Brinkman Flow of a Conducting Fluid in an Annular Porous Channel of Variable Permeability in the Presence of Magnetic Field.

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Abstract

Laminar fully developed flow of an incompressible, viscous and electrically conducting fluid in an annular channel filled with a porous medium of variable permeability in the presence of transverse magnetic field is studied. Three different cases of permeability variation are considered. Brinkman model is used to study the flow. Numerical solutions of the problem are obtained using Galerkin method. Quantities such as velocity and volumetric flow rate are obtained and exhibited graphically. The influence of the various parameters like porous media shape parameter and Hartmann Number on flow are discussed. It is found that these parameters have strong effect on the flow.

Subject class [2010]:
Keywords: Brinkman Flow, Variable Permeability, Porous Media, Galerkin's Method, MHD Flow.

1 Introduction:

The study of fluid flow in a porous media is a topic of long standing interest for researchers due to its numerous applications in many fields such as biomechanics, physical sciences, and chemical engineering, industrial processes and natural phenomena. A number of theoretical and experimental models have been developed to describe fluid flow through porous medium. Brinkman flow model is one of them. Flow in porous channels of different sections has been extensively investigated in last few years. Kaviany (1985) initiated the study of flow through a porous channel. He considered laminar flow through a porous channel bounded by two parallel plates maintained at a constant and equal temperature and obtained an analytical solution using the Brinkman flow model. Nakayama (1988) investigated fully developed flow and heat transfer in a Darcy - Brinkman porous channel bounded by two parallel walls subjected to uniform heat flux. Vafai and Kim (1989) considered convection in a porous channel bounded by parallel plates based on the Brinkman - Forchheimer - extended Darcy model and obtained solutions for velocity and temperature field. Haji-Sheikh and Vafai (2004) considered flow and heat transfer in parallel plate channels, circular pipes, and elliptical passages filled with a Darcy - Brinkman medium and obtained velocity and temperature field. Wang (2008) studied fully developed laminar forced convection inside a semi-circular channel filled with a Brinkman-Darcy porous medium. He obtained analytical solutions for flow and constant flux heat transfer using a mixture of Cartesian and cylindrical coordinates. Wang (2010) investigate a fully developed flow and constant flux heat transfer in super-elliptic ducts filled with a porous medium. He obtained numerical solution using Ritz method for Darcy-Brinkman flow to determine the velocity and temperature fields. Wang (2011) considered the similar problem for regular polygonal ducts. He uses the boundary collocation method to solve the problem.

Flow through channels having corrugated or wavy wall has attracted the attention of few investigators. Akyildiz and Bellout (2010) consider a fully developed flow of a Newtonian fluid in a porous-saturated corrugated pipe on the basis of a Brinkman model. They found numerical solution of the problem using a spectral Galerkin method. The effects of both the Darcy number and corrugation on the velocity field are discussed and presented graphically. Ng and Wang (2010) studied pressure-driven flow through a porous channel that is laterally bounded by corrugated walls. They assume that the corrugations are periodic sinusoidal waves of small amplitude and examine the effects of the corrugations and permeability of the channel on the flow using perturbation analysis. Gray et al. (2013) presented an analytical solution for the steady Darcy flow of an incompressible fluid through a homogeneous, isotropic porous medium filling a channel bounded by symmetric wavy walls. The channel walls change from parallel planes, to small amplitude sine waves, to large amplitude nonsinusoidal waves as certain parameters are increased. Expressions are obtained for the stream function, specific discharge, piezometric head, and pressure.

Pantokratoras and Fang (2010) investigate the Poiseuille and Couette flow in a fluid-saturated Darcy -Brinkman porous medium channel with an electrically conducting fluid under the action of a magnetic and electric field. Exact analytical solutions are derived for fluid velocity. Zhao et al. (2010) extend the previous work Pantokratoras and Fang (2010) to the case with a Darcy-Brinkman - Forchheimer porous medium. Most of the work done on the problems of flow through porous media is related to homogeneous porous media. There are many situations where the porous medium is heterogeneous in nature, for example, water flow in rocks, filtration processes, porous aggregate, and reservoirs. In all such cases, permeability distribution of a medium is nonuniform. Scant literature is available on the fluid flow problems through porous media of variable permeability. Some authors have dealt with the problems of this kind. Govender (2006) presented a theoretical analysis of the fluid motion in a curved porous channel for the specific case of monotonic permeability variation in the vertical direction. He presented solutions in terms of the curvature ratio which is shown to affect the flow patterns. Nield and Kuznetsov (2000) consider the linear permeability variation and study the effect of this variation on forced convection in a parallel plate channel or circular duct using Darcy model. Verma and Datta (2012a) considered fully developed flow in an annular porous channel for the specific case of radial permeability variation. They found analytical solution for three special cases of permeability variation and obtained velocity, volumetric flow rate, average velocity and stress on the boundaries. They also discussed the influence of various parameters on the flow. Verma and Datta (2012 b) studied Poiseuille and Couette flow between two parallel plate channel filled with a porous medium of variable permeability and obtained exact expressions for the velocity and rate of volume flow. Srivastava and Satya Deo (2013) studied the Poiseuille and Couette flow of an electrically conducting fluid through a porous medium of variable permeability under the transverse magnetic field. They use Brinkman equation for flow through porous medium and obtained numerical solution for velocity and volumetric flow rate using Galerkin's method. They also discussed the influence of various parameters on the flow. Mohammadein and El-Shaer (2004), Kim and Yuan (2005), Verma and Datta (2012 c) and Vadasz (1993) also studied the problem of flow through variable permeability porous media.

In the present paper we have investigated the steady laminar flow of a viscous, incompressible and electrically conducting fluid in an annular tube filled with a fluid-saturated porous medium of variable permeability in the presence of transverse applied magnetic field. Brinkman model is used with no slip condition on the boundary. We assume that permeability (k) varies with radial distance and three cases of permeability variation are investigated. Case I, when permeability is constant, $k = k_o$; case II, when permeability variation is linear, $k = k_o r$ and case III, when variation is quadratic, $k = k_o r^2$. Numerical solutions for all the cases of permeability variations are obtained by using Galerkin method. The problem is new in the literature.

2 Mathematical Formulation:

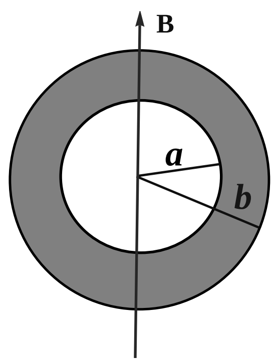

Fig. 1: Cross-section of annular tube filled with porous medium with transverse applied magnetic field.

For the steady state fully developed situation there is unidirectional flow in the z^* direction inside an annular tube with an impermeable wall at $r^* = a$ and $r^* = b$, $(b > a)$ as illustrated in figure 1. A transverse magnetic field B of uniform intensity is applied. We follow Srivastava and Satya Deo (2013) and assume that Magnetic Reynolds number is small and there is no external electric field therefore induced current can be neglected also internal causes such as separation of charges or polarization do not rise to induced electric field. The Lorentz force acting on the fluid in this case will be $\sigma B^2 u^*$ and is opposite to the direction of flow. The governing Brinkman (1947) equation for the present problem is

$$\mu_e\left(\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*}\right) - \frac{\mu}{k}u^* - \sigma B^2 u^* = \frac{\partial p^*}{\partial z^*} \tag{2.1}$$

where u^* is the fluid velocity, μ_e is the effective viscosity, μ is the fluid viscosity, k is the permeability of the porous medium, σ is the electrical conductivity of fluid and $\frac{\partial p^*}{\partial z^*}$ is the constant applied pressure gradient. In the present work we follow Brinkman (1947) and Chikh et al. (1995) and assume that $\mu_e = \mu$. Therefore, Eq.(2.1) becomes

$$\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*} - \frac{u^*}{k} - \frac{\sigma B^2 u^*}{\mu} = \frac{1}{\mu}\frac{\partial p^*}{\partial z^*} \tag{2.2}$$

Now we define dimensionless variables as follows

$$r = \frac{r^*}{a}, u = \frac{\mu u^*}{a^2(-\frac{\partial p^*}{\partial z^*})} \tag{2.3}$$

Equation (2.2) in dimensionless variables is

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{a^2}{k}u - M^2u = -1 \tag{2.4}$$

$M = Ba\sqrt{\frac{\sigma}{\mu}}$ is Hartmann Number and is dimensionless.

3 Solution and Results:

Now we will solve the governing equation of motion (2.4) for three cases of permeability variation. Case I, when permeability is constant, $k = k_o$; case II, when permeability variation is linear, $k = k_o r$ and case III, when variation is quadratic, $k = k_o r^2$. Here k_o is characteristic permeability, which may be taken as permeability on the surface of an inner cylinder. The boundary conditions for the present problem are

$$u(1) = 0 = u(q),\ q = b/a \tag{3.1}$$

3.1 Case-I

when permeability of the medium is constant i.e. $k = k_0$, the governing equation of motion (2.4) is

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - (\alpha^2 + M^2)u = -1 \tag{3.2}$$

where $\alpha^2 = a^2/k_0$ is the porous media shape parameter. Using the Galerkin's method solution of Eq. (3.2) satisfying boundary conditions (3.1) is taken as

$$u = C_1(r-1)(r-q) + C_2(r-1)^2(r-q) \tag{3.3}$$

where C_1 and C_2 are constants which are determined by Galerkin's method as

$$C_1 = \frac{-7[56 + 238q - 126q^2 + (3 + 8q^2 + 4q^3 + q^4)(\alpha^2 + M^2)]}{\Delta_1} \tag{3.4}$$

$$C_2 = \frac{-14(1+q)\{-35+(-1+q)^2(\alpha^2+M^2)\}}{\Delta_1} \tag{3.5}$$

where,

$$\begin{aligned} \Delta_1 \;=\; & [7(1+q)\{10+(-1+q)^2(\alpha^2+M^2)\}\{28+84q+(3+q-7q^2+5q^3)(\alpha^2+M^2)\} \\ & -2\{21+49q+(3-2q-5q^2+4q^3)(\alpha^2+M^2)\}^2] \end{aligned} \tag{3.6}$$

Dimensionless volumetric flow rate through the channel is given by

$$Q = 2\pi \int_1^q urdr \tag{3.7}$$

Substituting u from Eq.(3.3) in the above equation, after integration, we get

$$Q = -\frac{\pi}{30}[5C_1(q-1)^2(q+1) + C_2(q-1)^3(3q+2)] \tag{3.8}$$

for $q = 2$, Q is

$$Q = -\frac{\pi}{30}(15C_1 + 8C_2) \tag{3.9}$$

where C_1 and C_2 are given by Eq.(3.4) and (3.5), respectively.

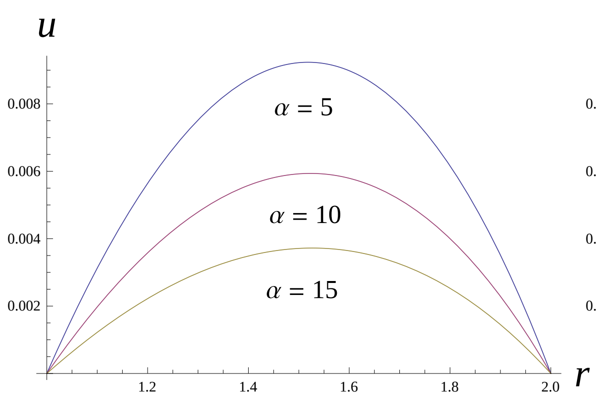

Fig. 2: Velocity profiles for permeability variation $k = k_0$ when $q = 2$, $M = 10$ for different values of α.

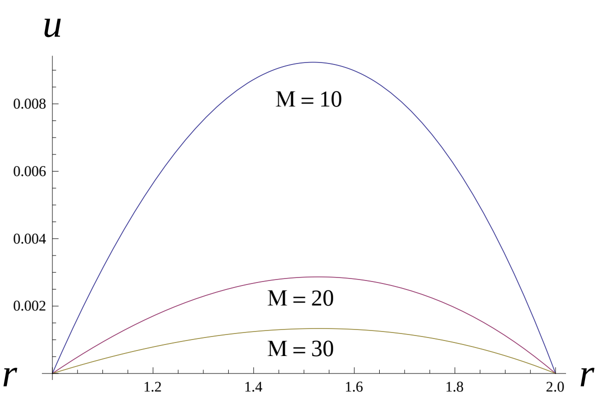

Fig. 3: Velocity profiles for permeability variation $k = k_0$ when $q = 2$ and $\alpha = 5$ for different values of M.

3.2 Case-II

when $k = k_o r$, i.e. permeability variation within the channel is linear, the governing equation of motion (2.4) is given by

$$r\frac{d^2u}{dr^2} + \frac{du}{dr} - (\alpha^2 + rM^2)u = -r \tag{3.10}$$

where $\alpha^2 = a^2/k_0$ is the porous media shape parameter. Using the Galerkin's method solution of Eq. (3.10) satisfying boundary conditions (3.1) is taken as k

$$u = C_3(r-1)(r-q) + C_4(r-1)^2(r-q) \tag{3.11}$$

where C_3 and C_4 are constants which are determined by Galerkin's method as
(3.12)

$$C_3 = \frac{-98(4+17q+9q^2) - 14(6-13q+8q^2-q^3)\alpha^2 - 7(3-8q^2+4q^3+q^4)M^2}{\Delta_2}$$

$$C_4 = \frac{490(1+q) - 98(q-1)^2\alpha^2 - 14(q-1)^2(q+1)M^2}{\Delta_2} \tag{3.13}$$

where,

$$\begin{aligned}\Delta_2 \;=\; & [-2\{7\alpha^2 + 4q^3M^2 + q^2(7\alpha^2 - 5M^2) + q(49 - 14\alpha^2 - 2M^2) + 3(7+M^2)\}^2 \\ & +7\{10 + 2\alpha^2 + (1+q^3)M^2 + q^2(2\alpha^2 - M^2) - q(-10 + 4\alpha^2 + M^2)\}\{28 + \\ & 8\alpha^2 + 3M^2 + 5q^3M^2 + q^2(8\alpha^2 - 7M^2) - q(-84 + 16\alpha^2 + M^2)\}]\end{aligned} \tag{3.14}$$

Dimensionless volumetric flow rate through the channel when permeability variation is $k = k_o r$ is obtained by using Eq.(3.7), with velocity u given by Eq.(3.11), as

$$Q = -\frac{\pi}{30}[5C_3(q-1)^2(q+1) + C_4(q-1)^3(3q+2)] \tag{3.15}$$

for $q = 2$, Q is

$$Q = -\frac{\pi}{30}(15C_3 + 8C_4) \tag{3.16}$$

where C_3 and C_4 are given by Eq. (3.12) and (3.13), respectively.

3.3 Case-III

when permeability variation within the channel is quadratic i.e. $k = k_o r^2$, the governing equation of motion (2.4) is given by

$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - (\alpha^2 + r^2M^2)u = -r^2 \tag{3.17}$$

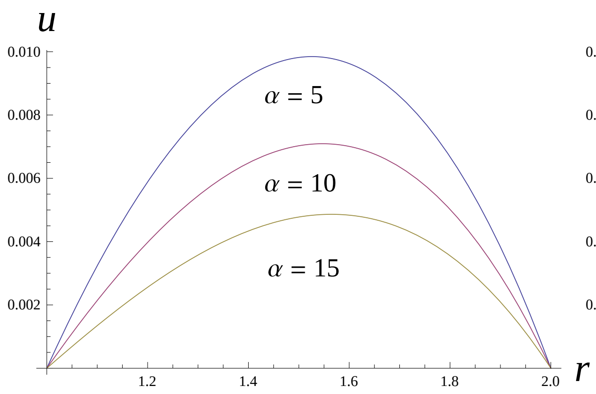

Fig. 4: Velocity profiles for permeability variation $k = k_0 r$ when $q = 2$, $M = 10$ for different values of α.

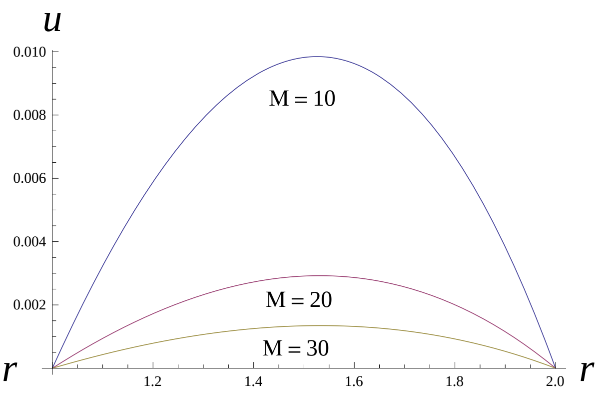

Fig. 5: Velocity profiles for permeability variation $k = k_0 r$ when $q = 2$ and $\alpha = 5$ for different values of M.

where $\alpha^2 = a^2/k_0$ is the porous media shape parameter. Using the Galerkin's method solution of Eq.(3.17) satisfying boundary conditions (3.1) is taken as

$$u = C_5(r-1)(r-q) + C_6(r-1)^2(r-q) \tag{3.18}$$

where C_5 and C_6 are constants which are determined by Galerkin's method as

$$C_5 = \frac{\begin{array}{c}-84(11+37q+86q^2+58q^3+18q^4) - 84(5-8q-q^2+6q^3-2q^4)\alpha^2 \\ -14(3+4q-4q^2-12q^3+5q^4+4q^5)\end{array}}{\Delta_3} \tag{3.19}$$

$$C_6 = \frac{588(1+q)[(1+3q-14q^2) - (-1+2q+14q^2)\alpha^2 + (q^2-1)^2M^2)]}{\Delta_3} \tag{3.20}$$

where,

$$\begin{aligned}\Delta_3 = {} & [2352 + 84\alpha^2 + 302M^2 + 5(1+q^8)M^4 + 2q^6M^2(151+26\alpha^2-8M^2) \\ & -16q^5M^2(-25+9\alpha^2+M^2) + 4\alpha^2(399+13M^2) - 4q^3\{-1785+84\alpha^4 \\ & +376M^2 + 4M^4 + 8\alpha^2(63+M^2)\} + 2q^2\{8148+252\alpha^4+25M^2-8M^4 \\ & +6\alpha^2(70+9M^2)\} + 2q^4\{1176+42\alpha^4+25M^2+27M^4+6\alpha^2(133+9M^2\} \\ & -4q\{84\alpha^4+36\alpha^2(14+M^2) - 5(357+20M^2)\}]\end{aligned} \tag{3.21}$$

Dimensionless volumetric flow rate through the channel when permeability variation is $k = k_o r^2$ is obtained by using Eq.(3.7), with velocity u given by Eq.(3.18), as

$$Q = -\frac{\pi}{30}[5C_5(q-1)^2(q+1) + C_6(q-1)^3(3q+2)] \tag{3.22}$$

for $q = 2$, Q is

$$Q = -\frac{\pi}{30}(15C_5 + 8C_6) \tag{3.23}$$

where C_5 and C_6 are given by Eq.s (3.19) and (3.20), respectively.

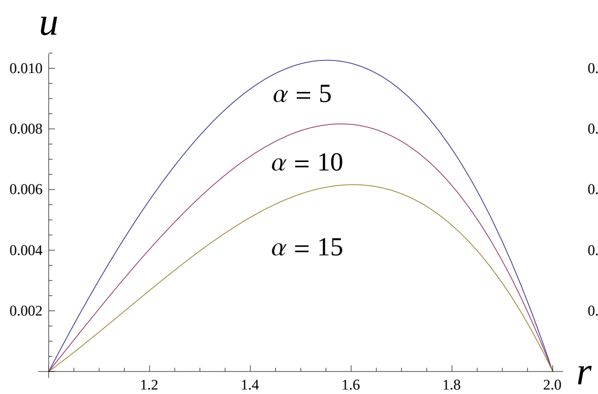

Fig. 6: Velocity profiles for permeability variation $k = k_0 r^2$ when $q = 2$, $M = 10$ for different values of α.

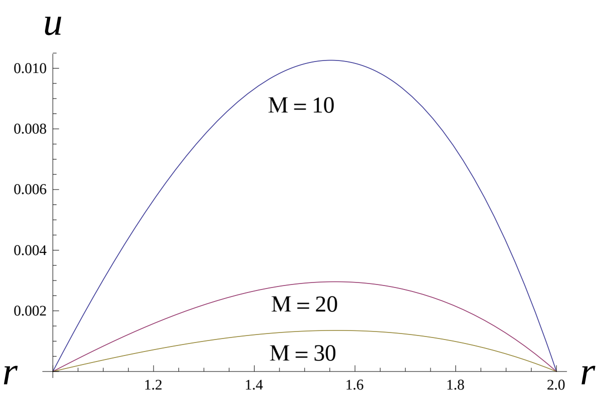

Fig. 7: Velocity profiles for permeability variation $k = k_0 r^2$ when $q = 2$ and $\alpha = 5$ for different values of M.

4 Discussion

Fig. 2, 4 and 6 represent the effects of porous media shape parameter α on the velocity for $M = 10$ and $q = 2$ when the permeability variation within the channel is $k = k_0$, $k = k_0 r$, and $k = k_o r^2$, respectively. We find that as the value of α increases, velocity decreases for all the cases of permeability variation. This happens due to the fact that an increase in α is caused by a decrease in permeability. Fig. 2 shows that when permeability of the medium within the channel is constant i.e. $k = k_0$, velocity profile is symmetrical with respect to the mid point between the annular gap. Fig. 4 and 6 reveal that velocity profile gets more and more asymmetric with the position of maximum velocity shifting towards the outer cylinder when permeability of the medium is variable i.e. $k = k_0 r$ and $k = k_0 r^2$. This is because the permeability of the channel is greater near the outer cylinder than near the inner cylinder. Fig. 3, 5 and 7 represents the effects of Hartmann number M on the fluid velocity for $\alpha = 5$ and $q = 2$ when the permeability variation within the channel is $k = k_0$, $k = k_0 r$, and $k = k_o r^2$, respectively. Figures reveal that as M increases, velocity u decreases for all the cases of permeability variation. This is because increase in the Hartmann number leads to an increase in the Lorentz force opposing the flow.

Fig. 8 and 10 shows the variation of fluid velocity u and rate of volume flow Q with α for fixed values of M and q. We find that as the value of α increases, u and Q decreases

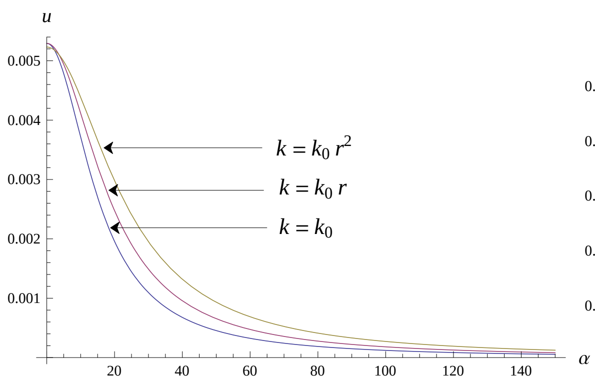

Fig. 8: Variation of velocity with α for different cases of permeability variation when $M = 15$, $q = 2$ and $r = 1.5$.

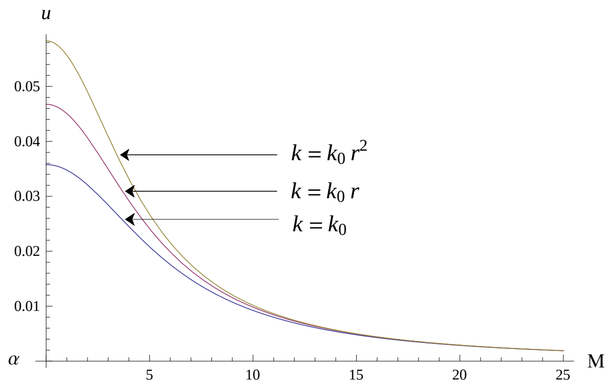

Fig. 9: Variation of velocity with M for different cases of permeability variation when $\alpha = 5$, $q = 2$ and $r = 1.5$.

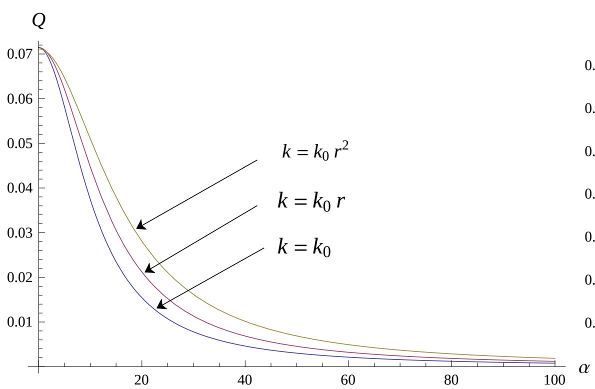

Fig. 10: Variation of rate of volume flow with α for different cases of permeability variation when $M = 10$ and $q = 2$.

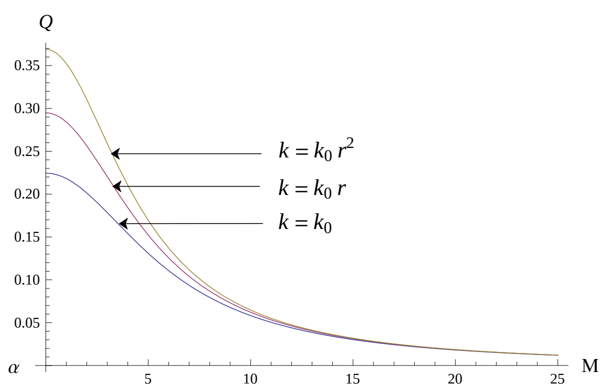

Fig. 11: Variation of rate of volume flow with M for different cases of permeability variation when $\alpha = 5$ and $q = 2$.

for all the cases of permeability variation. This is because an increase in α is caused by a decrease in permeability.

Fig. 9 and 11 represents the variation of fluid velocity u and rate of volume flow Q with M for fixed values of α and q. Figures reveal that as M increases, u and Q decreases for all the cases of permeability variation. This is because increase in the Hartmann number M leads to an increase in the Lorentz force opposing the flow.

5 Conclusion

Numerical solutions using Galerkin's method are obtained for the flow of viscous, incompressible, electrically conducting fluid through an annular porous channel in the presence of transverse magnetic field for three cases of permeability variations. It is observed that distribution of permeability in the porous channel and applied magnetic field affects the flow considerably. When permeability distribution in the channel is uniform i.e. $k = k_0$, velocity profile is symmetrical and when permeability varies according to the law $k = k_0 r$ and $k = k_o r^2$, asymmetry appears. We found that as α increases, fluid velocity and rate of volume flow decreases for all the cases of permeability variation. Effect of increase in Hartmann number (magnetic field) is to decrease the velocity and volume flow rate. This is because increase in the Hartmann number leads to an increase in the Lorentz force opposing the flow.

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