Rotation of a Spherical Particle with Porous Core in a Concentric Spherical Cavity

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Abstract

An investigation for the slow rotational motion of a spherical particle composed of a porous core of constant permeability k_1 and a porous concentric shell with constant permeability k_2, enclosed in a concentric cavity filled with viscous fluid. Stokes and Brinkman equations are employed to obtain the flow fields in clear and porous fluid regions respectively. The torque on the composite sphere is evaluated and wall correction factor is also obtained. Graphs are drawn to show effects of various parameters on wall correction factor and discussed.

Subject class [2010]:
Keywords: Composite sphere, porous core, spherical cavity, permeability, torque, wall correction factor.

1 INTRODUCTION

The problem addressed in this paper is to obtain the torque and wall effects on the slow rotational motion of a spherical particle composed of a porous core of constant permeability k_1 covered by a concentric shell of permeability k_2, enclosed in a concentric cavity filled with viscous fluid. The flow inside cavity wall and outside spherical particle is governed by the Stokes' equation. The flow within the porous shell and porous core are governed by Brinkman equations. Boundary conditions e.g. no slip and matching conditions are employed on flow governing equations to obtain solution of the problem. Our objective here is to determine the torque exerted on the composite spherical particle. The wall correction factor is evaluated and its variation is studied numerically. Also, we study the variation of angular velocity of fluid particle in different region inside the cavity with respect to radial distance.
The problem has many applications in nature e.g. various regions inside the earth, transport phenomena in environment, flotation, sedimentation, electroph oresis, spray drying, agglomeration and motion of blood cells in an artery.

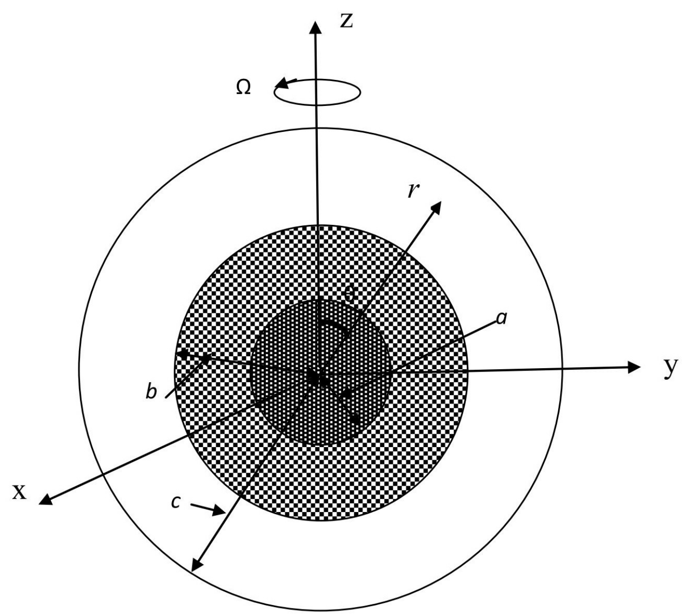

Fig. 1: Figure for the rotation of spherical cavity with enclosing concentric composite sphere

2 MATHEMATICAL FORMULATION

Referring to Fig.1, consider the slow rotational motion of a non-deformable composite sphere of radius b, consisting of a homogeneous porous core of radius a and permeability k_1 covered by a homogeneous porous shell of thickness $b-a$ with permeability k_2 in a concentric spherical cavity of radius c filled with an incompressible Newtonian fluid. We shall suppose that spherical cavity rotates with constant angular velocity Ω in the positive z direction. Let us introduce a spherical co-ordinate system (r, θ, ϕ) with the origin located at the cavity center and the line $\theta = 0$ (z-axis) as the axis of rotation. The Reynolds number is assumed to be sufficiently small so that the inertial terms in the fluid momentum equation can be neglected, in comparison with the viscous terms. The porous core region $(r \leqslant a)$, the porous shell region $(a \leqslant r \leqslant b)$, and the region outside composite sphere and inside spherical cavity $(b \leqslant r \leqslant c)$, are denoted as regions I, II and III respectively. Because of axial symmetry only the non-zero azimuthal components $v_{\phi i}(i = 1, 2, 3)$ of velocities appear in region I, II and III respectively. The fluid flow in regions I and II is governed by Brinkman equation and fluid flow in region III is governed by Stokes equation. The equation of continuity is satisfied in all three regions automatically. The relevant equations are expressible as:
Brinkman:

$$\frac{\partial}{\partial r}(r^2 \frac{\partial v_{\phi 1}}{\partial r}) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \frac{\partial v_{\phi 1}}{\partial \theta}) - (\frac{1}{\sin^2\theta} + \frac{r^2}{k_1})v_{\phi 1} = 0 (r \leqslant a) \tag{2.1}$$

$$\frac{\partial}{\partial r}(r^2 \frac{\partial v_{\phi 2}}{\partial r}) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \frac{\partial v_{\phi 2}}{\partial \theta}) - (\frac{1}{\sin^2\theta} + \frac{r^2}{k_2})v_{\phi 2} = 0 (a \leqslant r \leqslant b) \tag{2.2}$$

Stokes:

$$\frac{\partial}{\partial r}(r^2 \frac{\partial v_{\phi 3}}{\partial r}) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \frac{\partial v_{\phi 3}}{\partial \theta}) - \frac{v_{\phi 3}}{\sin^2\theta} = 0 (b \leqslant r \leqslant c) \tag{2.3}$$

The subscripts 1, 2 and 3 refers to the physical quantities in regions I, II and III respectively. Here, we have assumed that the fluid has the same viscosity inside and outside the composite sphere [4]. The effective viscosity and fluid viscosity are, in general different. For high porosity material, Brinkman assumed that both are same [1,2].

3 BOUNDARY CONDITIONS

The following boundary conditions are used to analyze the flow in the three regions: [5 and 9 - 11]
The no singularity condition is imposed on the center of porous core $(r = 0)$

$$v_{\phi 1} = 0 \tag{3.1}$$

The two matching conditions are imposed on the surface of porous core (r=a)

$$v_{\phi 1} = v_{\phi 2} \tag{3.2}$$

$$\tau_{r\phi 1} = \tau_{r\phi 2} \tag{3.3}$$

The boundary conditions at the outer surface of the porous surface layer ($r = b$) due to the continuity of velocity and stress components, which is physically realistic and mathematically consistent for the present problem

$$v_{\phi 2} = v_{\phi 3} \tag{3.4}$$

$$\tau_{r\phi 2} = \tau_{r\phi 3} \tag{3.5}$$

The no-slip boundary condition at the spherical cavity surface ($r = c$) is

$$v_{\phi 3} = \Omega c \sin\theta \tag{3.6}$$

Here, $\tau_{r\phi i}$ are the shear stresses for the fluid flow relevant to the particle surfaces. Due to above boundary conditions and the flow is axisymmetric, it is clear radial and polar velocity components are zero and the pressure is constant everywhere.

4 SOLUTION OF THE PROBLEM AND DE TERMINATION OF ARBITRARY CONSTA NTS

The boundary condition (9) suggest that solution of Equations (1) to (3) of the form $v_{\phi i} = f_i(r)\sin\theta (i = 1, 2, 3)$ and then Equation (1) to (3), provide

$$r^2 f_1^{''}(r) + 2r f_1^{'}(r) - (2 + \frac{r^2}{k_1}) f_1(r) = 0 \tag{4.1}$$

$$r^2 f_2^{''}(r) + 2r f_2^{'}(r) - (2 + \frac{r^2}{k_2}) f_2(r) = 0 \tag{4.2}$$

$$r^2 f_3^{''}(r) + 2r f_3^{'}(r) - 2 f_3(r) = 0 \tag{4.3}$$

The boundary conditions (4) to (9) reduce to

$$f_1(0) = 0 \tag{4.4}$$

$$f_1(a) = f_2(a) \tag{4.5}$$

$$f_1^{'}(a) = f_2^{'}(a) \tag{4.6}$$

$$f_2(b) = f_3(b) \tag{4.7}$$

$$f_2'(b) = f_3'(b) \tag{4.8}$$

$$f_3(c) = \Omega c \tag{4.9}$$

The general solutions of Equations (10) to (12) with satisfying boundary conditions (13) to (18) are of the form

$$f_1(r) = A(\frac{\sqrt{k_1}}{r}\cosh\frac{r}{\sqrt{k_1}} - \frac{k_1}{r^2}\sinh\frac{r}{\sqrt{k_1}}) \tag{4.10}$$

$$f_2(r) = (B(\frac{\sqrt{k_2}}{r}\cosh\frac{r}{\sqrt{k_2}} - \frac{k_2}{r^2}\sinh\frac{r}{\sqrt{k_2}}) + C(\frac{\sqrt{k_2}}{r}\sinh\frac{r}{\sqrt{k_2}} - \frac{k_2}{r^2}\cosh\frac{r}{\sqrt{k_2}}) \tag{4.11}$$

$$f_3(r) = \frac{D}{r^2} + Er \tag{4.12}$$

The constants A, B, C, D and E are determined from equations (14)-(18) as follows

$$\begin{aligned} A &= (-3abc^3\Omega\sqrt{k_2})/((b^3-c^3)\sinh((a-b)/\sqrt{k_2})(\sqrt{k_1}\sinh(a/\sqrt{k_1}) \\ &\quad -a\cosh(a/\sqrt{k_1}))\sqrt{k_1} + \cosh((a-b)/\sqrt{k_2})((a(b^3-c^3)+3b^2k_1) \\ &\quad \sinh(a/\sqrt{k_1}) - 3ab^2\cosh(a/\sqrt{k_1}))\sqrt{k_1})\sqrt{k_2} + \sinh((a-b)/\sqrt{k_2}) \\ &\quad ((3ab^2-b^3+c^3+3bk_1)\sinh(a/\sqrt{k_1}) - 3ab\sqrt{k_1}\cosh(a/\sqrt{k_1}))k_2 \\ &\quad +3(a-b)b\cosh((a-b)/\sqrt{k_2})\sinh(a/\sqrt{k_1}){k_2}^{3/2} - 3b{k_2}^2 \\ &\quad \sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2})) \end{aligned} \tag{4.13}$$

$$\begin{aligned} B &= -(3bc^3\Omega(a\sqrt{k_2}\sinh(a/\sqrt{k_1})\sinh(a/\sqrt{k_2}) - \cosh(a/\sqrt{k_2})(a\sqrt{k_1} \\ &\quad \cosh(a/\sqrt{k_1}) + (k_2-k_1)\sinh(a/\sqrt{k_1}))))/((b^3-c^3)\sinh((a-b)/ \\ &\quad \sqrt{k_2})(a\cosh(a/\sqrt{k_1}) - \sqrt{k_1}\sinh(a/\sqrt{k_1}))\sqrt{k_1} + \cosh((a-b)/ \\ &\quad \sqrt{k_2})(3ab^2\sqrt{k_1}\cosh(a/\sqrt{k_1}) + (a(c^3-b^3)-3b^2k_1)\sinh(a/\sqrt{k_1})) \\ &\quad \sqrt{k_2} + \sinh((a-b)/\sqrt{k_2})(3ab\sqrt{k_1}\cosh(a/\sqrt{k_1}) - (3ab^2-b^3+ \\ &\quad c^3+3bk_1)\sinh(a/\sqrt{k_1}))k_2 - 3b(a-b){k_2}^{3/2}\cosh((a-b)/\sqrt{k_2}) \\ &\quad \sinh(a/\sqrt{k_1}) + 3b{k_2}^2\sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2})) \end{aligned} \tag{4.14}$$

$$\begin{aligned} C &= -(3bc^3\Omega(-a\sqrt{k_2}\cosh(a/\sqrt{k_2})\sinh(a/\sqrt{k_1}) + \sinh(a/\sqrt{k_2})(a\sqrt{k_1} \\ &\quad \cosh(a/\sqrt{k_1}) + (k_2-k_1)\sinh(a/\sqrt{k_1}))))/((b^3-c^3)\sinh((a-b)/ \\ &\quad \sqrt{k_2})(a\cosh(a/\sqrt{k_1}) - \sqrt{k_1}\sinh(a/\sqrt{k_1}))\sqrt{k_1} + \cosh((a-b)/ \\ &\quad \sqrt{k_2})(3ab^2\sqrt{k_1}\cosh(a/\sqrt{k_1}) + (a(c^3-b^3)-3b^2k_1)\sinh(a/\sqrt{k_1})) \\ &\quad \sqrt{k_2} + \sinh((a-b)/\sqrt{k_2})(3ab\sqrt{k_1}\cosh(a/\sqrt{k_1}) - (3ab^2-b^3+ \\ &\quad c^3+3bk_1)\sinh(a/\sqrt{k_1}))k_2 - 3b(a-b){k_2}^{3/2}\cosh((a-b)/\sqrt{k_2}) \\ &\quad \sinh(a/\sqrt{k_1}) + 3b{k_2}^2\sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2})) \end{aligned} \tag{4.15}$$

$$
\begin{aligned}
D \;=\; & -(bc^3\Omega(b^2\sinh((a-b)/\sqrt{k_2})(a\cosh(a/\sqrt{k_1})-\sqrt{k_1}\sinh(a/\sqrt{k_1})) \\
& \sqrt{k_1}-b\cosh((a-b)/\sqrt{k_2})((ab+3k_1)\sinh(a/\sqrt{k_1})-3a\sqrt{k_1}\cosh(\\
& a/\sqrt{k_1}))\sqrt{k_2}+\sinh((a-b)/\sqrt{k_2}))((b(b-3a)-3k_1)\sinh(a/\sqrt{k_1})+ \\
& 3a\sqrt{k_1}\cosh(a/\sqrt{k_1}))k_2+3(b-a){k_2}^{3/2}\cosh((a-b)/\sqrt{k_2})\sinh(a/ \\
& \sqrt{k_1})+3{k_2}^2\sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2}))))/((b^3-c^3)\sinh((a- \\
& -b)/\sqrt{k_2})(\sqrt{k_1}\sinh(a/\sqrt{k_1})-a\cosh(a/\sqrt{k_1}))\sqrt{k_1}+\cosh((a- \\
& b)/\sqrt{k_2})((a(b^3-c^3)+3b^2k_1)\sinh(a/\sqrt{k_1})-3ab^2\sqrt{k_1}\cosh(a/\sqrt{k_1})) \\
& \sqrt{k_2}+k_2\sinh((a-b)/\sqrt{k_2})((3ab^2-b^3+c^3+3bk_1)\sinh(a/\sqrt{k_1}) \\
& -3ab\sqrt{k_1}\cosh(a/\sqrt{k_1}))+3b(a-b){k_2}^{3/2}\cosh((a-b)/\sqrt{k_2})\sinh(a/ \\
& \sqrt{k_1})-3b{k_2}^2\sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2}))
\end{aligned} \tag{4.16}
$$

$$
\begin{aligned}
E \;=\; & -(c^3\Omega(\sinh((a-b)/\sqrt{k_2})(-a\cosh(a/\sqrt{k_1})+\sqrt{k_1}\sinh(a/ \\
& \sqrt{k_1}))\sqrt{k_1}+a\sqrt{k_2}\cosh((a-b)/\sqrt{k_2})\sinh(a/\sqrt{k_1})-k_2 \\
& \sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2})))/((b^3-c^3)\sinh((a-b)/\sqrt{k_2}) \\
& (\sqrt{k_1}\sinh(a/\sqrt{k_1})-a\cosh(a/\sqrt{k_1}))\sqrt{k_1}+\cosh((a-b)/\sqrt{k_2}) \\
& ((a(b^3-c^3)+3b^2k_1)\sinh(a/\sqrt{k_1})-3ab^2\sqrt{k_1}\cosh(a/\sqrt{k_1})) \\
& \sqrt{k_2}+k_2\sinh((a-b)/\sqrt{k_2})((3ab^2-b^3+c^3+3bk_1)\sinh(a/\sqrt{k_1}) \\
& -3ab\sqrt{k_1}\cosh(a/\sqrt{k_1}))+3b(a-b){k_2}^{3/2}\cosh((a-b)/\sqrt{k_2}) \\
& \sinh(a/\sqrt{k_1})-3b{k_2}^2\sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2}))
\end{aligned} \tag{4.17}
$$

5 EVALUATION OF TORQUE ON THE COMPOSITE SPHERE

Evaluation of torque on composite sphere is important in the applications of the flow problem we are investigating. Torque on the sphere is the couple that tends to rotate it. The torque now evaluated by integrating in moment of tangential stress as follows

$$T = 2\pi\int_0^\pi r^3\tau_{r\phi(3)}|_{r=b}\sin^2\theta \tag{5.1}$$

So, we get

$$T = -8\pi\eta D \tag{5.2}$$

where D is given by equation (25).

6 SPECIAL CASES

(i) When $(b/c) \to 0$ then by equation (27), we obtained the torque on the particle in an unbounded fluid as follows

$$T^{\infty} = -8\pi\eta D^{\infty} \tag{6.1}$$

where

$$\begin{aligned} D^{\infty} \;=\; & (b\Omega(b^2 \sinh((a-b)/\sqrt{k_2})(a\cosh(a/\sqrt{k_1}) - \sqrt{k_1}\sinh(a/\sqrt{k_1})) \\ & \sqrt{k_1} - b\sqrt{k_2}\cosh((a-b)/\sqrt{k_2})(-3a\sqrt{k_1}\cosh(a/\sqrt{k_1}) + (ab+3k_1) \\ & \sinh(a/\sqrt{k_1}))\sqrt{k_2} + \sinh((a-b)/\sqrt{k_2})((b(b-3a) - 3k_1)\sinh(a/ \\ & \sqrt{k_1}) + 3a\sqrt{k_1}\cosh(a/\sqrt{k_1}))k_2 + 3(b-a){k_2}^{3/2}\cosh((a-b)/\sqrt{k_2}) \\ & \sinh((a/\sqrt{k_1}) + 3\sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2}){k_2}^2)) \\ & /(\sinh((a-b)/\sqrt{k_2})(\sqrt{k_1}\sinh(a/\sqrt{k_1}) - a\cosh(a/\sqrt{k1}) \\ & \sqrt{k_1} + a\cosh((a-b)/\sqrt{k_2})\sinh(a/\sqrt{k_1})\sqrt{k_2} - \\ & \sinh(a/\sqrt{k_1})\sinh((a-b)/\sqrt{k_2})k_2) \end{aligned} \tag{6.2}$$

(ii) The wall correction factor W (the ratio of the actual couple experienced by the particle in the container and the couple on a particle in an infinite expanse of fluid) is

$$W = T/T^{\infty} = D/D^{\infty} \tag{6.3}$$

(iii) When $k_1 = k_2 = k$, then Equations (28), (29) and (31) become

$$\begin{aligned} T_k \;=\; & -8\pi\eta(c^3\Omega(b^3 + 3bk - 3b^2\sqrt{k}\coth(b/\sqrt{k}))/(b^3 - c^3 + 3bk \\ & -3b^2\sqrt{k}\coth(b/\sqrt{k}))) \end{aligned} \tag{6.4}$$

$$T_k^{\infty} = 8\pi\eta\Omega(b^3 + 3bk - 3b^2\sqrt{k}\coth(b/\sqrt{k})) \tag{6.5}$$

$$W^{\infty} = T/T^{\infty} = D/D^{\infty} \tag{6.6}$$

Equations (32), (33) and (34) describe the rotation of porous sphere of radius b in a cavity of radius c, in an unbounded fluid and wall correction factor respectively. These corresponds to Khe's papers [6-7].

(iv) When $k \to 0$, Equations (32), (33) and (34) reduces as follows

$$T_0 = (8\pi\eta\Omega b^3 c^3)/(c^3 - b^3) \tag{6.7}$$

$$T_0^{\infty} = 8\pi\eta\Omega b^3 \tag{6.8}$$

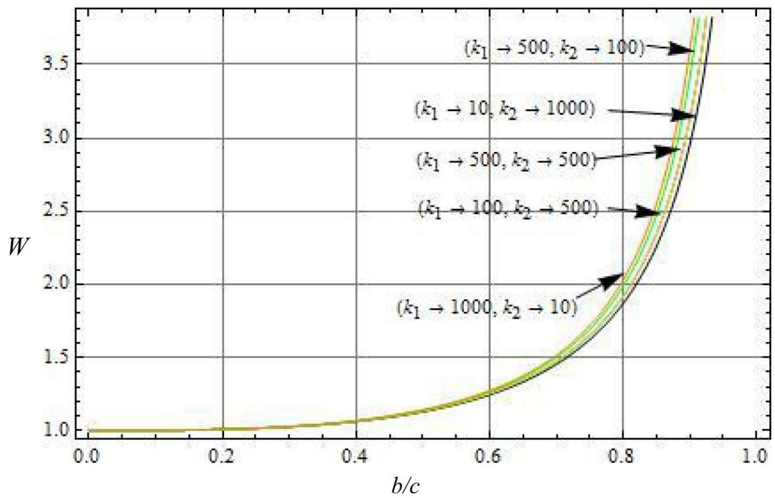

Fig. 2: Variation of W with b/c for various values of k_1 and k_2 with $a = 100$ and $b = 1000$

(6.9) $$W_0^{\infty} = c^3/(c^3 - b^3)$$

Equations (35), (36) and (37) describe the rotation of solid sphere of radius b in a cavity of radius c, in an unbounded fluid and wall correction factor respectively.
(v) When $k \to \infty$, Equations (32), (33) and (34) reduce to

(6.10) $$T_{\infty} = 0$$

(6.11) $$T_{\infty}^{\infty} = 0$$

Results (38) and (39) correspond to the rotation of spherical cavity of radius c containing only fluid and no particle.
(vi) When $b \to a$,then we get results for porous sphere of radius a.
It be noted that in the above superscript represents the radius of cavity and subscript permeability.

7 RESULTS AND DISCUSSION

In this section we discuss effects of separation parameter b/c on wall correction factor W and of radial distance r on $v_{\phi i}/(\Omega r \sin\theta)$ graphically. This is done for various values of parameters as depicted on the diagrams.

In figs. 2, 3 and 4, the graphs of the wall correction factor W of composite sphere at the center of spherical cavity with respect to b/c for various values of k_1 and k_2 for constant ratio $b/a = 10$ are depicted. In these graphs we describe the three types of variations of the wall correction factor W. Firstly we see that the wall correction factor W increases when b/c increases. Secondly, in each of three figures (2, 3 and 4) when ratios of k_1/k_2

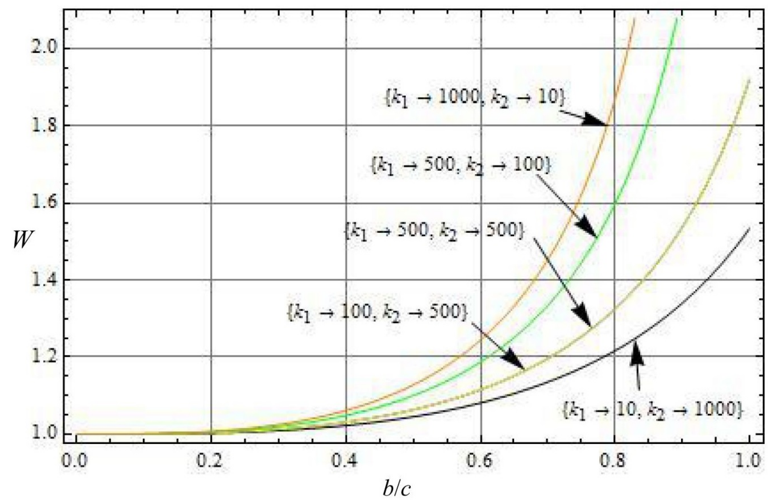

Fig. 3: Variation of W with b/c for various values of k_1 and k_2 with$a = 10$ and $b = 100$

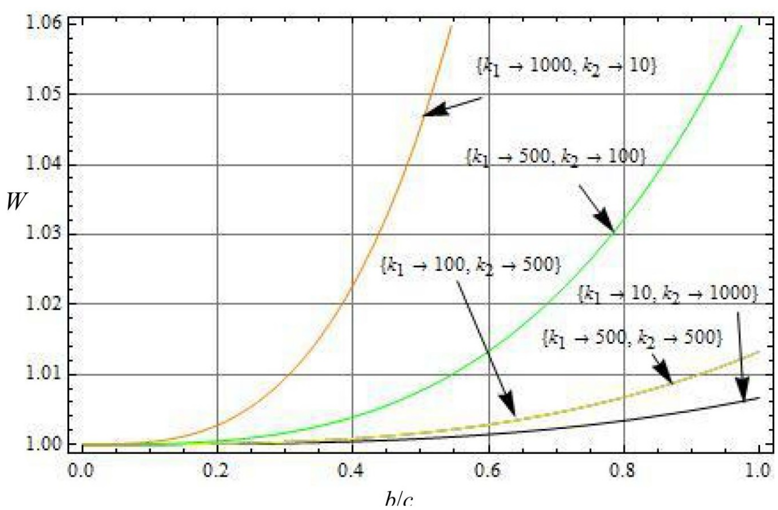

Fig. 4: Variation of W with b/c for various values of k_1 and k_2 with $a = 1$ and $b = 10$.

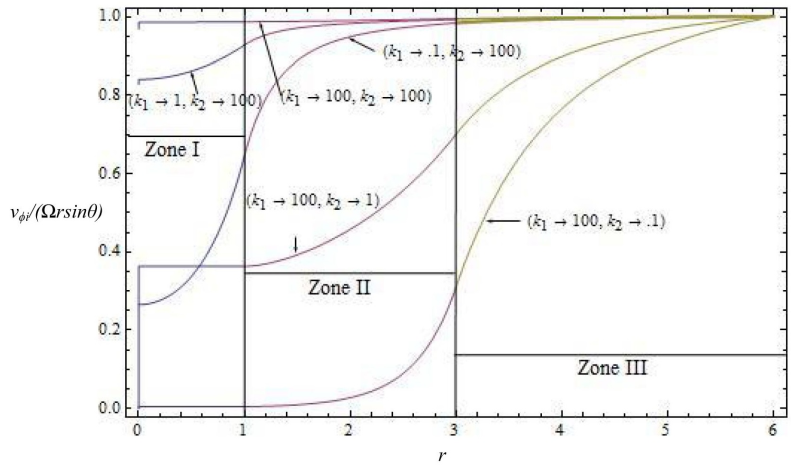

Fig. 5: Variation of $v_{\phi i}/(\Omega r \sin\theta)$ with radial distance r for various values of k_1 and k_2 with $a = 1, b = 3$ and $c = 6$

decreases W also decreases. Finally when we compare figures 2, 3 and 4, it is seen that as size of particle decreases then W decreases for fixed values of $b/c, k_1$ and k_2. So, we can be seen that our results are agreement with our physical expectations.
Fig. 5 presents the variation of $v_{\phi i}/(\Omega r \sin\theta)(i = 1, 2, 3)$ with radial distance r for fixed value of a, b, c and various values of the k_1 and k_2. In order to illustrate the effect of permeability, when $k_1 = k_2$ then the variation of $v_{\phi i}/(\Omega r \sin\theta)$ is not perturb because in this case composite sphere behave as a porous sphere of constant permeability and also in this case sphere is more permeable than other cases so variation in each zone approximate same. In other cases it is clear from fig. 5 variation in curves for $v_{\phi i}/(\Omega r \sin\theta)$ change as from convex to concave or concave to convex according to permeability of each zone (here rate of change from concave to convex or convex to concave directly proportional to difference between permeabilities of two joint regions). So, we can be seen that our results are agreement with our physical expectations.

8 CONCLUSION

An analytic solution of the governing equations for the problem of the motion of a composite sphere in a spherical cavity filled with an incompressible Newtonian fluid has been obtained. Brinkman model is used in porous region and Stokes' equations in the clear fluid region to solve the problem. An expression for the hydrodynamic torque on the composite sphere in the spherical cavity is obtained. The wall effect is computed and presented graphically from the limiting case of nearly porous sphere to solid sphere. It has been found that the wall correction factor of the composite sphere is increasing function of separation parameter (ratio of radius of composite sphere to spherical cavity). The analysis assumes that composite sphere and its core are non-deformable. We believe that our results provide useful insight into the actual phenomena of the motion of a composite sphere in a spherical cavity.

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