X,Z)Y] \\
& +\epsilon(1+\delta\beta)[\eta(Y)g(X,Z)-\eta(X)g(Y,Z)]\xi \\
& +(1+\delta\beta)\eta(Z)[\eta(X)Y-\eta(Y)X] \\
& +\alpha[g(\phi X,Z)Y-g(\phi Y,Z)X-g(Y,Z)\phi X \\
& +g(X,Z)\phi Y].
\end{aligned}
$$

Taking the inner product of equation (3.36) with ξ and using equation (2.4), we get

$$
\begin{aligned}
(3.37)\qquad \epsilon\eta(R(X,Y)Z) = & \delta\beta[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)] \\
& +\epsilon\alpha[g(\phi X,Z)\eta(Y)-g(\phi Y,Z)\eta(X)].
\end{aligned}
$$

Using equation (2.4) in equation (3.37), we get

$$
(3.38)\qquad R(X,Y)Z=\epsilon\delta\beta[g(Y,Z)X-g(X,Z)Y]+\alpha[g(\phi X,Z)Y-g(\phi Y,Z)X].
$$

Theorem 3.3. *If the curvature tensor $\bar{R}$ of a semi-symmetric metric connection in an $(\epsilon, \delta)-$trans-Sasakian manifold M vanishes, then the $(\epsilon, \delta)-$trans sasakian manifold is of constant curvature if $\alpha = 0$.*

Now, equation (3.2) putting value $Z = \xi$ and using equations (2.1), (2.2), (2.4), (2.10), we get

$$\begin{aligned} \bar{R}(X,Y)\xi &= (\alpha^2 - \beta^2 - \delta\beta)[\eta(Y)X - \eta(X)Y] \\ &+ \epsilon\alpha(1 + 2\delta\beta)[\eta(Y)\phi X - \eta(X)\phi Y] \\ &+ \epsilon[(Y\alpha)\phi X - (X\alpha)\phi Y] + \delta[(Y\beta)\phi^2 X - (X\beta)\phi^2 Y] \\ &+ 2\alpha\beta(\delta - \epsilon)g(\phi X, Y)\xi. \end{aligned} \tag{3.39}$$

Replace $Y = \xi$ in equation (3.39) and using equations (2.1), (2.2), (2.4), we get

$$\begin{aligned} \bar{R}(X,\xi)\xi &= (\alpha^2 - \beta^2 - \epsilon\beta)[X - \eta(X)\xi] \\ &+ \epsilon[\alpha(1 + 2\beta) + (\xi\alpha)]\phi X + \delta(\xi\beta)\phi^2 X]. \end{aligned} \tag{3.40}$$

Now, again replace $X = \xi$ in equation (3.39) and using equations (2.1), (2.2) and (2.4), we get

$$\begin{aligned} \bar{R}(\xi,Y)\xi &= (\alpha^2 - \beta^2 - \epsilon\beta)[\eta(Y)\xi - Y] - \epsilon[\alpha(1 + 2\delta\beta) \\ &+ (\xi\alpha)]\phi Y - \delta(\xi\beta)\phi^2 Y. \end{aligned} \tag{3.41}$$

Replace $Y = X$ in equation (3.41), we get

$$\begin{aligned} \bar{R}(\xi,X)\xi &= -(\alpha^2 - \beta^2 - \epsilon\beta)[X - \eta(X)\xi] \\ &- \epsilon[\alpha(1 + 2\delta\beta) + (\xi\alpha)]\phi X - \epsilon(\xi\beta)\phi^2 X. \end{aligned} \tag{3.42}$$

From equations (3.40) and (3.42), we get

$$\bar{R}(X,\xi)\xi = -\bar{R}(\xi,X)\xi. \tag{3.43}$$

Theorem 3.4. *Let M be an $n-$dimensional $(\epsilon, \delta)-$trans-Sasakian manifold with a semi-symmetric metric connection, then*

$$\begin{aligned} \bar{R}(X,Y)\xi &= (\alpha^2 - \beta^2 - \delta\beta)[\eta(Y)X - \eta(X)Y] \\ &+ \epsilon\alpha(1 + 2\delta\beta)[\eta(Y)\phi X - \eta(X)\phi Y] \\ &+ \epsilon[(Y\alpha)\phi X - (X\alpha)\phi Y] + \delta[(Y\beta)\phi^2 X - (X\beta)\phi^2 Y] \\ &+ 2\alpha\beta(\delta - \epsilon)g(\phi X, Y)\xi. \end{aligned} \tag{3.44}$$

Lemma 3.1. *Let M be an $n-$dimensional $(\epsilon, \delta)-$trans-Sasakian manifold with a semi-symmetric metric connection, then*

$$(\bar{\nabla}_X\eta)(Y) = \epsilon(1+\delta\beta)g(X,Y) - (1+\delta\beta)\eta(X)\eta(Y) - \alpha g(\phi X, Y), \tag{3.45}$$

and

$$\bar{\nabla}_X\xi = (1+\delta\beta)[X - \eta(X)\xi] - \epsilon\alpha(\phi X). \tag{3.46}$$

Proof. By the covariant differentiation of $\eta(Y)$ with respect X, we have

$$\begin{aligned} \bar{\nabla}_X\eta(Y) &= (\bar{\nabla}_X\eta)(Y) + \eta(\bar{\nabla}_X Y), \\ (\bar{\nabla}_X\eta)(Y) &= \bar{\nabla}_X\eta(Y) - \eta(\bar{\nabla}_X Y). \end{aligned}$$

By using equation (2.4) and g is $\epsilon-$metric, we get

$$(\bar{\nabla}_X\eta)(Y) = \epsilon g(Y, \bar{\nabla}_X\xi). \tag{3.47}$$

We putting value $Y = \xi$ in equation (2.18) and using equation (2.4), we have

$$\bar{\nabla}_X\xi = \nabla_X\ \xi + \eta(\xi)X - \epsilon g(X,\xi)\xi.$$

$$\bar{\nabla}_X\xi = \nabla_X\ \xi + X - \eta(X)\xi. \tag{3.48}$$

From equation (3.47) and using equations (2.4), (2.8), (3.48), we get

$$(\bar{\nabla}_X\eta)(Y) = \epsilon(1+\delta\beta)g(X,Y) - (1+\delta\beta)\eta(X)\eta(Y) - \alpha g(\phi X, Y).$$

From equations (2.8) and (3.48), we get

$$\bar{\nabla}_X\xi = (1+\delta\beta)[X - \eta(X)\xi] - \epsilon\alpha(\phi X).$$

□

4 Quasi-projectively flat $(\epsilon, \delta)-$trans-Sasakian manifold with respect to semi-symmetric metric connection

Let M be an $n-$dimensional $(\epsilon, \delta)-$trans-Sasakian manifold. If there exists a one to one correspondence between each coordinate neighbourhood of M and a domain in Euclidean space such that any geodesic of the $(\epsilon, \delta)-$trans-Sasakian manifold corresponds to a straight line in the Euclidean space, then M is said to be locally projectively flat. The projective curvature tensor $\bar{P}$ with respect to semi-symmetric metric connection is defined by

$$\bar{P}(X,Y)Z = \bar{R}(X,Y)Z - \frac{1}{(n-1)}[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y]. \tag{4.1}$$

Definition 4.1. An $(\epsilon,\delta)-$trans-Sasakian manifold M is said to be quasi-projectively flat with respect to semi-symmetric metric connection, if

$$g(\bar{P}(\phi X,Y)Z,\phi U)=0, \tag{4.2}$$

where $\bar{P}$ is the projective curvature tensor with respect to semi-symmetric metric connection.

From equation (4.1) taking inner product with U, we get

$$\begin{aligned} g(\bar{P}(X,Y)Z,U) &= g(\bar{R}(X,Y)Z,U)-\frac{1}{(n-1)} \\ &\quad [\bar{S}(Y,Z)g(X,U)-\bar{S}(X,Z)g(Y,U)]. \end{aligned} \tag{4.3}$$

Replace $X=\phi X$ and $U=\phi U$ in equation (4.3), we get

$$\begin{aligned} g(\bar{P}(\phi X,Y)Z,\phi U) &= g(\bar{R}(\phi X,Y)Z,\phi U)-\frac{1}{(n-1)} \\ &\quad [\bar{S}(Y,Z)g(\phi X,\phi U)-\bar{S}(\phi X,Z)g(Y,\phi U)]. \end{aligned} \tag{4.4}$$

From equations (4.2) and (4.4), we have

$$g(\bar{R}(\phi X,Y)Z,\phi U)=\frac{1}{(n-1)}[\bar{S}(Y,Z)g(\phi X,\phi U)-\bar{S}(\phi X,Z)g(Y,\phi U)]. \tag{4.5}$$

Now, using equations $(2,1),(2.4)$, (3.17) and (3.21) in equation (4.5), we have

$$\begin{aligned} & g(R(\phi X,Y)Z,\phi U) \\ = \; & \frac{1}{(n-1)}[S(Y,Z)g(\phi X,\phi U)-S(\phi X,Z)g(Y,\phi U)] \\ & -\frac{\epsilon(1+\delta\beta)}{(n-1)}g(\phi X,Z)g(Y,\phi U)+\frac{\epsilon(1+\delta\beta)}{(n-1)}g(Y,Z)g(\phi X,\phi U) \\ & -\frac{(1+\delta\beta)}{(n-1)}\eta(Y)\eta(Z)g(\phi X,\phi U)+\frac{\epsilon\alpha}{(n-1)}\eta(X)\eta(Z)g(Y,\phi U) \\ & -\frac{\alpha}{(n-1)}g(X,Z)g(Y,\phi U)-\frac{\alpha}{(n-1)}g(\phi Y,Z)g(\phi X,\phi U) \\ & +\alpha g(Y,Z)g(X,\phi U)+\alpha g(\phi X,Z)g(\phi Y,\phi U). \end{aligned} \tag{4.6}$$

Let $\{e_1,e_{2,}............,e_{n-1,}\xi\}$ be a local orthonormal basis of vector fields on $(\epsilon,\delta)-$trans-Sasakian manifold M and $\{\phi e_{1,}\phi e_{2,}............,\phi e_{n-1,}\ \xi\}$ is a also local orthonormal basis of vector fields on (ϵ,δ)-trans-Sasakian manifold M. Now, putting $X=U=e_i$ in equation

(4.6), and taking summation over $i, 1 \leq i \leq n-1$, we have

$$
\begin{aligned}
(4.7) \qquad & \sum_{i=1}^{n-1} g(R(\phi e_i, Y)Z, \phi e_i) \\
= \; & \frac{1}{(n-1)} \sum_{i=1}^{n-1} [S(Y,Z) g(\phi e_i, \phi e_i) - S(\phi e_i, Z) g(Y, \phi e_i)] \\
& - \frac{\epsilon(1+\delta\beta)}{(n-1)} \sum_{i=1}^{n-1} g(\phi e_i, Z) g(Y, \phi e_i) + \frac{\epsilon(1+\delta\beta)}{(n-1)} \sum_{i=1}^{n-1} g(Y,Z) g(\phi e_i, \phi e_i) \\
& - \frac{(1+\delta\beta)}{(n-1)} \sum_{i=1}^{n-1} \eta(Y)\eta(Z) g(\phi e_i, \phi e_i) + \frac{\epsilon\alpha}{(n-1)} \sum_{i=1}^{n-1} \eta(e_i)\eta(Z) g(Y, \phi e_i) \\
& - \frac{\alpha}{(n-1)} \sum_{i=1}^{n-1} g(e_i, Z) g(Y, \phi e_i) - \frac{\alpha}{(n-1)} \sum_{i=1}^{n-1} g(\phi Y, Z) g(\phi e_i, \phi e_i) \\
& + \alpha \sum_{i=1}^{n-1} g(Y,Z) g(e_i, \phi e_i) + \alpha \sum_{i=1}^{n-1} g(\phi e_i, Z) g(\phi Y, \phi e_i).
\end{aligned}
$$

Now, using equations $(2.1), (2.2), (2.4), (2.17)$, and $(3.8) - (3.16)$ in equation (4.7), we have

$$
\begin{aligned}
(4.8) \qquad S(Y,Z) &= [(n-1)(\alpha^2 - \beta^2) + \epsilon(n-2)(1+\delta\beta) - \epsilon\delta(n-1)(\xi\beta)] g(Y,Z) \\
&\quad + [(n-1)(\epsilon\alpha^2 - \delta\beta^2) - (n-1)(\alpha^2 - \beta^2) + \epsilon(n-1)(\xi\beta) \\
(4.9) \qquad &\quad - (\xi\beta) - (n-2)(1+\delta\beta)]\eta(Y)\eta(Z) \\
&\quad + [\alpha - 2(n-1)\epsilon\delta\alpha\beta - (n-1)(\xi\alpha)] g(\phi Y, Z) \\
&\quad - \eta(Y)(\phi Z)\alpha - (n-2)(Z\beta)\eta(Y)
\end{aligned}
$$

If $\alpha = 0$ and $\beta = constant$ in equation (4.8), we get

$$
\begin{aligned}
(4.10) \qquad S(Y,Z) &= [(n-1)(\alpha^2 - \beta^2) + \epsilon(n-2)(1+\delta\beta)] g(Y,Z) \\
&\quad + [(n-1)(\epsilon\alpha^2 - \delta\beta^2) - (n-1)(\alpha^2 - \beta^2) \\
&\quad - (n-2)(1+\xi\beta)]\eta(Y)\eta(Z).
\end{aligned}
$$

Therefore,

$$S(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z),$$

where

$$a = (n-1)(\alpha^2 - \beta^2) + \epsilon(n-2)(1+\delta\beta),$$

and

$$b = (n-1)(\epsilon\alpha^2 - \delta\beta^2) - (n-1)(\alpha^2 - \beta^2) - (n-2)(1+\xi\beta)].$$

This result shows that the manifold under the consideration is an $\eta-$ Einstein manifold. Thus, we can state the following theorem

Theorem 4.1. *An $n-$dimensional quasi projectively flat $(\epsilon,\delta)-$trans-Sasakin manifold M with respect to a semi-symmetric metric connection is $\eta-$Einstein manifold if $\alpha = 0$ and $\beta = constant$.*

5 $\phi-$Projectively flat $(\epsilon,\delta)-$trans Sasakian manifold with respect to a semi-symmetric metric connection

Let M be an $(\epsilon,\delta)-$trans-Sasakian manifold with respect to a semi-symmetric metric connection is said be $\phi-$projectively flat if

$$\phi^2(\bar{P}(\phi X,\phi Y)\phi Z) = 0 \tag{5.1}$$

where $\bar{P}$ is the Projective curvature tensor of the $(\epsilon,\delta)-$trans-Sasakian manifold with respect to a semi-symmetric metric connection. Suppose M be a $\phi-$Projectively flat $(\epsilon,\delta)-$trans-Sasakian manifold with respect to a semi-symmetric metric connection . It is know that $\phi^2(\bar{P}(\phi X,\phi Y)\phi Z) = 0$ holds if and only if

$$g(\bar{P}(\phi X,\phi Y)\phi Z),\phi U) = 0 \tag{5.2}$$

for any $X,Y,Z,U \in TM$.

Replace $Y = \phi Y$ and $U = \phi U$ in the equation (4.4), we have

$$\begin{aligned} g(\bar{p}(\phi X, Y\phi)\phi Z,\phi U) &= g(\bar{R}(\phi X,\phi Y)\phi Z,\phi U) - \frac{1}{(n-1)} \\ &\quad [\bar{S}(\phi Y,\phi Z)g(\phi X,\phi U) - \bar{S}(\phi X,\phi Z)g(\phi Y,\phi U)] \end{aligned} \tag{5.3}$$

From equations (5.2) and (5.3), we have

$$\begin{aligned} g(\bar{R}(\phi X,\phi Y)\phi Z,\phi U) &= \frac{1}{(n-1)}[\bar{S}(\phi Y\phi, Z)g(\phi X,\phi U) \\ &\quad -\bar{S}(\phi X,\phi Z)g(\phi Y,\phi U)] \end{aligned} \tag{5.4}$$

Using equations $(2.1), (2.2), (2.4), (2.5), (3.2)$ and (3.17) in equation (5.4), we have

$$\begin{aligned} & g(R(\phi X,\phi Y)\phi Z,\phi U) \\ = & \frac{1}{(n-1)}[S(\phi Y,\phi Z)g(\phi X,\phi U) - S(\phi X,\phi Z)g(\phi Y,\phi U)] \\ & +\frac{\epsilon(1+\delta\beta)}{(n-1)}g(\phi Y,\phi Z)g(\phi X,\phi U) - \frac{\epsilon(1+\delta\beta)}{(n-1)}g(\phi X,\phi Z)g(\phi Y,\phi U) \\ & +\frac{\alpha}{(n-1)}g(Y,\phi Z)g(\phi X,\phi U) - \frac{\alpha}{(n-1)}g(X,\phi Z)g(\phi Y,\phi U) \\ & +\alpha g(\phi Y,\phi Z)g(X,\phi U) - \alpha g(\phi X,\phi Z)g(Y,\phi U). \end{aligned} \tag{5.5}$$

Let $\{e_1, e_2, ..., e_{-1}, \xi\}$ be a local orthonormal basis of vector fields on (ϵ, δ)-trans-Sasakian manifold M and $\{\phi e_1, \phi e_2, ..., \phi e_{n-1}, \xi\}$ is a also local orthonormal basis of vector fields on $(\epsilon, \delta)-$trans-Sasakian manifold M. Now, replace $X = U = e_i$ in equation (5.5), and taking summation over $i, 1 \leq i \leq n-1$, we get

$$
\begin{aligned}
(5.6) \qquad & \sum_{i=1}^{n-1} g(R(\phi e_i, \phi Y)\phi Z, \phi e_i) \\
= \; & \frac{1}{(n-1)} \sum_{i=1}^{n-1} [S(\phi Y, \phi Z) g(\phi e_i, \phi e_i) - S(\phi e_i, \phi Z) g(\phi Y, \phi e_i)] \\
& + \frac{\epsilon(1+\delta\beta)}{(n-1)} \sum_{i=1}^{n-1} [g(\phi Y, \phi Z) g(\phi e_i, \phi e_i) - g(\phi e_i, \phi Z) g(\phi Y, \phi e_i)] \\
& + \frac{\alpha}{(n-1)} \sum_{i=1}^{n-1} g(Y, \phi Z) g(\phi e_i, \phi e_i) - \frac{\alpha}{(n-1)} \sum_{i=1}^{n-1} g(e_i, \phi Z) g(\phi Y, \phi e_i) \\
& + \alpha \sum_{i=1}^{n-1} g(\phi Y, \phi Z) g(e_i, \phi e_i) - \alpha \sum_{i=1}^{n-1} g(\phi e_i, \phi Z) g(Y, \phi e_i).
\end{aligned}
$$

Now, using equations $(2.1), (2.2), (2.4), (2.5), (2.17)$ and $(3.8) - (3.16)$ in equation (5.6), we have

$$
\begin{aligned}
(5.7) \qquad & S(Y, Z) \\
= \; & [\epsilon(n-2)(1+\delta\beta) + (n-1)(\alpha^2 - \beta^2) - \epsilon\delta(n-1)(\xi\beta)] g(Y, Z) \\
& + [(n-1)(\epsilon\alpha^2 - \delta\beta^2) - \epsilon(n-1)(\alpha^2 - \beta^2) + \delta(n-1)(\xi\beta) \\
& - 2(\xi\beta) - (n-2)(1+\beta)] \eta(Y)\eta(Z) \\
& + [\alpha - 2\epsilon\delta\alpha\beta(n-1) - (n-1)(\xi\alpha)] g(\phi Y, Z) \\
& - [(\phi Z)\alpha + \epsilon(n-2)(Z\beta)]\eta(Y) - [(\phi Y)\alpha + (n-2)(Y\beta)]\eta(Z).
\end{aligned}
$$

If $\alpha = 0$ and $\beta = constant$ in equation (5.7), we get

$$
\begin{aligned}
(5.8) \qquad S(Y, Z) \; = \; & [\epsilon(n-2)(1+\delta\beta) - (n-1)\epsilon\beta^2] g(Y, Z) \\
& + [\epsilon(n-1)\beta^2 - \delta(n-1)\beta^2 - (2-n)(1+\delta\beta)] \eta(Y)\eta(Z).
\end{aligned}
$$

Therefore

$$S(Y, Z) = ag(Y, Z) + b\eta(Y)\eta(Z),$$

where

$$a = \epsilon(n-2)(1+\delta\beta) - (n-1)\epsilon\beta^2,$$

and

$$b = [\epsilon(n-1)\beta^2 - \delta(n-1)\beta^2 - (2-n)(1+\delta\beta)].$$

This result shows that the manifold under the consideration is an $\eta-$ Einstein manifold. Thus, we can state the following theorem

Theorem 5.1. *An $n-$dimensional ϕ-projectively $(\epsilon,\delta)-$trans-Sasakin manifold M with a semi-symmetric metric connection is a $\eta-$Einstein manifold if $\alpha=0$ and $\beta=constant$.*

6 Weyl conformal curvature tensor on $(\epsilon,\delta)-$trans-Sasakian manifold with a semi-symmetric metric connection

The weyl conformal curvature tensor $\bar{C}$ of type $(1,3)$ of M an $n-$dimensional $(\epsilon,\delta)-$trans-Sasakian manifold a with semi-symmetric metric connection $\bar{\nabla}$ is given by [14]

$$\begin{aligned}\bar{C}(X,Y)Z &= \bar{R}(X,Y)Z-\frac{1}{(n-2)}[\bar{S}(Y,Z)X-\bar{S}(X,Z)Y+g(Y,Z)\bar{Q}X \\ &\quad -g(X,Z)\bar{Q}Y]+\frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)X-g(X,Z)Y],\end{aligned} \tag{6.1}$$

where $\bar{Q}$ is the Ricci operator with respect to the semi-symmetric metric connection $\bar{\nabla}$. Let M be an $n-$dimensional $(\epsilon,\delta)-$trans-Sasakian manifold . The Weyl conformal curvature tensor $\bar{C}$ of M with respect to the semi-symmetric metric connection $\bar{\nabla}$ is define above equation (6.1). Now, taking inner product with U in equation (6.1), we get

$$\begin{aligned}& g(\bar{C}(X,Y)Z,U) \\ =\ & g(\bar{R}(X,Y)Z,U)-\frac{1}{(n-2)}[\bar{S}(Y,Z)g(X,U)-\bar{S}(X,Z)g(Y,U) \\ & +g(Y.Z)g(\bar{Q}X,U)-g(X.Z)g(\bar{Q}Y,U)]+\frac{\bar{r}}{(n-1)(n-2)} \\ & [g(Y,Z)g(X,U)-g(X,Z)g(Y,U)],\end{aligned} \tag{6.2}$$

Using equations $(2.4),(3.2),(3.17),(3.18)$ and (3.20) in equation (6.2), we get

$$\begin{aligned}& \bar{C}(X,Y,Z,U) \\ =\ & g(R(X,Y)Z,U)-\frac{1}{(n-2)}[S(Y,Z)g(X,U)-S(X,Z)g(Y,U) \\ & +g(Y,Z)g(QX,U)-g(X,Z)g(QY,U)]+\frac{r}{(n-1)(n-2)} \\ & [g(Y,Z)g(X,U)-g(X,Z)g(Y,U)],\end{aligned} \tag{6.3}$$

where $g(\bar{C}(X,Y)Z,U)=\bar{C}(X,Y,Z,U)$ and $g(R(X,Y)Z,U)=C(X,Y,Z,U)$ are Weyl curvature tensor with respect to semi-symmetric metric connection and metric connection respectively. We have

$$\bar{C}(X,Y,Z,U)=C(X,Y,Z,U) \tag{6.4}$$

where

$$
\begin{aligned}
(6.5)\qquad & C(X,Y,Z,U) \\
= \; & g(R(X,Y)Z,U) - \frac{1}{(n-2)}[S(Y,Z)g(X,U) - S(X,Z)g(Y,Z) \\
& + g(Y,Z)g(QX,U) - g(X,Z)g(QY,U)] + \frac{r}{(n-1)(n-2)} \\
& [g(Y,Z)g(X,U) - g(X,Z)g(Y,U)].
\end{aligned}
$$

Theorem 6.1. *The Weyl conformal curvature tensor of an $(\epsilon,\delta)-$trans-Sasakian manifold M with respect to a metric connection is equal to the weyl conformal curvature of with respect to semi-symmetric metric connection.*

7 On an $(\epsilon,\delta)-$trans-Sasakian manifold with weyl conformal flat conditions with a semi-symmetric metric connection

Let us consider that the $(\epsilon,\delta)-$trans-Sasakian manifold M with respect to the semi-symmetric metric connection is Weyl conformally flat, that is $\bar{C}=0$. Then from equation (6.1), we get

$$
\begin{aligned}
(7.1)\qquad \bar{R}(X,Y)Z \;=\; & \frac{1}{(n-2)}[\bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)\bar{Q}X \\
& - g(X,Z)\bar{Q}Y] - \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y].
\end{aligned}
$$

Now, taking the inner product of equation (7.1) with U . Then, we get

$$
\begin{aligned}
(7.2)\qquad g(\bar{R}(X,Y)Z,U) \;=\; & \frac{1}{(n-2)}[\bar{S}(Y,Z)g(X,U) - \bar{S}(X,Z)g(Y,U) \\
& + g(Y.Z)g(\bar{Q}X,U) - g(X.Z)g(\bar{Q}Y,U)] \\
& - \frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)g(X,U) \\
& - g(X,Z)g(Y,U)].
\end{aligned}
$$

Using equations $(2.4), (3.2), (3.17), (3.18)$ and (3.20) in equation (7.2), we get

$$
\begin{aligned}
(7.3)\qquad g(R(X,Y)Z,U) \;=\; & \frac{1}{(n-2)}[S(Y,Z)g(X,U) - S(X,Z)g(Y,U) \\
& + g(Y,Z)g(QX,U) - g(X,Z)g(QY,U)] \\
& - \frac{r}{(n-1)(n-2)}[g(Y,Z)g(X,U) \\
& - g(X,Z)g(Y,U)],
\end{aligned}
$$

Putting $X = U = \xi$ in equation (7.3) and using equations (2.2), (2.3) and (2.4), we get

$$\begin{aligned}(7.4)\qquad & g(R(\xi, Y)Z, \xi) \\ = \ & \frac{1}{(n-2)}[\epsilon S(Y, Z) - \epsilon\eta(Y)S(\xi, Z) + g(Y, Z)S(\xi, \xi) \\ & -\epsilon\eta(Z)S(Y, \xi)] - \frac{r}{(n-1)(n-2)}[\epsilon g(Y, Z) - \eta(Y)\eta(Z)].\end{aligned}$$

Now, using equations (2.12), (2.13) and (2.17), we get

$$\begin{aligned}(7.5)\qquad & S(Y, Z) \\ = \ & [(n-2)(\alpha^2 - \beta^2) - \epsilon(\epsilon\alpha^2 - \delta\beta^2) + \epsilon\delta\beta(n-1)(\xi\beta) \\ & -\epsilon\delta(n-2)(\xi\beta) + \frac{r}{(n-1)}]g(Y, Z) \\ & +[2(n-2)(\epsilon\alpha^2 - \delta\beta^2) - (n-2)(\alpha^2 - \beta^2) - 2(\xi\beta) \\ & +\delta(n-2)(\xi\beta) - n\delta\beta - \frac{\epsilon r}{(n-1)}]\eta(Y)\eta(Z) \\ & -[2\epsilon\delta(n-2)\alpha\beta + (n-2)(\xi\alpha)]g(\phi Y, Z) \\ & -[\epsilon\delta(n-2)(Z\beta)\xi + (n-2)(Z\beta) + (\phi Z)\alpha]\eta(Y) \\ & -[(n-2)(Y\beta) + (\phi Y)\alpha]\eta(Z) + \epsilon\delta(n-2)(Z\beta)Y.\end{aligned}$$

If $\alpha = 0$ and $\beta = constant$ in equation (7.5), we get

$$\begin{aligned}(7.6)\qquad & S(Y, Z) \\ = \ & [\epsilon\delta(n-1)\beta^2 - (n-2)\beta^2 + \frac{r}{(n-1)}]g(Y, Z) \\ & +[(n-2)\beta^2 - 2(n-1)\delta\beta^2 - \delta n\beta - \frac{\epsilon r}{(n-1)}]\eta(Y)\eta(Z).\end{aligned}$$

Therefore

$$S(Y, Z) = ag(Y, Z) + b\eta(Y)\eta(Z),$$

where

$$a = [\epsilon\delta(n-1)\beta^2 - (n-2)\beta^2 + \frac{r}{(n-1)}],$$

and

$$b = [(n-2)\beta^2 - 2(n-1)\delta\beta^2 - \delta n\beta - \frac{\epsilon r}{(n-1)}].$$

This shows that M is an $\eta-$Einstein manifold. Thus, we can state as follows:

Theorem 7.1. *Let M be an $n-$dimensional Weyl conformally flat $(\epsilon, \delta)-$trans-Sasakian manifold with respect to a semi symmetric metric connection $\bar{\nabla}$ is an $\eta-$Einstein manifold if $\alpha = 0$ and $\beta =$constant.*

Now, taking equation (6.1), we have

$$\begin{aligned}(7.7)\qquad &\bar{C}(X,Y)Z\\ =\ &\bar{R}(X,Y)Z-\frac{1}{(n-2)}[\bar{S}(Y,Z)X-\bar{S}(X,Z)Y+g(Y,Z)\bar{Q}X\\ &-g(X,Z)\bar{Q}Y]+\frac{\bar{r}}{(n-1)(n-2)}[g(Y,Z)X-g(X,Z)Y].\end{aligned}$$

Using equations (2.16), (3.2), (3.17), (3.18) and (3.20) in equation (7.7), we get

$$\begin{aligned}(7.8)\qquad &\bar{C}(X,Y)Z\\ =\ &C(X,Y)Z+\epsilon(1+2\delta\beta)[g(X,Z)Y-g(Y,Z)X\ ]\\ &+\epsilon(1+\delta\beta)[\eta(X)g(Y,Z)-\eta(Y)g(X,Z)]\xi\\ &+(1+\delta\beta)\eta(Z)[\eta(Y)X-\eta(X)Y\ ]\\ &-\alpha[g(\phi X,Z)Y-g(\phi Y,Z)X-g(Y,Z)\phi X\ +g(X,Z)\phi Y]\\ &-\frac{1}{(n-2)}[(n-2)(1+\delta\beta)\eta(Y)\eta(Z)-(\epsilon(1+2\delta\beta)(n-2)\\ &+\delta\beta)g(Y,Z)X+\alpha(n-2)g(\phi Y,Z)X+(\epsilon(1+2\delta\beta)(n-2)\\ &+\delta\beta)g(X,Z)Y-\alpha(n-2)g(\phi X,Z)Y+\delta\beta)g(Y,Z)X\\ &-(n-2)(1+\delta\beta)\eta(X)\eta(Z)Y-\epsilon(1+2\delta\beta)(n-2)\\ &+\epsilon(n-2)(1+\delta\beta)g(Y,Z)\eta(X)\xi+\alpha(n-2)g(Y,Z)\phi X\\ &+(\epsilon(1+2\delta\beta)(n-2)+\delta\beta)g(X,Z)Y\\ &-\epsilon(n-2)(1+\delta\beta)g(X,Z)\eta(Y)\xi-\alpha(n-2)g(X,Z)\phi Y]\\ &-\frac{2\epsilon\delta\beta+\epsilon(1+2\delta\beta)(n-2)}{(n-2)}[g(Y,Z)X-g(X,Z)Y].\end{aligned}$$

Let X and Y are orthogonal basis to ξ . Putting the value $Z=\xi$ and using equations (2.1), (2.2) and (2.4) in (7.8), we get

$$(7.9)\qquad \bar{C}(X,Y)\xi=C(X,Y)\xi.$$

Theorem 7.2. *An $n-$dimensional $(\epsilon,\delta)-$trans-Sasakian manifold M is Weyl $\xi-$conformally flat with respect to the semi-symmetric metric connection if and only if the manifold is also weyl $\xi-$conformally flat with respect to the metric connection provided that the vector fields are horizontal vector fields.*

8 On an $(\epsilon,\delta)-$trans-Sasakian manifold with a semi-symmetric metric connection satisfying $\bar{R}.\bar{S}=0$

Now, suppose that M be $n-$dimensional $(\epsilon,\delta)-$trans-Sasakian manifold with a semi-symmetric metric connection $\bar{\nabla}$ satisfying the condition

$$(8.1)\qquad \bar{R}(X,Y).\bar{S}=0.$$

Then, we have

$$\bar{S}(\bar{R}(X,Y)Z,U)+\bar{S}(Z,\bar{R}(X,Y)U)=0. \tag{8.2}$$

Now, replace $X=\xi$ in equation (8.2) and using equations (2.11), (3.2), we have

$$\begin{aligned}
&(\alpha^2-\beta^2)g(Y,Z)\bar{S}(\xi,U)-(\alpha^2-\beta^2)\eta(Z)\bar{S}(Y,U)\\
&+\epsilon g(\phi Y,Z)\bar{S}(gard\alpha,U)-\delta g(Y,Z)\bar{S}(gard\beta,U)\\
&+\epsilon\delta\eta(Y)\eta(Z)\bar{S}(grad\beta,U)+\epsilon(Z\alpha)\bar{S}(\phi Y,U)\\
&-\delta(Z\beta)\eta(Y)\bar{S}(\xi,U)+\epsilon(U\alpha)\bar{S}(\phi Y,Z)\\
&+\delta(Z\beta)\bar{S}(Y,U)-\delta(Z\beta)\eta(Y)\bar{S}(\xi,U)\\
&+2\epsilon\delta\alpha\beta \bar{S}(\xi,U)g(\phi Z,Y)+2\epsilon\delta\alpha\beta\eta(Z)\bar{S}(\phi Y,U)\\
&+\delta\beta\eta(Z)\bar{S}(Y,U)-\epsilon\delta\beta g(Y,Z)\bar{S}(\xi,U)\\
&+\alpha g(\phi Y,Z)\bar{S}(\xi,U)+2\epsilon(\delta-\epsilon)\alpha\beta\eta(Y)\bar{S}(\phi Z,U)\\
&-(\alpha^2-\beta^2)\eta(U)\bar{S}(Y,Z)-\epsilon\alpha\eta(Z)\bar{S}(\phi Y,U)\\
&+(\alpha^2-\beta^2)g(Y,U)\bar{S}(\xi,Z)+\epsilon g(\phi Y,U)\bar{S}(grad\beta,Z)\\
&-\delta\bar{S}(grad\beta,Z)g(Y,U)-\epsilon\delta\beta g(Y,U)\bar{S}(\xi,Z)\\
&+\epsilon\delta\bar{S}(gard\beta,Z)\eta(Y)\eta(U)+\delta(U\beta)\bar{S}(Y,Z)\\
&-\delta(U\beta)\eta(Y)\bar{S}(\xi,Z)+2\epsilon\delta\alpha\beta g(\phi U,Y)\\
&+\delta\beta\eta(U)\bar{S}(Y,Z)+2\epsilon\delta\alpha\beta\eta(U)\bar{S}(\phi Y,Z)\\
&+2\epsilon(\delta-\epsilon)\alpha\beta\eta(U)\bar{S}(\phi U,Z)-\epsilon\alpha\eta(U)\bar{S}(\phi Y,Z)\\
&+\alpha g(\phi Y,U)\bar{S}(\xi,Z)\\
=\;&0.
\end{aligned} \tag{8.3}$$

Using equations $(2.1)-(2.5),(2.12),(2.13),(3.17)$ and $(3.21)-(3.26)$ in equation (8.3), we get

$$
\begin{aligned}
& [(\alpha^2-\beta^2)-\delta\beta-\delta(\xi\beta)]S(Y,Z) \\
= & [(n-1)(\alpha^2-\beta^2)\{(\epsilon\alpha^2-\delta\beta^2)-\delta\beta-(\xi\beta)\}-\epsilon\delta(n-1)(\epsilon\alpha^2-\delta\beta^2)\{(\xi\beta)+\beta\} \\
& +\beta(n-1)(\xi\beta)(1+\epsilon\delta)+\epsilon\delta(\xi\beta)^2+\delta(n-2)(grad\beta)^2+\delta(\phi grad\beta)\alpha-\epsilon\beta(\xi\beta) \\
& +\epsilon(n-2)\beta^2+\epsilon\delta\beta(\alpha^2-\beta^2)-\epsilon(n-2)\alpha(\xi\alpha)-2\epsilon\delta\alpha^2\beta(n-2) \\
& +\epsilon(n-2)(1+2\delta\beta)\{(\alpha^2-\beta^2)-\delta(\xi\beta)-\delta\beta\}+\epsilon\alpha^2(n-2)]g(Y,Z) \\
& +[(n-1)(\alpha^2-\beta^2)\{\delta\beta-(\epsilon\alpha^2-\delta\beta^2)+\epsilon(\epsilon\alpha^2-\delta\beta^2)-\epsilon\delta\beta\}+(\epsilon-\delta)(\xi\beta)^2 \\
& +(\alpha^2-\beta^2)\{(\xi\beta)(1-\epsilon)-(n-2)(1+2\delta\beta)\}-\epsilon\delta(n-2)(grad\beta)^2 \\
& -\epsilon\delta(\phi grad\beta)\alpha+\alpha(n-2)(\xi\alpha)+\{\delta(n-2)(1+\delta\beta)-\delta\beta+(n-1)\beta\}(\xi\beta) \\
& -\epsilon\beta(n-2)(\xi\beta)+2\delta\alpha^2\beta(n-2)+\delta\beta(n-2)(1+\delta\beta)-\alpha^2(n-2)]\eta(Y)\eta(Z) \\
& +[(n-1)(\epsilon\alpha^2-\delta\beta^2)\{(\xi\alpha)+2\epsilon\delta\beta-\alpha\}+\alpha(\xi\beta)\{(n-1)-\delta(n-2)\} \\
& -\{\epsilon\delta\beta(n-1)+(\xi\beta)-(n-2)(1+2\delta\beta)-\delta\beta\}(\xi\alpha)-\epsilon(n-2)g(grad\alpha,grad\beta) \\
& -\epsilon(\phi grad\alpha)-2\epsilon\alpha\beta^2(n-1)-2\epsilon\delta\alpha\beta(n-1)(\xi\beta)+2\delta\alpha\beta+\alpha(n-2)(\alpha^2-\beta^2) \\
& +2\delta\alpha\beta(n-2)(1+2\delta\beta)+2\alpha\beta^2-\alpha(n-2)(1+2\delta\beta)]g(\phi Z,Y)+[\epsilon(\xi\alpha)+2\epsilon\delta\alpha\beta \\
& -\epsilon\alpha]S(\phi Y,Z)+[2\delta(n-2)(Z\beta)(\xi\beta)+\{\delta(\xi\beta)+\delta\beta-\epsilon(\alpha^2-\beta^2)\}(\phi Z)\alpha \\
& -2\epsilon(\delta-\epsilon)(n-2)\alpha\beta(\phi Z)\beta-2\epsilon(\delta-\epsilon)(\phi^2 Z)\alpha+(n-2)(Z\beta)\{\delta\beta-\epsilon(\alpha^2-\beta^2)\}] \\
& \eta(Y)+[(n-2)(\alpha^2-\beta^2)(Y\beta)+(\alpha^2-\beta^2)(\phi Y)\alpha-2\epsilon\alpha\beta(\phi^2 Y)\alpha-2(n-2)\epsilon\delta\alpha\beta(\phi Y)\beta \\
& -\delta\beta(n-2)(Y\beta)-\delta\beta(\phi Y)\alpha+\epsilon(\phi^2 Y)\alpha^2+\epsilon\alpha(n-2)(\phi Y)\beta]\eta(Z) \\
& -\epsilon(n-2)(Z\alpha)(\phi Y)\beta-\delta(n-2)(Z\beta)(Y\beta)-\epsilon(Z\alpha)(\phi^2 Y)\alpha-\delta(Z\beta)(\phi Y)\alpha.
\end{aligned}
$$

If $\alpha=0$ and $\beta=constant$ in equation (8.3), we get

$$S(Y,Z)=ag(Y,Z)+b\eta(Y)\eta(Z) \tag{8.4}$$

where

$$a=-[\frac{\delta(n-1)\beta^4+\{(\epsilon+\delta)(n-1)-\epsilon\delta\}\beta^3+\delta(n-2)(grad\beta)^2-2\epsilon\delta\beta(n-2)\beta^2-\epsilon\delta\beta(n-2)(1+2\delta\beta)}{\beta(\delta+\beta)}]$$

and

$$b=[\frac{(1-\epsilon)\delta(n-1)\beta^4+(1-\epsilon)\delta(n-1)\beta^3+\epsilon\delta(n-2)(grad\beta)^2-(n-2)(1+\delta\beta)\beta^2-\delta\beta(n-2)(1+\delta\beta)}{\beta(\delta+\beta)}].$$

This shows that M is an $\eta-$Einstein manifold. Thus, we can state the following:

Theorem 8.1. *If an (ϵ,δ)-trans-Sasakian manifold with a semi symmetric metric connection $\bar{\nabla}$ satisfies $\bar{R}.\bar{S}=0$, then the (ϵ,δ)-trans-Sasakian manifold is an $\eta-$ Einstein manifold if $\alpha=0$ and $\beta=constant$.*

9 On an $(\epsilon,\delta)-$trans-Sasakian manifold with respect to a semi-symmetric metric connection satisfies $\bar{P}.\bar{S}=0$

Now, we consider an (ϵ,δ)-trans-Sasakian manifold with a semi-symmetric connection $\bar{\nabla}$ satisfying

$$(\bar{P}(X,Y).\bar{S})(Z,U)=0. \tag{9.1}$$

where $\bar{P}$ is the Projective curvature tensor and $\bar{S}$ is the Ricci tensor with a semi-symmetric metric connection . Then, we have

$$\bar{S}(\bar{P}(X,Y)Z,U)+\bar{S}(Z,\bar{P}(X,Y)U)=0. \tag{9.2}$$

Replace $X=\xi$ in the equation (9.2), we get

$$\bar{S}(\bar{P}(\xi,Y)Z,U)+\bar{S}(Z,\bar{P}(\xi,Y)U)=0.$$

Replace $X=\xi$ in the equation (4.1), we get

$$\bar{P}(\xi,Y)Z=\bar{R}(\xi,Y)Z-\frac{1}{(n-1)}[\bar{S}(Y,Z)\xi-\bar{S}(\xi,Z)Y. \tag{9.3}$$

Using equations $(2.1), (2.2), (2.4), (2.11), (3.2), (3.17)$ and (3.23) in equation (9.3), we get

$$
\begin{aligned}
(9.4)\qquad & (\alpha^2-\beta^2)g(Y,Z)\bar{S}(\xi,U)-(\alpha^2-\beta^2)\eta(Z)\bar{S}(Y,U)+\delta\beta\eta(Z)\bar{S}(Y,U) \\
& \epsilon\bar{S}(grad\alpha,U)g(\phi Z,Y)+\epsilon(Z\alpha)\bar{S}(\phi Y,U)+\delta(Z\beta)\bar{S}(Y,U) \\
& -\delta\bar{S}(grad\beta,U)g(Y,Z)+\epsilon\delta\bar{S}(grad\beta,U)\eta(Y)\eta(Z)-\epsilon\alpha\eta(Z)\bar{S}(\phi Y,U) \\
& -\delta(Z\beta)\eta(Y)\bar{S}(\xi,U)+2\epsilon(\delta-\epsilon)\alpha\beta\eta(Y)\bar{S}(\phi Z,U)+\alpha g(\phi Z,Y)\bar{S}(\xi,U \\
& +2\epsilon\delta\alpha\beta g(\phi Z,Y)\bar{S}(\xi,U)+2\epsilon\delta\alpha\beta\eta(Z)\bar{S}(\phi Y,U)-\epsilon\delta\beta g(Y,Z)\bar{S}(\xi,U) \\
& +\frac{\epsilon(n-2)(1+2\delta\beta)}{(n-1)}g(Y,Z)\bar{S}(\xi,U)-\frac{(n-2)(1+\delta\beta)}{(n-1)}\eta(y)\eta(Z)\bar{S}(\xi,U) \\
& -\frac{1}{n-1}S(Y,Z)\bar{S}(\xi,U)-\frac{\alpha(n-2)}{(n-1)}g(\phi Y,Z)\bar{S}(\xi,U)-\delta\beta\eta(Z)\bar{S}(Y,U) \\
& +\frac{\epsilon\delta\beta}{n-1}g(Y,Z)\bar{S}(\xi,U)+(\epsilon\alpha^2-\delta\beta^2)\eta(Z)\bar{S}(Y,U)+\epsilon(U\alpha)\bar{S}(\phi Y,Z) \\
& -\frac{1}{(n-1)}(\xi\beta)\eta(Z)\bar{S}(Y,U)-\frac{(n-2)}{(n-1)}(Z\beta)\bar{S}(Y,U)+\delta(U\beta)\bar{S}(Y,Z) \\
& -\frac{1}{(n-1)}(\phi Z)\alpha\bar{S}(Y,U)+(\alpha^2-\beta^2)g(Y,U)\bar{S}(\xi,U) \\
& -(\alpha^2-\beta^2)\eta(U)\bar{S}(Y,U)+\epsilon g(\phi U,Y)\bar{S}(grad\alpha,z)-\epsilon\alpha\eta(U)\bar{S}(\phi Y,Z) \\
& -\delta g(Y,U)\bar{S}(grad\beta,z)+\epsilon\delta\eta(Y)\eta(U)\bar{S}(grad\beta,z)+\alpha g(\phi Y,U)\bar{S}(\xi,Z) \\
& -\delta(U\beta)\eta(Y)\bar{S}(\xi,Z)+\delta\beta\eta(U)\bar{S}(Y,Z)+2\epsilon\delta\alpha\beta\eta(U)\bar{S}(\phi Y,Z) \\
& +2\epsilon\delta\alpha\beta g(\phi U,Y)\bar{S}(\xi,Z)+2\epsilon(\delta-\epsilon)\alpha\beta\eta(Y)\bar{S}(\phi U,Z)-\delta\beta\eta(U)\bar{S}(Y,Z) \\
& -\epsilon\delta\beta g(Y,U)\bar{S}(\xi,Z)+\frac{\epsilon(n-2)}{(n-1)}(1+2\delta\beta)g(Y,U)\bar{S}(\xi,Z) \\
& -\frac{1}{(n-1)}S(Y,U)\bar{S}(\xi,Z)-\frac{(n-2)}{(n-1)}(1+\delta\beta)\eta(Y)\eta(U)\bar{S}(\xi,Z) \\
& +\frac{\epsilon\delta\beta}{(n-1)}g(Y,U)\bar{S}(\xi,Z)-\frac{\alpha(n-2)}{(n-1)}g(\phi Y,U)\bar{S}(\xi,Z) \\
& +(\epsilon\alpha^2-\delta\beta^2)\eta(U)\bar{S}(Y,Z)-\frac{1}{(n-1)}(\xi\beta)\eta(U)\bar{S}(Y,Z) \\
& -\frac{(n-2)}{(n-1)}(U\beta)\bar{S}(Y,Z)-\frac{1}{(n-1)}(\phi U)\alpha\bar{S}(Y,Z) \\
= \;& 0.
\end{aligned}
$$

Putting the value $U = \xi$ and using equations $(2.1)-(2.5), (3.17)$ and $(3.21)-(3.26)$ in equation (9.4), we get

$$
\begin{aligned}
(9.5) \qquad & [(\alpha^2-\beta^2)-\delta\beta-\delta(\xi\beta)]S(Y,Z) \\
= \quad & [(n-1)(\alpha^2-\beta^2)\{(\epsilon\alpha^2-\delta\beta^2)-\delta\beta-(\xi\beta)\}-\epsilon\delta(n-1)(\epsilon\alpha^2-\delta\beta^2)(\xi\beta) \\
& +\beta(n-1)(\xi\beta)-\epsilon\delta(\xi\beta)^2-\delta(\phi grad\beta)\alpha-\delta(n-2)(grad\beta)^2 \\
& +\epsilon(n-2)(1+2\delta\beta)\{(\alpha^2-\beta^2)-(\xi\beta)-\delta(1+\beta)\}+\epsilon\delta\beta(\alpha^2-\beta^2) \\
& -\epsilon\alpha(n-2)(\xi\alpha)-\epsilon\beta(\xi\beta)-2(n-2)\epsilon\delta\alpha^2\beta+\epsilon\alpha^2(n-2) \\
& +\frac{(n-2)^2}{(n-1)}\epsilon(1+2\delta\beta)(\xi\beta)+\frac{(n-2)}{(n-1)}\epsilon\delta\beta(\xi\beta)]g(Y,Z) \\
& +[(1+\epsilon)(n-1)(\alpha^2-\beta^2)(\epsilon\alpha^2-\delta\beta^2)-(\alpha^2-\beta^2)\{\delta\beta(n-1)+(\xi\beta) \\
& +(n-2)(Y\beta)+\epsilon\delta\beta(n-1)+\epsilon+(n-2)(1+\delta\beta)\}-\epsilon\beta(n-1)(\xi\beta) \\
& +\delta(n-1)(\epsilon\alpha^2-\delta\beta^2)(\xi\beta)-\delta(\xi\beta)^2-\epsilon\delta(\phi grad\beta)\alpha+\alpha(n-2)(\xi\alpha) \\
& -\epsilon\delta(n-2)(grad\beta)^2+(n-2)(1+\delta\beta)\{(\xi\beta)+\delta\beta\} \\
& -\frac{(n-2)^2}{(n-1)}(1+2\delta\beta)(\xi\beta)]\eta(Y)\eta(Z)+[\epsilon(\xi\alpha)+2\epsilon\delta\alpha\beta-\epsilon\alpha]S(\phi Y,Z) \\
& +[(n-1)(\epsilon\alpha^2-\delta\beta^2)\{(\xi\alpha)+2\epsilon\delta\alpha\beta\}+\alpha(\epsilon\alpha^2-\delta\beta^2)-\epsilon\delta\beta(n-1)(\xi\alpha) \\
& -(\xi\alpha)(\xi\beta)-\epsilon(n-2)g(grad\alpha,grad\beta)-\epsilon(\phi grad\alpha)\alpha-2\epsilon(n-1)\alpha\beta^2 \\
& -2\epsilon\delta\alpha\beta(n-1)(\xi\beta)-\alpha(n-2)(\alpha^2-\beta^2)-(n-2)(1+2\delta\beta)(\xi\alpha) \\
& -\delta\beta(\xi\alpha)+\delta\beta(n-2)(\xi\beta)-2(n-2)\delta\alpha\beta(1+2\delta\beta)+\alpha(n-2)(1+2\delta\beta) \\
& -\frac{\alpha(2n-3)}{(n-1)}(\xi\beta)-\frac{\alpha(n-2)^2}{(n-1)}(\xi\beta)]g(\phi Y,Z)+[2\delta(n-2)(Z\beta)(\xi\beta) \\
& -2\epsilon(\delta-\epsilon)\alpha\beta(\phi^2 Z)\alpha-2\epsilon\alpha\beta(n-2)(\delta-\epsilon)(\phi Z)\beta+\delta\beta(n-2)(Z\beta) \\
& -\epsilon(n-2)(\alpha^2-\beta^2)(Z\beta)-\epsilon(\alpha^2-\beta^2)(\phi Z)\alpha+\delta(\xi\beta)(\phi Z)\alpha \\
& +\delta\beta(\phi Z)\alpha]\eta(Y)+[\epsilon\alpha^2(\phi^2 Y)\alpha-(n-2)(\alpha^2-\beta^2)(Y\beta)-(\alpha^2-\beta^2)(\phi Z)\alpha \\
& -2\epsilon\delta\alpha\beta(\phi^2 Y)\alpha-2\epsilon\delta\alpha\beta(n-2)(\phi Y)\beta-\delta\beta(n-2)(Y\beta)-\delta\beta(\phi Y)\alpha \\
& +\epsilon\alpha(n-2)(\phi Y)\beta]\eta(Z)-\epsilon(Z\alpha)(\phi^2 Y)\alpha-\epsilon(n-2)(Z\alpha)(\phi Y)\beta \\
& -(n-2)(Z\beta)(Y\beta)-\delta(Z\beta)(\phi Y)\alpha.
\end{aligned}
$$

If $\alpha = 0$ and $\beta = constant$ in equation (9.6), we get

$$
(9.6) \qquad S(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z)
$$

where
$a = \frac{(n-1)\epsilon\beta^4-(n-2)\beta^2(\epsilon+\beta)+(n-1)\beta^3-(n-2)\beta(1+\epsilon\beta+(n-1)\epsilon\beta+(n-2)(grad\beta)^2}{\beta(\epsilon+\beta)}$
and
$b = \frac{(n-2)\beta(\epsilon+\beta)+(n-2)\beta^2-(n-2)\epsilon(grad\beta)^2}{\beta(\epsilon+\beta)}$.

This result shows that the manifold under the consideration is an $\eta-$Einstein manifold. Thus,we can state the following theorem

Theorem 9.1. *If an $(\epsilon, \delta)-$trans-Sasakian manifold with a semi symmetric metric connection $\bar{\nabla}$ satisfies $\bar{P}.\bar{S} = 0$, then the $(\epsilon, \delta)-$trans-Sasakian manifold is an $\eta-$ Einstien manifold if $\alpha = 0$ and $\beta = constant$.*

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