Slip and Chemical Reaction Effects On MHD Natural Convection Flow Past a Vertical Channel Under the Influence of Radiation and Heat Source Through Porous Medium

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Abstract

The goal of present article is to discuss the effects of heat and mass transfer with slip and chemical reaction on MHD free convection boundary layer flow past a vertical porous channel under the influence of heat source, viscous dissipation and constant suction with uniform magnetic field, which is applied normal to the surface. The governing boundary layer equations are formulated and transformed into a set of ordinary differential equations using non-dimensional parameters. The perturbation technique is employed to solve the partial differential equations to examination the velocity profile, derive the skin friction, heat and mass transfer rates and other indices. The effect of several physical parameters affecting the velocity, temperature, and concentration has been explained and discussed graphically.

Subject class [2010]:76S05, 76Sxx, 35Q35, 65L10
Keywords: Slip effect; MHD; Heat source; Porous medium; Viscous dissipation; Chemical reaction.

1 Introduction

There are many transport processes in nature and in many industries where flows with natural convection currents due to the temperature difference are affected by the differences in concentration or material constitutions. In nature, flows are existing, which are generated not only by the temperature differences but also by concentration differences as a result the rate of heat transfer takes place. Various transport processes that exist in industries in which the simultaneous phenomenon of heat and mass transfer that takes place, as a outcome of combined buoyancy effect of thermal diffusion and diffusion thermo chemical species. Such type of phenomenon frequently exists in chemically processed industries such as polymer production and food processing [1]. Natural convective flows in different geometries are of large interest in a number of industrial applications for example geothermal systems, fiber and granular insulation. Beyond that convective flow along porous media has applications in thermal energy storage, oil extraction, geothermal energy recovery and flow through filtering devices. These days magnetohydrodynamics (MHD) is very much attracting the notice of the many researchers due to its applications in engineering and geophysics. Ibrahim *et al.* [2] have investigated the study the mixed convection on MHD flow of Casson fluid over a nonlinearly permeable stretching sheet with thermal radiation, viscous dissipation, heat source/sink, chemical reaction and suction. Lopez *et al.* [3] have examined the heat transfer and entropy generation in a MHD flow of nanofluid through a porous vertical microchannel in the presence of nonlinear radiative heat flux. Narasu *et al.* [4] have analyzed an analytical solution of free convective unsteady fluid flow under the influence of thermal diffusion and chemical reaction over a vertical permeable plate under the influence of heat source dependent in slip flow regime Sahoo [5] has investigated an Unsteady flow of an electically conducting and incompressible viscoelastic liquid of the Walter model with simultaneous heat and mass transfer near an oscillating porous plate in slip flow regime in the presence of a transverse magnetic field of uniform strength. Ayano and Demeke [6] have studied the magneto micropolar fluid over a stretched semi-infinite vertical and porous surface under the influence of heat absorption, Hall and ion-slip effects, first-order chemical reaction, and radiation effects. Raju *et al.* [7] have investigated the regular perturbation solution of MHD natural convective, dissipative boundary layer flow past a porous vertical sheet in the presence of chemical reaction, constant suction and thermal radiation under the influence of uniform magnetic field which is applied normal to the surface. Raju and varma [8], Magyari et al. [9], Ravikumar *et al.* [10], Chamkha [11], Hayat *et al.* [12], Makinde and Mhone [13] etc. are the few to make a mention who contributed in this area. When high temperature acquired in some engineering devices, for an example, gas can be ionized and so becomes a very good electrical conductor. The ionized gas or plasma interacts along the magnetic and alters heat transfer and friction characteristic. On account of, some fluids can also release and occupy thermal radiation, hence it is of notice to study the result of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also useful of emitting and absorbing thermal radiation. This is of so interest because heat transfer by thermal radiation is becoming of very high importance when we are perturbed along higher operating temperatures and space applications. Soundalgekar and Takhar [14] have investigated the result of radiation on the free convection flow of a gas past a semi-infinite plate applying the Cogley Vincentine Gilles equilibrium model. For the same gas Takhar *et al.* [15] have studied the radiation effect on the MHD free convection flow past a semi-infinite vertical plate. Later, Hossain *et al.* [16] have investigated the effect of radiation on free convection from a porous vertical plate. Muthucumarswamy and Kumar [17] have considered the thermal radiation effects on moving infinite vertical plate in the presence of variable temperature. Mazumdar and Deka [18] have investigated MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. Mass transfer and radiation outcomes on a natural convection flow with a porous medium bounded by a vertical surface has been examined by Raju *et al.* [19] Manivannan *et al.* [20] have studied radiation and chemical reaction effects on isothermal vertical oscillating plate along with variable mass diffusion.

2 Formulation of the problem

This study assumes a viscous, incompressible, electrically conducting and radiating fluid passing through a porous medium with heat source and slip velocity occupying a semi-infinite region of the space bounded by a vertical infinite surface. The x^* axis is taken along the surface in an vertical direction and the y^* axis is in a normal position to it. A uniform magnetic field B_0 is applied in a direction perpendicular to the surface. The properties of the fluid are supposed to be constant with the exception of the density in the body force term. In addition, a chemically reactive species is assumed to be emitted from the vertical surface into a hydrodynamic flow field. It diffuses into the fluid, where it undergoes a homogenous chemical reaction. The reaction is assumed to take place entirely in the stream. Then the fully developed flow under the above assumed conditions through a highly porous medium is governed by the following set of equations:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{2.1}$$

$$v^*\frac{\partial u^*}{\partial y^*} = \nu\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T(T^* - T^*_\infty) + g\beta_C(C^* - C^*_\infty) - \frac{\sigma B_0^2 u^*}{\rho} - \frac{\nu}{k_p}u^* \tag{2.2}$$

$$v^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\nu}{C_p}(\frac{\partial u^*}{\partial y^*})^2 - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} + Q^*(T^* - T^*_\infty) \tag{2.3}$$

$$v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - k_p C^* \tag{2.4}$$

In this work, we have considered the level species concentration is very low; therefore, the heat generated due to chemical reaction is neglected. The boundary conditions are given as follows:

at

$$y^* = 0: \qquad u^* = L_1\frac{\partial u^*}{\partial y^*}, \qquad T^* = T_w, \qquad C^* = C_w \tag{2.5}$$

at

$$y^* \to \infty: \qquad u^* \to 0, \qquad T^* \to T_\infty, \qquad C^* \to C_\infty \tag{2.6}$$

From Eq. (1), we have

$$v^* = -v_0 = constant \tag{2.7}$$

In an attempt Cogley *et al.* [21] proved that the optically thick limit for a non-gray gas near equilibrium as presented below:

$$\frac{\partial q_r}{\partial y^*} = 4(T^* - T^*_\infty)\int_0^\infty K_{\lambda w}\left(\frac{de_{b\lambda}}{dT^*}\right)d\lambda = 4I_1(T^* - T^*_\infty) \tag{2.8}$$

Now introducing the following non-dimensional quantities,

$$\begin{aligned}
&u = \frac{u^*}{v_0}, y = \frac{v_0 y^*}{\vartheta}, M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \alpha = \frac{Q\nu^2}{k v_0^2}, F = \frac{4I_1\nu}{k v_0^2}\\
&\Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D}, k_0 = \frac{\nu k_c}{v_0^2}, k = \frac{k_p v_0^2}{\nu^2}, E = \frac{v_0^2}{C_p(T_w - T^*_\infty)}\\
&Gr = \frac{\nu g \beta_T (T_w - T^*_\infty)}{v_0^3}, Gm = \frac{\nu g \beta C(C_w - C^*_\infty)}{v_0^3}, N = \frac{L_1 v_0}{\nu}\\
&\theta = \frac{(T^* - T^*_\infty)}{(T_w - T^*_\infty)}, C = \frac{(C^* - C^*_\infty)}{(C_w - C^*_\infty)}
\end{aligned} \tag{2.9}$$

The non-dimensional form of the governing Eqs. (2.2) - (2.4) reduce to

$$u^{''} + u^{'} = Gr\theta - GmC + M_1 u \tag{2.10}$$

$$\theta^{''} + \Pr\theta^{'} = -\Pr E u^{'2} + (F + \alpha)\theta \tag{2.11}$$

$$C^{''} + ScC^{'} = k_0 ScC \tag{2.12}$$

The corresponding boundary conditions are given by

$$\begin{aligned}
&y = 0: \quad u = N\frac{\partial u}{\partial y}, \quad \theta = 1, \quad C = 1\\
&y \to \infty: \quad u \to 0, \quad \theta \to 0, \quad C \to 0
\end{aligned} \tag{2.13}$$

3 Method of solution

Eqs. (2.10) to (2.12) represent a set of partial differential equations that can be solved in closed form. However, these equations can be solved analytically. In order to solve the coupled nonlinear system of Eqs. (2.10) to (2.12) with the boundary conditions (2.13), the following simple perturbation is used. The governing Eqs.(2.10) to (2.12) are expanded in powers of Eckert number $E(<< 1)$

$$u = u_0 + Eu_1 + o(E^2) \tag{3.1}$$

$$\theta = \theta_0 + E\theta_1 + o(E^2) \tag{3.2}$$

$$C = C_0 + EC_1 + o(E^2) \tag{3.3}$$

Substituting the above expressions in Eqs. (2.10) to (2.12) and equating the coefficients of the like powers of E, and neglecting the terms of higher order, the following equations are obtained

Zeroth order equations are:

$$u_0^{''} + u_0^{'} = Gr\theta_0 - GmC_0 + M_1u_0 \tag{3.4}$$

$$\theta_0^{''} + \Pr\theta_0^{'} = -\Pr Eu_0^{'2} + (F+\alpha)\theta_0 \tag{3.5}$$

$$C_0^{''} + ScC_0^{'} = k_0ScC_0 \tag{3.6}$$

and the first order equations are:

$$u_1^{''} + u_1^{'} = Gr\theta_1 - GmC_1 + M_1u_1 \tag{3.7}$$

$$\theta_1^{''} + \Pr\theta_1^{'} = -\Pr Eu_1^{'2} + (F+\alpha)\theta_1 \tag{3.8}$$

$$C_1^{''} + ScC_1^{'} = k_0ScC_1 \tag{3.9}$$

The corresponding boundary conditions are:

$$\begin{aligned} y &= 0: \quad u_0 = N\frac{\partial u_0}{\partial y},\ u_1 = N\frac{\partial u_1}{\partial y},\ \ \theta_0 = 1,\ \theta_1 = 0\ ,\ C_0 = 1, C_1 = 0 \\ y &\to \infty: \quad u_0 \to 0,\ \ u_1 \to 0\ ,\ \ \theta_0 \to 0,\ \theta_1 \to 0,\ \ C_0 \to 0,\ \ C_1 \to 0 \end{aligned} \tag{3.10}$$

Solving Eqs. (3.4) to (3.9) under the boundary conditions (3.10), we get

$$u_0 = m_{11}e^{m_6y} + m_{12}e^{m_2y} + m_{13}e^{m_4y} \tag{3.11}$$

$$u_1 = m_{50}e^{m_6 y} + m_{37}e^{m_{30}y} + m_{38}e^{m_{20}y} + m_{39}e^{m_{22}y} + m_{40}e^{m_{24}y} + m_{41}e^{m_{26}y} + m_{42}e^{m_{28}y} \tag{3.12}$$

$$\theta_0 = e^{m_2 y} \tag{3.13}$$

$$\theta_1 = m_{36}e^{m30y} + m_{31}e^{m_{20}y} + m_{32}e^{m_{22}y} + m_{33}e^{m_{24}y} + m_{34}e^{m_{26}y} + m_{35}e^{m_{28}y} \tag{3.14}$$

$$C_0 = e^{m_4 y} \tag{3.15}$$

$$C_1 = 0 \tag{3.16}$$

Substituting the solutions of equations (3.11) to (3.16) in (3.1) to (3.3), we obtain

$$u = m_{11}e^{m_6 y} + m_{12}e^{m_2 y} + m_{13}e^{m_4 y} + E(m_{50}e^{m_6 y} + m_{37}e^{m_{30}y} + m_{38}e^{m_{20}y} + m_{39}e^{m_{22}y} + m_{40}e^{m_{24}y} + m_{41}e^{m_{26}y} + m_{42}e^{m_{28}y}) \tag{3.17}$$

$$\theta = e^{m_2 y} + E(m_{36}e^{m30y} + m_{31}e^{m_{20}y} + m_{32}e^{m_{22}y} + m_{33}e^{m_{24}y} + m_{34}e^{m_{26}y} + m_{35}e^{m_{28}y}) \tag{3.18}$$

$$C = e^{m_4 y} \tag{3.19}$$

The skin friction(τ) , which is defined as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = m_6 m_{11} + m_2 m_{12} + m_4 m_{13} + E(m_6 m_{50} + m_{30}m_{37} + m_{20}m_{32} + m_{22}m_{39} + m_{24}m_{40} + m_{26}m_{41} + m_{28}m_{42}) \tag{3.20}$$

The Nusselt number (Nu) , which decides the rate of heat transfer as given below

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -m_2 - E(m_{26}m_{30} + m_{20}m_{31} + m_{22}m_{32} + m_{24}m_{33} + m_{26}m_{34} + m_{28}m_{35}) \tag{3.21}$$

The Sherwood number (Sh), which decides the rate of mass transfer as given below

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -m_4 \tag{3.22}$$

4 Results and discussion

In this section, representative numerical results are computed for velocity, temperature, concentration, skin friction coefficient, Nusselt number and Sherwood number for different values of various parameters. Fig. 1 shows that when the value of chemical reaction parameter increases, the corresponding value of velocity also increases. Fig. 2 shows that when the permeability of porous medium is increased, the corresponding value of velocity also increases. It can be observed from Fig. 3 that if the value of radiation parameter is increased, the corresponding value of velocity decreases. It can be seen from Fig. 4 that the value of slip parameter is increased, the corresponding value of skin friction coefficient decreases. From Fig. 5, we can see that increase in the value of radiation parameter, leads to a decrease in the corresponding value of temperature. Fig. 6 shows that increase in the value of Pr leads to a decrease in the the corresponding value of temperature, and increase in the value of M, leads to a decrease in the corresponding value of the temperature. It can be seen from Fig. 7 that increase in the value of radiation parameter leads to a increase in the corresponding value of Nusselt number. It can be noticed from Fig. 8 that increase in the value of chemical reaction parameter leads to a decrease in the corresponding value of concentration. Finally, from Fig. 9, we see that increase in the values of Sc and k_0, leads to a increase in the corresponding value of Sherwood number.

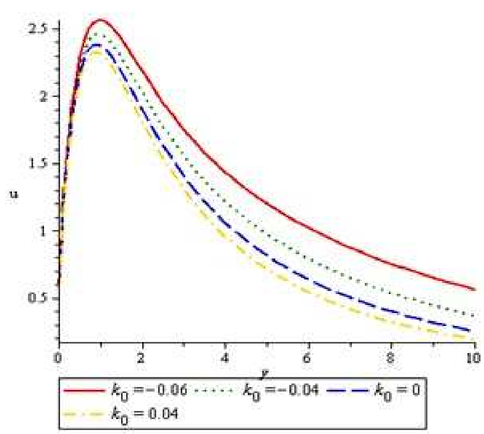

Fig. 1: The Influence of k_0 on the velocity descriptions for $Pr = 0.71, N = 0.1, k = 1, Gr = 5, \alpha = 0, Gm = 5, M = 1, F = 0.05$ and $Sc = 0.22$.

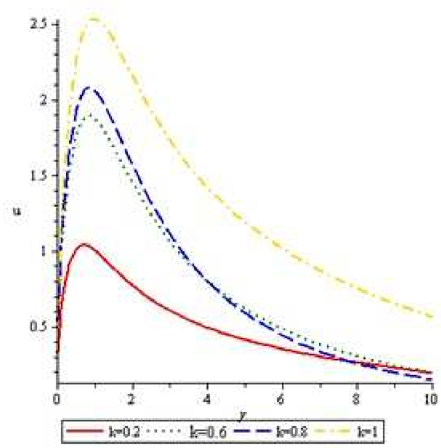

Fig. 2: The Influence of k on the velocity descriptions for $Pr = 0.71, N = 0.1, k_0 = -0.05, Gr = 5, \alpha = 0.02, Gm = 5, M = 1, F = 0.06$ and $Sc = 0.22$

5 Conclusion

In this paper, the author has analyzed the effects of velocity slip, chemical reaction, radiation on MHD natural convection flow through a porous medium bounded by a vertical surface. The numerical computation is conducted by using Maple. The effect of various

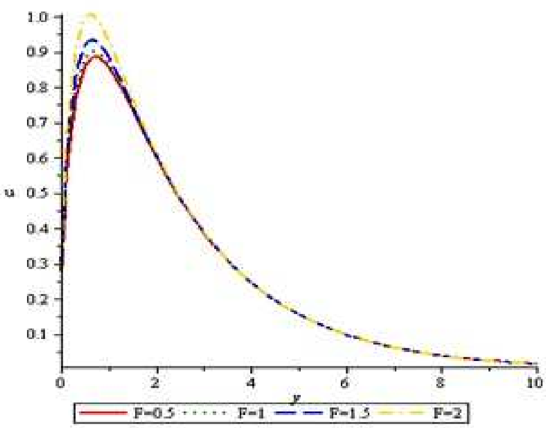

Fig. 3: The Influence of F on the velocity descriptions for $Pr = 0.71, N = 0.1, k_0 = 0.05,$
$Gr = 5, \alpha = 0.02, Gm = 5, M = 2, k = 1$ and $Sc = 0.22$.

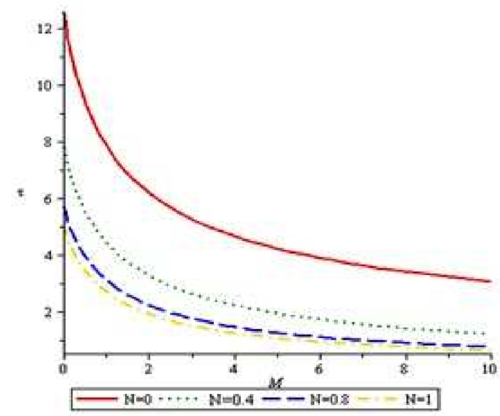

Fig. 4: The Influence of k on the velocity descriptions for $Pr = 0.71, F = 0.06, k_0 = -0.05, Gr = 5, \alpha = 0.02, Gm = 5, M = 2, k = 1.5$ and $Sc = 0.22$

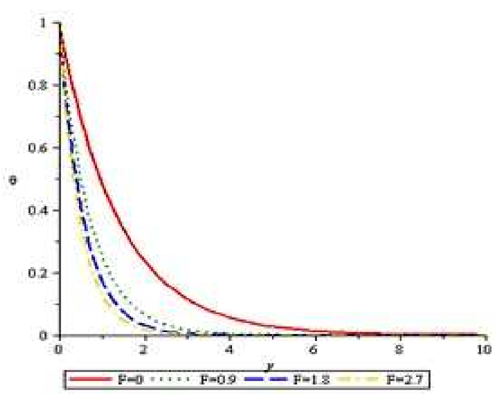

Fig. 5: The Influence of F on the temperature descriptions for $Pr = 0.71, Gr = 5, k_0 = 1.5, \alpha = 0.01, Gm = 5, M = 1, k = 1.5$ and $Sc = 0.22$.

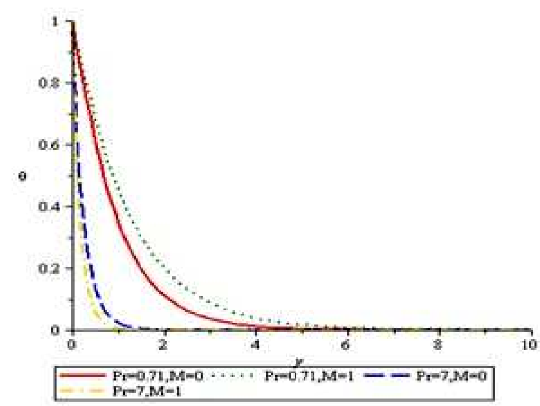

Fig. 6: The Influence of Pr and M on the temperature descriptions for $F = 0.5, k_0 = 1, Gr = 10, \alpha = 0.01, Gm = 10, k = 1$ and $Sc = 0.22$

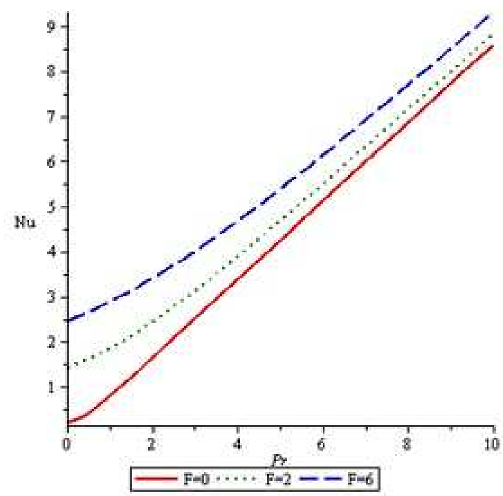

Fig. 7: The Influence of F on the Nusselt number descriptions for $\alpha = 0.05, Gr = 10, k_0 = 1, Gm = 10, M = 1, k = 1$ and $Sc = 0.22$.

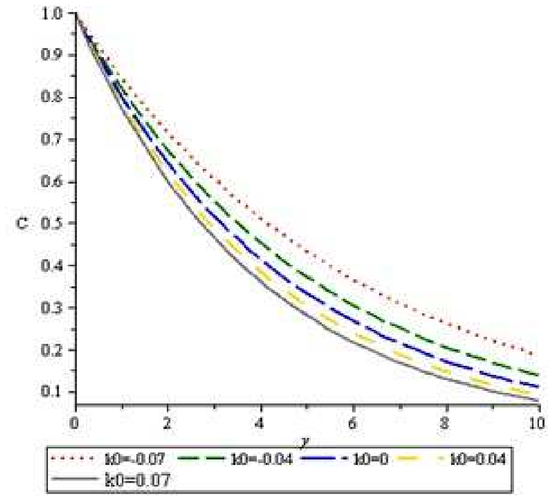

Fig. 8: The Influence of k_0 on concentration descriptions for $Sc = 0.22$

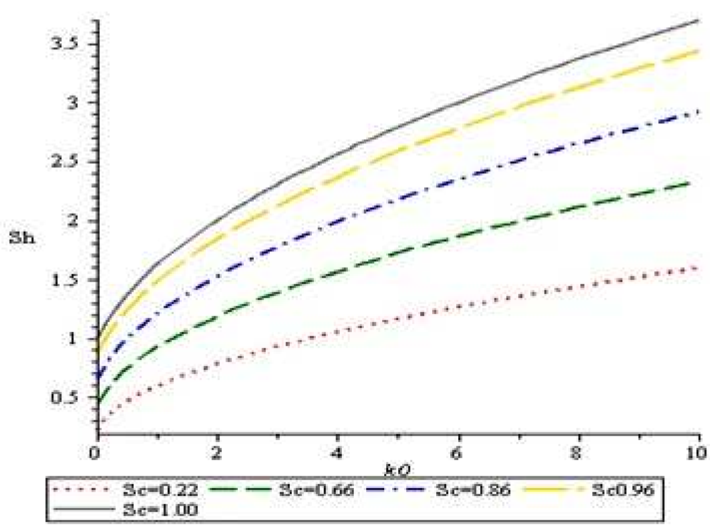

Fig. 9: The Influence of Sc on Sherwood number.

parameters on velocity, temperature, concentration, Skin friction coefficient, Nusselt number and Sherwood number for different values of various parameters are shown graphically.

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