Coupled and Random Fixed Point by Contraction of Integral Type.

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Abstract

The present paper contains two parts. In the first part, the existence of coupled fixed point in G-metric space using integral type of contraction has been established . The second part contains the stochastic existence of fixed point in separable Banach space using such integral type of contraction.

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1 Introduction

Mustafa and Sims [1] introduced a new structure of generalized metric spaces, which is called G-metric spaces, as a generalization of metric spaces, to develop fixed point theory for various mappings.
Branaciarai [2] first introduced fixed point results using the following type basic integral contraction
$\int_0^{(d(fx,fy))} \alpha(t)dt \le c\int_0^{(d(x,y))} \alpha(t)dt.$
Latter on some authors [9,10] proved different results on fixed point theory using integral type of contraction. .Taking a clue of these concepts we established our first result on the existence of coupled fixed point in G-metric space in section 2.The next section deals with the basic notion of probability theory.
Probabilistic operator theory is branch of mathematics is concern with the study of operator valued random variable or simply random operator and their properties.The development of theory of random operator is of interest in its own right as a probabilistic generalization. In a recent work Saha and Debnath[5] have studied almost sure type of random fixed point theorem of class of mapping over Hilbert space.
In this section we establish a result on existence of random fixed point theory in separable Banach space using integral type of contraction.
Definition 1.1 Let X be a nonempty set $G : X \times X \times X \to R^+$ be a function satisfying

the following properties.
(i) $G(x,y,z) = 0$ if $x = y = z$.
(ii)$0 \leq G(x,x,y)$ $\forall x, y \in X$ with $x \neq y$.
(iii)$G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $z \neq y$.
(iv)$G(x,y,z) = G(x,z,y) = G(y,z,x) = \dots$ (symmetry in all three variables).
(v) $G(x,x,y) \leq G(x,x,z) + G(z,z,y)$ for all x, y, z X. (triangular inequality)
(vi)$G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$ (rectangle inequality).
Then the function G is called a $G-$ metric on X and the pair (X,G) is called a $G-$ metric space.

Definition 1.2 If f is map on X and $f(x) = x$ for some x is called fixed point and if f is a map from $X \times X$ to X then (x,y) is called coupled fixed point of f if $f(x,y) = x$ and $f(y,x) = y$ holds for some $x, y \in X$.

Definition 1.3 Let (X,G) be a $G-$metric space and let (x_n) be a sequence in X then we say (x_n) is convergent if there exist a point $x \in X$ such that for any $o < \epsilon$ there exists a positive integer N such that $G(x, x_m, x_n) < \epsilon$ for all $n, m \geq N$ and x is called limit of the sequence (x_n), at that time we say (x_n) converges to x w.r.t $G-metric$.

Definition 1.4 Let (X,G) be a G- metric space, a sequence (x_n) is called Cauchy sequence w.r.t $G-metric$ if, for every $\epsilon > 0$, there is a positive integer N such that $G(x_n, x_m, x_m) < \epsilon$ for all $n, m \geq N$.

Definition 1.5 Let (X,G) be a G- metric space, it is said to be complete if every Cauchy sequence is convergent in X w.r.t$G-metric$. Every $G-metric$ on X will define a metric on X by $dG(X,y) = G(x,y,y) + G(y,x,x)$, for all x,y$\in X$.

Definition 1.6 Let (X_1, G_1), (X_2, G_2) be two G- metric spaces. Then a function$f : X_1 \to X_2$ is G-continuous at a point$x \in X$ if and only if it is sequentially continuous at x , that is, whenever x_n is converges to x w.r.t $G_1 - metric$ on X then $f(x_n)$ is converges to f (x) w.r.t G_2 metric space on X_2.

Lemma 1.1 If (X,G) is a $G-metric$ space and x_n is sequence in X then $(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4)$ and the converse holds if (x_n) is Cauchy sequence.
(1) x_n converges to x w.r.t $G-metric$.
(2)$G(x_n, x_m, x) < \epsilon$ as n ,m $\geq$ N for some N.
(3)$G(x_n, x_m, x) < \epsilon$ as $n \geq N$ for some N.
(4) $G(x_n, x, x)$ as $n \geq N$ for some N.

Lemma 1.2 Let (X,G) be a G metric space .Then, for any $x,y,z,a \in X$ it follows that
(1) If $G(x,y,z) = 0$ then $x = y = z$.
(2) $G(x,y,z) \leq G(x,x,y) + G(x,x,z)$
(3) $G(x,y,y) \leq 2G(y,x,x)$,
(4) $G(x,y,z) \leq G(x,a,z) + G(a,y,z)$,

Now in the context of integral convergence we have the following lemmas of Branaciarai[2]

Lemma 1.3 [4] Let $r(t) \in \phi$ defined in [3] and (r_n) is +ve sequence of real number such that $\lim r_n = a$ then $lim \int_0^{r_n} r(t)dt = \int_0^a r(t)dt$
Lemma 1.4 [4] Let $r(t) \in \phi$ defined in [3] and (r_n) is +ve sequence of real number such

that $lim \int_0^{r_n} r(t)dt = 0$ then $\lim r_n = 0$

2 Main result

Theorem 2.1 Let (X, G) be a complete $G - metric$ space. f is a continuous map from $X \times X$ to X and f satisfied the following integral contraction condition
$\int_0^{Gf(x_1,y_1),f(x_2,y_2),f(x_3,y_3)} \alpha(t)dt \leq a \int_0^{G(x_1,x_2,x_3)} \alpha(t)dt$ and
$\int_0^{Gf(x_1,y_1),f(x_2,y_2),f(x_3,y_3)} \alpha(t)dt \leq b \int_0^{G(x_1,x_2,x_3)} \alpha(t)dt$
where $a, b \in (0, 1)$.
$\alpha(t)$ is a Lebesge integral function on every compact subset of R and $x_1, x_2, x_3, y_1, y_2, y_3 \in X$ then f has coupled fixed point in X.

Proof Let (x_0, y_0) be a arbitrary point in $X \times X$. Now define $x_1 = f(x_0, y_0)$ and $y_1 = f(y_0x_0)$ similar way define the sequence $x_{n+1} = f(x_n, y_n)$ and $y_{n+1} = f(y_n, x_n)$.
Now $\int_0^{G(x_{n+1},x_{n+1},x_n)} \alpha(t)dt = \int_0^{G((f(x_n,y_n),f(x_n,y_n)f(x_{n-1},y_n-1))} \alpha(t)dt$
$\leq a \int_0^{G((f(x_{n-1},y_{n-1}),f(x_{n-1},y_{n-1})f(x_{n-2},y_{n-2}))} \alpha(t)dt \ldots.\ a^n \int_0^{G(x_1,x_1,x_0)} \alpha(t)dt.$
Now take limit n tends to ∞ we get
$lim \int_0^{G(x_{n+1},x_{n+1},x_n)} = 0.$
$\Rightarrow G(x_{n+1}, x_{n+1}, x_n) = 0$(by Lemma 1.3) . we have to show (x_n) is a Cauchy sequence . suppose (x_n) is not Cauchy . Then there exist $\epsilon > 0$ such that we have sequences of natural numbers (m(k)) and (l(k)) for every natural number $k, m(k) \geq l(k) \geq k$ and $G(x_{m(k)}, x_{m(k)}, x_{l(k)}) > \epsilon$
Corresponding to l(k) we choose m(k) to be the smallest for which $G(x_{m(k)-1}, x_{m(k)-1}, x_{l(k)}) < \epsilon$ holds. Now
$\epsilon \leq G(x_{m(k)}, x_{m(k)}, x_{l(k)})$
$\leq G(x_{m(k)}, x_{m(k)}, x_{m(k)-1}) + G(x_{m(k)-1}, x_{m(k)-1}, x_{l(k)})$
$\leq \epsilon + G(x_{m(k)-1}, x_{m(k)-1}, x_{l(k))}$. (Using the rectangle inequality)
taking k tends to ∞ in the above inequality and we have

$limG(x_{m(k)}, x_{m(k)}, x_{l(k)})$
$= \epsilon = limG(x_{m(k)-1}, x_{m(k)-1}, x_{l(k)-1})$.

Now
$\lim \int_0^{G(x_{m(k)},x_{m(k)},x_{l(k)})} \alpha(t)dt$
$\leq lim \int_0^{G(x_{m(k)},x_{m(k)},x_{l(k)})} \alpha(t)dt$
$\Rightarrow \int_0^{limG(x_{m(k)},x_{m(k)},x_{l(k)})} \alpha(t)dt$
$\leq \int_0^{limG(x_{m(k)},x_{m(k)},x_{l(k)})} \alpha(t)dt \Rightarrow \int_0^{\epsilon} \alpha(t)dt < a \int_0^{\epsilon} \alpha(t)$
$.\Rightarrow 1 \leq a.$
which is a contradiction. Hence (x_n) is a Cauchy sequence.
Since X is complete (x_n) converges to some x in X. similar way we can show (y_n) converges to some y in X.

Now we claim to (x, y) be coupled fixed point of f in $X \times X$. Since (x_n) converges to x we have $\lim G(x_n, x_n, x) = 0 = \lim G(x_{n+1}, x_{n+1}, x)$ as n tends to ∞.
Now by Lemma 1.3 and integral contraction we have $\int_0^{(G(x_{n+1},x_{n+1},f(x,y)))} \alpha(t)dt \leq alim \int_0^{(G(x_n,x_n,x))} \alpha(t)dt =$

$a\int_0^{limG(x_n,x_n,x)}\alpha(t)dt = 0 = \lim G(x_n,x_n,f(x,y)) = 0$ (by Lamma 1.4).
Hence x_n converges to f(x,y) . since limit is unique therefore f(x,y)=x .
similar way we can show f(y,x)=y . ◇

Theorem 2.2 Let (X,G) be a complete G- metric space . f be a continuous map from $X\times X$ to X and f satisfies the following integral contraction condition
$\int_0^{Gf(x_1,y_1),f(x_2,y_2),f(x_3,y_3)}\alpha(t)dt \le a\int_0^{G(x_1,x_2,x_3)}\alpha(t)dt + b\int_0^{G(y_1,y_2,x_3)}\alpha(t)dt$
where $a+b\in(0,1)$.α is a Lebesgue integral function on every compact subset of R and x,y,u,v,w,z are arbitrary points in X with
$\lim\binom{n}{0}a^nk+\binom{n}{1}a^{n-1}bk+\cdots+\binom{n}{n}b^n = 0$
(where k is $\max(\int_0^{G((x_1,x_1,x_0))}\alpha(t)dt\ ,\ \int_0^{G((y_1,y_1,y_0,)}\alpha dt)))$.
then f has coupled fixed point.

Proof Let (x_n) and (y_n) be two sequences that we have mentioned in theorem 2.1. Now we claim that$\int_0^{G((x_{n+1},x_{n+1},x_n))}\alpha(t)dt \le \binom{n}{0}a^nk+\binom{n}{1}a^{n-1}bk+\cdots+\binom{n}{n}b^n$ and
$\int_0^{G((y_{n+1},y_{n+1},y_n))}\alpha(t)dt \le \binom{n}{0}a^nk+\binom{n}{1}a^{n-1}bk+\cdots+\binom{n}{n}b^n$.
We prove it by method of induction.
Let p_n is statement which stands for $\int_0^{G((x_{n+1},x_{n+1},x_n)}\alpha(t)dt \le \binom{n}{0}a^nk+\binom{n}{1}a^{n-1}bk+\cdots+\binom{n}{n}b^n$ and $\int_0^{G((y_{n+1},y_{n+1},y_n))}\alpha(t)dt \le \binom{n}{0}a^nk+\binom{n}{1}a^{n-1}bk+\cdots+\binom{n}{n}b^n$
Now we have to show p_1 $\int_0^{G((x_2,x_2,x_1))}\alpha(t)dt \le ak+bk$ and $\int_0^{G((y_2,y_2,y_1))}\alpha(t)dt \le ak+bk$ is true.
From the above integral contraction we get
$\int_0^{G((x_2,x_2,x_1))}\alpha(t)dt \le a\int_0^{G((x_1,x_1,x_0)}\alpha(t)dt + b\int_0^{G((y_1,y_1,y_0)}\alpha(t)dt = ak+bk.$
similarly way we can show
$\int_0^{G((y_2,y_2,y_1))}\alpha(t)dt \le ak+bk.$
Suppose it is true for n.
that is
$\int_0^{G((x_{n+1},x_{n+1},x_n)}\alpha(t)dt \le \binom{n}{0}a^nk+\binom{n}{1}a^(n-1)bk+\cdots+\binom{n}{n}b^n$ and
$\int_0^{G((y_{n+1},y_{n+1},y_n)}\alpha(t)dt \le \binom{n}{0}a^nk+\binom{n}{1}a^(n-1)bk+\cdots+\binom{n}{n}b^n$
Now we require that p_{n+1} is true .
From the condition of theorem we have
$\int_0^{G((x_{n+2},x_{n+2},x_{n+1}))}\alpha(t)dt$
$\le a\int_0^{(G((x_{n+1},x_{n+1},x_n)))}\alpha(t)dt + b\int_0^{(G((y_{n+1},y_{n+1},y_n))}\alpha(t)dt$
$\le a(\binom{n}{0}a^nk+\binom{n}{1}a^{n-1}bk+\cdots+\binom{n}{n}b^n) + b(\binom{n}{0}a^nk+\binom{n}{1}a^{n-1}bk+\cdots+\binom{n}{n}b^n) =$
$a^{n+1}k+\binom{n}{1}a^nbk+\cdots+\binom{n}{n}ab^n) + (\binom{n}{0}ba^nk+\binom{n}{1}a^{n-1}bk+\cdots+\binom{n}{n}b^{n+1}) =$
$\binom{n}{0}a^{n+1}k+\binom{n}{1}a^nbk+\cdots+\binom{n}{n}kb^{n+1}$ (From induction hypothesis and pascal formula).

Hence it is true for all $n\in N$. Now taking limit n tends to ∞
we have $\lim\int_0^{(G((x_{n+1}],x_{n+1},x_n))}\alpha(t)dt = 0$(from condition of a,b in hypothesis of theorem).
$\lim G(x_{n+1},x_{n+1},x_n) = 0$ (by Lemma-1.3). Now we claim that $((x_n))$ is a Cauchy sequence .
suppose (x_n) is not Cauchy . Then there exist $\epsilon > 0$ such that we have sequences of

natural numbers $(m(k))$ and $(l(k))$ for every natural number k, $m(k) \geq l(k) \geq k$ and $G(x_{m(k)}, x_{m(k)}, x_{l(k)}]) > \epsilon$

Here we choose $m(k)$ corresponding to $l(k)$ be the smallest +ve integer for which $G(x_{m(k)-1}, x_{m(k)-1}, x_{l(k))}) < \epsilon$ holds.
Now $\epsilon < G(x_{m(k)}, x_{m(k)}, x_{l(k)}) \leq G(x_{m(k)}, x_{m(k)}, x_{m(k)-1}) + G(x_{m(k)-1}, x_{m(k)-1}, x_{l(k)})$
$\leq \epsilon + G(x_{m(k)-1}, x_{m(k)-1}, x_l(k))$ (Using the rectangle inequality).
Taking $k \to \infty$ in the above inequality and we have $\lim G(x_{m(k)}, x_{m(k)-1}, x_{m(k)-1}) = \epsilon$
$= \lim G(x_{m(k)-1}, x_{m(k)-1}, x_{m(k)-1})$ and similar way we can show that

$\lim G(y_{m(k)-1}, y_{m(k)-1}, y_{l(k)}) = \epsilon$
$= \lim G(y_{m(k)}, y_{l(k)-1}, y_{m(k)-1})$.

Applying Lemma 1.4 and lim n tends to ∞ we have
$\lim \int_0^{G(x_{m(k)+1}, x_{m(k)+1}, x_{m(k)})} \alpha(t)dt$
$\leq lima \int_0^{G(x_{m(k)}, x_{m(k)}, x_{m(k)-1})} \alpha(t)dt + blim \int_0^{G(y_{m(k)}, y_{m(k)}, y_{m(k)-1})} \alpha(t)dt$
$\leq a \int_0^{limG(x_{m(k)}, x_{m(k)}, x_{m(k)-1})} \alpha(t)dt + b \int_0^{limG(y_{m(k)}, y_{m(k)}, y_{m(k)-1})} \alpha(t)dt.$
$\Rightarrow \int_0^{\epsilon} \alpha(t)dt \leq a \int_0^{\epsilon} \alpha(t)dt + b \int_0^{\epsilon} \alpha(t)dt.$
$\Rightarrow 1 \leq a + b.$
which is a contradiction.
.Hence (x_n) is a Cauchy sequence.
Therefore (x_n) converges to some x in X (since X is complete w.r.t G-metric).
Similarly (y_n) converges to y in X.
Now we have to show that (x,y) is coupled fixed point of f. Now
$\lim \int_0^{(G(x_{n+1}, x_{n+1}, f(x,y)))} \alpha(t)dt$
$\leq alim \int_0^{(G(x_n, x_n, x))} \alpha(t)dt + blim \int_0^{G(y_n, y_n, y))} \alpha(t)dt.$
$.\Rightarrow \lim \int_0^{(G(x_{n+1}, x_{n+1}, f(x,y)))} \alpha(t)dt.$
$\leq a \int_0^{lim(G(x_n, x_n, x))} \alpha(t)dt + b \int_0^{limG(y_n, y_n, y))} \alpha(t)dt.$
$\Rightarrow \lim \int_0^{(G(x, x_{n+1}, f(x,y)))} \alpha(t)dt \leq 0+0 = 0($ since x_n, y_n convergence to x, y w.r.t G-metric).
$\lim G(x_n, x_n, f(x,y)) = 0$ (by Lemma 1.3) (x_n) convergence to $f(x,y)$.
since limit is unique $f(x,y) = x$.In a similar way we can show that $f(y,x) = y$. $\diamond$

3 Basic notions of probabilistic space

First we give basic definition on probabilistic space [6].

Definition 3.1 β is called sigma algebra of Ω if β is a collection of subset s of Ω is closed under countable union and complements.

Definition 3.2 $[\Omega, \beta, P]$ is called probabilistic space if β is a sigma algebra overΩ and P is a map from β to [0,1]

Definition 3.3 X is a Banach space is called separable if it has a countable dense subset

Definition 3.4 Let $[\Omega, \beta, P]$ is a probabilistic space . x is a map from Ω to R then it is called random variable if $x^{-1}(B) \in \beta$ where B is a Borel set of R .

Definition 3.5 A mapping $F : \Omega \times X \to$ Y is said to be a random mapping if $F(\omega, x) = Y(\omega)$ is a Y -valued random variable for every $x \in X$.

Definition 3.6 A mapping$F : \Omega \times X \to$ Y is said to be a continuous random mapping if the set of all $\omega \in \Omega$ for which $F(\omega, x)$ is a continuous function of x has measure one.

Definition 3.7 An equation of the type$F(\omega, x(\omega)) = x(\omega)$ where $F : \Omega \times X \to$ to X is a random mapping is called a random fixed point equation.

Definition 3.8 Any mapping $x : \Omega \times X \to X$ which satisfies random fixed point equation $F(\omega, x(\omega)) = x(\omega)$ almost surely is said to be a wide sense solution of the fixed point equation.

Definition 3.9 Any X-valued random variable $x(\omega)$ which satisfies $P(\omega : F(\omega, x(\omega)) = x(\omega)) = 1$ is said to be a random solution of the fixed point equation or a random fixed point of F.

Definition 3.10 A function F :$\Omega \times X \times X \to$ to X satisfying $F(\omega, x(\omega), y(\omega) = x(\omega)$ and$F(\omega, y(\omega), x(\omega) = y(\omega)$ almost surely is called random coupled fixed point.

Now we give our results of this section

Theorem-3.1- Let $[\Omega, \beta, P]$ be probabilistic space and X is a separable Banach space and $F : \Omega \times X \to$ to Y is map satisfying $F(\omega, x)$ is continuous on x almost every where . $F(\omega, x,) = f(x)$ has integral contraction almost surely then F has random fixed point.

Proof Let $C_{xy} = \left\{\omega \in \Omega : \int_0^{(d(fx,fy))} \alpha(t)dt \leq c \int_0^{(d(x,y))} \alpha(t)dt\right\}$ and $A = \{\omega \in \Omega : F(\omega, x,) = f(x)\}$
$\Rightarrow$ P(C) = P(A) = 1.
From [3] we know if f satisfies the above integral contraction condition then it has fixed point on X .
Now we have to show
$\bigcap_{xy}(C_{xy} \bigcap A) = \bigcap_{st}(C_{st} \bigcap A)$
for all $x, y \in X$ and all$s, t \in S$
where S is countable dense subset of X(being X is separable). Since $S \subset X$ therefore this inclusion is
$\bigcap_{xy}(C_{xy} \bigcap A) \subset \bigcap st(C_{st} \bigcap A)$
is obvious.Now We have to show the opposite inclusion.

Let us suppose that $\omega \in C_{s,t}$ and x, y are two arbitrary points of X. Since S is separable we have two sequences (s_n) and (t_n) in S such that
$\lim s_n = x$ and $limt_n = y$.
Now $\int_0^{d(fx,fy))} \alpha(t)dt$
$= \int_0^{d(f(\lim s_n, f(\lim n)} \alpha(t)dt$
$= \int_0^{d \lim(fs_n, ft_n)} \alpha(t)dt$
$= \int_0^{limd(fs_n, ft_n)} \alpha(t)dt$ (since f is continuous and metric is also continuous)
$= lim \int_0^{d(fs_n, ft_n)} \alpha(t)dt$ (by Lemma 1.3)
$\leq limc \int_0^{d(s_n, t_n)} \alpha(t)dt$ (since $\omega \in Cs, t$ by integral contraction) $= limc \int_0^{d(s_n, t_n)} \alpha(t)dt =$

$c\int_0^{\lim d(s_n,t_n))} \alpha(t)dt = c\int_0^{d(\lim s_n \lim t_n)} \alpha(t)dt$ (again by Lemma 1.4)
$= c\int_0^{d(x,y))} \alpha(t)dt.$
$\Rightarrow \omega \in \bigcap_{xy}(C_{xy} \bigcap A). \Rightarrow \bigcap st(C_{st} \bigcap A) \subset (C_{xy} \bigcap A).$
$\Rightarrow \bigcap st(C_{st} \bigcap A) = \bigcap_{xy}(C_{xy} \bigcap A).$
Now $P(\bigcap_{xy}(C_{xy} \bigcap A)^c) = P((\bigcup_{xy} C_{xy}^c \bigcup A^c) \leq \sum_{xy} P(C_{xy}^c) + P(A^c) = 0+0 = 0.$
Hence it holds for almost surely. . $\diamond$

Conclusion The theorem 2.1 and 2.2 and 3.1 on existence of fixed point for integral contraction on G-metric space and separable Banach space respectively are the new addition in fixed point theory.

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