Flow of Two Immiscible Viscous Fluids in Porous Medium Between Two Parallel Plates.

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Abstract

In the present paper a mathematical analysis of pulsating MHD flow of two immiscible conducting and non-conducting incompressible viscous fluids in porous medium bounded by horizontal plates is presented. Fluids are flowing in porous media of different permeabilities and are assumed to obey Darcy's law. The no slip condition at the plates and matching conditions at the interface of both mediums are used. Separate expressions for velocity, mass flux and shear stress in both regions are obtained. The effects of governing flow parameters on the velocities and stresses are studied numerically and results are presented through graphs.

Subject class [2010]:76S99; 76T99
Keywords: Conducting fluid, Porous medium, Two fluid flow.

1 Introduction

Flows through Porous medium are of principal interest because they are quite prevalent in nature.Such flows have many scientific and engineering applications,viz., in the field of agriculture engineering to study the underground water resources, seepage of water in river beds; in chemical engineering for filtration and purification processes; in petroleum technology to study the movement of natural gases,oil and water through oil reservoirs.

Berman[12] was the first researcher who studied the problem of steady flow in an incompressible viscous fluid through a porous channel with rectangular cross section, when the Reynolds number is low and the perturbation solution assuming normal wall velocity to be equal was obtained. The Hartmann flow of a conducting fluid in a channel with a layer of non-conducting fluid between the upper channel wall and conducting fluid was studied by Shail[11]. He found that an increase in 30 percent could be achieve in the flow rate for suitable ratio of depths and viscosities of the two fluids. M.Sahimi[9] reported that microscopic and macroscopic descriptions of multiphase flow in porous media differ considerably from each other and both have their own characteristic problem. R.Hilfer[8] established macroscopic equations of motion for two phase immiscible displacement in porous media. Vafai and Kim[7] have studied and reported an exact solutions for forced convection in a channel filled with a porous medium. Chen S.C. and Vafai[6] have analysed free surface momentum and energy transport in porous medium in the absence and presence of surface tension effects.

Recently Singh and Rana [5] have studied the flow and heat transfer through a porous medium. They studied effect of periodic variation of suction velocity on the flow and heat transfer during the motion of a viscous incompressible fluid through a highly porous media. Z.Q.Chen[4] have investigated experimentally two phase flow and boiling heat transfer in channels packed with sintered copper bi-dispersed porous media. Chamakha[3] reported analytical solutions for the flow of two immiscible fluids in porous and non-porous parallel plate channels with magnetic field. Hartmann two fluid flow and heat transfer in a horizontal channel was studied by J.C.Umavathi etal.[2]. Hafeez and Ndikilar[1] presented exact solution for the steady laminar flow of viscous incompressible fluid between two parallel porous plates with bottom injection and top suction.

In this paper we consider MHD flow of two immiscible conducting and non-conducting incompressible viscous fluids in porous medium bounded by horizontal plates. Fluids are assumed to obey Darcy's law. The no slip condition at the plates and matching conditions at the interface of both mediums are used.The governing equations are solved analytically and separate expressions for velocity, mass flux and shear stress in both regions are obtained. The effects of governing flow parameters on the velocities and stresses are studied numerically and results are presented through graphs.Although the exact solutions for two phase flow in porous media can only be obtained with some assumptions, yet they are very helpful to verify practical datas.

2 Mathematical Formulation

The geometry under consideration is illustrated in Fig. 1, consists of two porous regions $-I\&II$ of permeabilities $K_1\&K_2$ respectively. The region $-h \leq y \leq 0$ is occupied by a conducting viscous fluid of density ρ_1, viscosity μ_1 and the region $0 \leq y \leq h$ is filled with a viscous fluid of density $\rho_2(< \rho_1)$ and viscosity μ_2.

We consider the fluids to be incompressible and the flow is unsteady, laminar and fully developed, driven only by a pulsatile pressure gradient

$$-\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_o e^{i\omega t}$$

where $\left(\frac{\partial p}{\partial x}\right)_s$ and $\left(\frac{\partial p}{\partial x}\right)_o$ are amplitudes of steady and oscillatory pulsations respectively and w is the frequency. It is noted that the non-conducting and conducting fluids are immiscible (that is there exist no mixing between the fluids) and the constitutive equations for both fluids are different. For instance, benzene is a non-conducting fluid and water is a conducting fluid and it is well known that benzene and water can not mix. Since our model is general, One can choose any two different fluids which are immiscible.

Assuming that non zero component of velocity is X-component, the governing equations of fluid flow are

Region-I

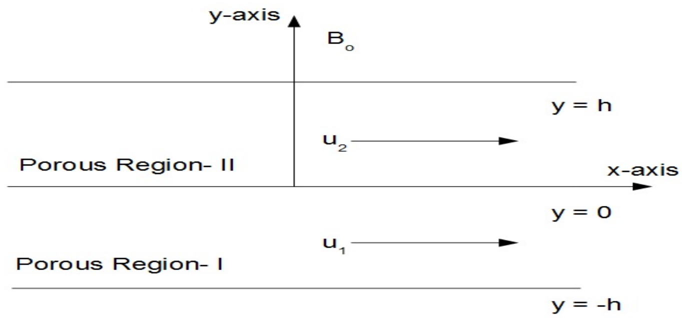


Fig. 1: Flow diagram.

Equation of mass balance

$$\frac{\partial u_1}{\partial x} = 0 \tag{2.1}$$

Equation of momentum balance

$$\rho_1 \frac{\partial u_1}{\partial t} = -\frac{\partial p}{\partial x} + \mu_1 \frac{\partial^2 u_1}{\partial y^2} - \frac{\mu_1}{K_1} u_1 - \sigma B_o^2 u_1 \tag{2.2}$$

Region-II

Equation of mass balance

$$\frac{\partial u_2}{\partial x} = 0 \tag{2.3}$$

Equation of momentum balance

$$\rho_2 \frac{\partial u_2}{\partial t} = -\frac{\partial p}{\partial x} + \mu_2 \frac{\partial^2 u_2}{\partial y^2} - \frac{\mu_2}{K_2} u_2 \tag{2.4}$$

Here σ is the electrical conductivity of the conducting fluid and B_o is the strength of applied magnetic field. Herein the velocities $u_1(y,t), u_2(y,t)$ are to satisfy the conditions

$$u_1 = 0 \quad \text{at } y = -h \tag{2.5}$$
$$u_2 = 0 \quad \text{at } y = h \tag{2.6}$$
$$u_1 = u_2 \quad \text{at } y = 0 \tag{2.7}$$
$$\mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} \quad \text{at } y = 0 \tag{2.8}$$

Equations (2.7&2.8) are the matching conditions at the interfaces of upper and lower porous media. In view of pulsating pressure gradient, let us assume that the velocities are in the form

$$u_i = u_{i1} + u_{i2} e^{i\omega t}, \qquad i = 1, 2$$

where u_{i1} and u_{i2} represent the steady and oscillatory parts of the velocities respectively.

3 Non-dimensionalization of flow quantities

We introduce following non dimensional quantities to make the governing equations and the boundary conditions dimensionless:

$$\begin{aligned} x^* &= \frac{x}{h}, y^* = \frac{y}{h}, u_i^* = \frac{u_i}{u}, u_{i1}^* = \frac{u_{i1}}{u}, \\ u_{i2}^* &= \frac{u_{i2}}{u}, t^* = \frac{tu}{h}, K_i^* = \frac{K_i}{h^2}, \tau_i^* = \frac{\tau_i}{\rho u^2} \\ w^* &= \frac{wh}{u}, p^* = \frac{p}{\rho u^2}, M = B_o h\sqrt{\frac{\sigma}{\mu_1}} \end{aligned}$$

After dropping the asterisks, governing equations of motion (2.2&2.4) are given by

$$\frac{\partial u_1}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{R_1}\frac{\partial^2 u_1}{\partial y^2} - \frac{1}{K_1 R_1}u_1 - \frac{M^2}{R_1}u_1 \tag{3.1}$$

$$\frac{\partial u_2}{\partial t} = -\alpha\frac{\partial p}{\partial x} + \frac{1}{R_2}\frac{\partial^2 u_2}{\partial y^2} - \frac{1}{K_2 R_2}u_2 \tag{3.2}$$

and boundary and interface conditions $(2.5 - 2.8)$ become

$$u_1 = 0 \quad \text{at } y = -1 \tag{3.3}$$

$$u_2 = 0 \quad \text{at } y = 1 \tag{3.4}$$

$$u_1 = u_2 \quad \text{at } y = 0 \tag{3.5}$$

$$\beta\frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial y} \quad \text{at } y = 0 \tag{3.6}$$

where

$$\begin{aligned} u_i &= u_{i1} + u_{i2}e^{iwt}, \qquad i = 1, 2 \\ -\frac{\partial p}{\partial x} &= \left(\frac{\partial p}{\partial x}\right)_s + \left(\frac{\partial p}{\partial x}\right)_o e^{iwt} \end{aligned}$$

are non-dimensional velocities and pressure gradient respectively.

$R_1 = \frac{\rho_1 hu}{\mu_1}, R_2 = \frac{\rho_2 hu}{\mu_2}$ are Reynolds numbers respectively in flow regions I, II , $M = B_o h\sqrt{\frac{\sigma}{\mu_1}}$ is the Hartmann number and $\beta = \frac{\mu_1}{\mu_2}$, $\alpha = \frac{\rho_1}{\rho_2}$ are non-dimensional parameters.

3.1 Steady flow

The governing equations of steady flow are given by

$$\frac{1}{R_1}\frac{d^2 u_{11}}{dy^2} - \left(\frac{1}{K_1 R_1} + \frac{M^2}{R_1}\right)u_{11} + P_s = 0 \tag{3.7}$$

$$\frac{1}{R_2}\frac{d^2 u_{21}}{dy^2} - \frac{1}{K_2 R_2}u_{21} + \alpha P_s = 0 \tag{3.8}$$

The boundary conditions to be satisfied by u_{i1} are

$$u_{11} = 0 \quad \text{at } y = -1 \tag{3.9}$$
$$u_{21} = 0 \quad \text{at } y = 1 \tag{3.10}$$
$$u_{11} = u_{21} \quad \text{at } y = 0 \tag{3.11}$$
$$\beta\frac{du_{11}}{dy} = \frac{du_{21}}{dy} \quad \text{at } y = 0 \tag{3.12}$$

3.2 Oscillatory flow

The governing equations of oscillatory flow are given by

$$\frac{1}{R_1}\frac{d^2u_{12}}{dy^2} - \left(\frac{1}{K_1R_1} + \frac{M^2}{R_1} + iw\right)u_{12} + P_o = 0 \tag{3.13}$$
$$\frac{1}{R_2}\frac{d^2u_{22}}{dy^2} - \left(\frac{1}{K_2R_2} + iw\right)u_{22} + \alpha P_o = 0 \tag{3.14}$$

The boundary conditions to be satisfied by u_{i2} are

$$u_{12} = 0 \quad \text{at } y = -1 \tag{3.15}$$
$$u_{22} = 0 \quad \text{at } y = 1 \tag{3.16}$$
$$u_{12} = u_{22} \quad \text{at } y = 0 \tag{3.17}$$
$$\beta\frac{du_{12}}{dy} = \frac{du_{22}}{dy} \quad \text{at } y = 0 \tag{3.18}$$

where $P_s = (\frac{\partial p}{\partial x})_s, P_o = (\frac{\partial p}{\partial x})_o$.

4 Solution of the problem

4.1 Steady flow solution

The solution of steady flow described in Section 3.1 is given by

$$u_{11}(y) = C_1\cosh(\sqrt{A_1}y) + C_2\sinh(\sqrt{A_1}y) + \frac{P_sR_1}{A_1} \tag{4.1}$$
$$u_{21}(y) = C_3\cosh(\sqrt{A_2}y) + C_4\sinh(\sqrt{A_2}y) + \frac{\alpha P_sR_2}{A_2} \tag{4.2}$$

where $A_1 = \left(\frac{1}{K_1} + M^2\right)$, $A_2 = \frac{1}{K_2}$ and $C_i, i = 1, 2, 3, 4$ are not reported for brevity.

4.2 Oscillatory flow solution

The solution of unsteady flow described in Section 3.2 is given by

$$u_{12}(y) = C_5 \cosh(\sqrt{A_3}y) + C_6 \sinh(\sqrt{A_3}y) + \frac{P_o R_1}{A_3} \tag{4.3}$$

$$u_{22}(y) = C_7 \cosh(\sqrt{A_4}y) + C_8 \sinh(\sqrt{A_4}y) + \frac{\alpha P_o R_2}{A_4} \tag{4.4}$$

where $A_3 = \left(\frac{1}{K_1} + M^2 + i\omega R_1\right)$, $A_4 = \left(\frac{1}{K_2} + i\omega R_2\right)$ and $C_i, i = 5, 6, 7, 8$ are not reported for brevity.

4.3 Pulsatile flow solution

The solution of pulsatile flow is given by

$$u_1 = u_{11} + u_{12}e^{i\omega t}, \qquad u_2 = u_{21} + u_{22}e^{i\omega t}$$

where u_{11}, u_{21}, u_{12}, u_{22} are known from steady flow and oscillatory flow solutions given in Eqs. (4.1), (4.2), (4.3) and(4.4) respectively.

4.4 Shear stress

The shear stress at lower and upper plates in non-dimensional form are respectively given as

$$\tau_1 = \frac{1}{R_1}\frac{\partial u_1}{\partial y} \quad \text{at } y = -1 \tag{4.5}$$

$$\tau_2 = \frac{1}{\beta R_1}\frac{\partial u_2}{\partial y} \quad \text{at } y = 1 \tag{4.6}$$

and the full expressions are not reported for brevity.

4.5 Mass flux

The instantaneous mass fluxes in both regions are respectively given as

$$Q_1 = \int_{-1}^{0} u_{11} dy + \left[\int_{-1}^{0} u_{12} dy\right] e^{iwt} \tag{4.7}$$

$$Q_2 = \int_{0}^{1} u_{21} dy + \left[\int_{0}^{1} u_{22} dy\right] e^{iwt} \tag{4.8}$$

	mod τ	$R_1 = 1$	$R_1 = 3$	$R_1 = 5$	$R_1 = 7$	$R_1 = 9$
$\omega t = 0$	LP	0.88505	0.87107	0.87572	0.87665	0.87604
	UP	2.36891	2.79983	2.63656	2.57756	2.52409
$\omega t = \frac{\pi}{4}$	LP	0.82004	0.80935	0.81652	0.82023	0.82243
	UP	2.40463	2.61344	2.49751	2.47673	2.45844
$\omega t = \frac{\pi}{2}$	LP	0.63242	0.62658	0.63514	0.64100	0.64558
	UP	1.93030	2.03600	1.98393	2.00345	2.02197

Tab. 1: Variation of τ with R_1 ($M = 5, \beta = 1.2, \alpha = 1.4, K_1 = 0.5, K_2 = 3, P_s = 2.5, P_O = 2, \omega = \frac{\pi}{8}$).

5 Results and discussions

The analytical solutions for velocity profiles in both regions for steady, oscillatory and pulsating flows are derived. The analytical solutions are evaluated numerically for different values of governing parameters and the results are depicted graphically in Figs. $2-5$. In numerical work we take $\alpha = 1.4$, $\beta = 1.2$, $R_1 = 5$, $K_1 = 0.5$, $K_2 = 3$, $P_o = 2.5$, $P_s = 2$ and $R_2 = \frac{\beta}{\alpha} R_1$.

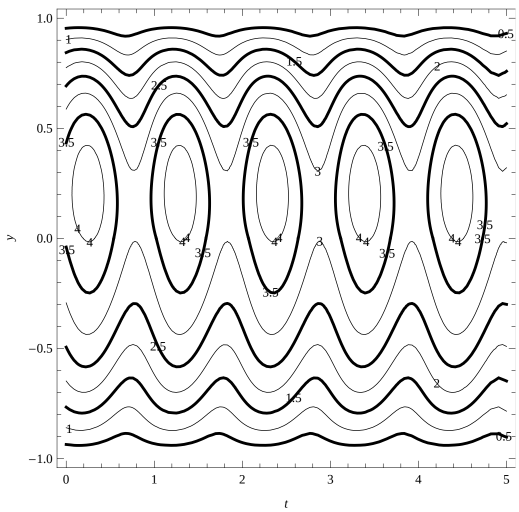
Fig.2 shows the variation of pulsating velocity with respect to y and t simultaneously for different values of Hartmann number M at $\omega = 6$. We observed that flatness in the velocity profile in Region-I increases as M increases. The variation in velocities u_1 at $y = -0.5$ and u_2 at $y = 0.5$ with respect to time t for different values of M is shown in Figs. 3(A) and 3(B). It is observed that as M increases velocities in both region decrease.

In Fig. 4, we see the variation of pulsating velocity with respect to time t for $M = 5$. In region-I (Fig. 4(A)), velocity decreases as we move from interface towards lower plate while in region-II (Fig. 4(B)), we see that velocity first increases then decreases as we move from interface towards upper plate. It happens due to no slip condition at upper plate.

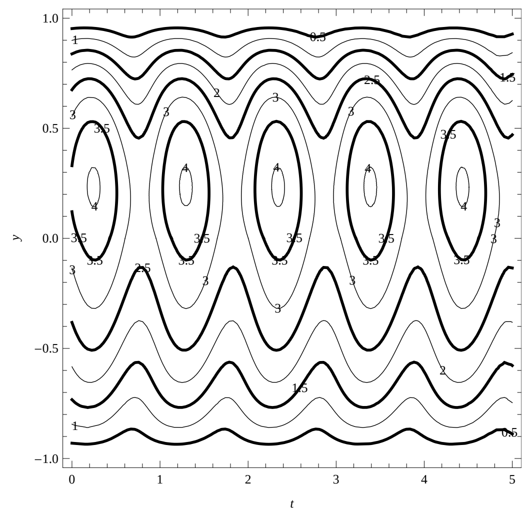
The complete velocity profile in both regions at a particular time instant with respect to y for different values of M is shown in Fig. 5(A). In figure it is seen that maximum velocity occurs in region-II. Fig. 5(B) depict variation in velocity with frequency parameter ωt. Decrement in velocity is noticed as a result of increment in ωt

The variation in shear stress τ with respect to different flow parameters at lower plate(LP) and upper plate(UP) is presented numerically through Tables $1-4$. In Table 1, we have presented the variation of shear stress with Reynolds number R_1 for the fixed values of other parameters. At lower plate, as R_1 is increasing through values 1 and 3, shear stress is decreasing and further an increase in R_1 results an increase in shear stress. While at the upper plate shear stress shows reverse trends.

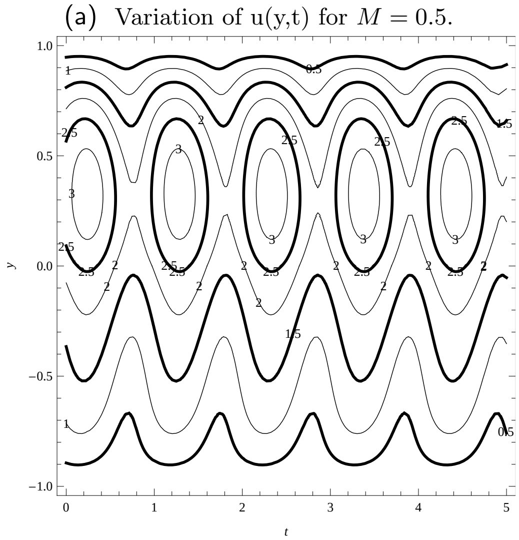
In Table 2, it is noticed that as the Hartmann number M is increasing shear stress at both plate is decreasing. In Table 3, it is seen that as the parameter α is increasing for $\omega t = 0$, the shear stress at the both plates are increasing while at $\omega t = \frac{\pi}{4}$ and $\frac{\pi}{2}$, the shear stress is decreasing at both plates. Table 4 shows that for all values of ωt, as β is increasing the shear stress at lower plate is increasing while shear stress at upper plate is decreasing.



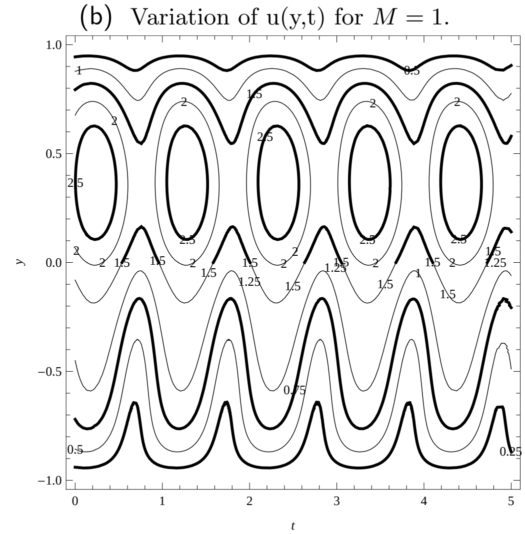
(a) Variation of u(y,t) for $M = 0.5$.



(b) Variation of u(y,t) for $M = 1$.



(c) Variation of u(y,t) for $M = 2.5$.

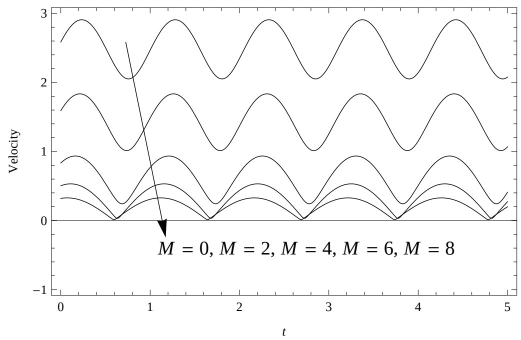


(d) Variation of u(y,t) for $M = 3.5$.

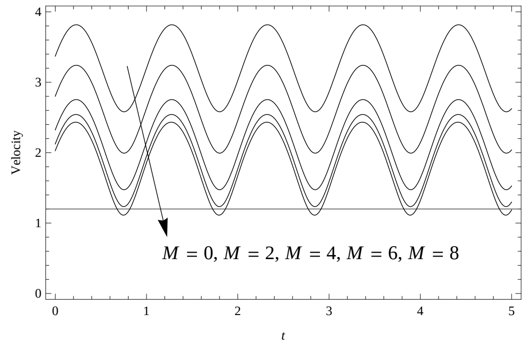
Fig. 2

	mod τ	$M = 1$	$M = 3$	$M = 5$	$M = 7$	$M = 9$
$\omega t = 0$	LP	2.70649	1.45453	0.88481	0.63303	0.49435
	UP	3.29837	2.81778	2.57464	2.45931	2.39351
$\omega t = \frac{\pi}{4}$	LP	2.56893	1.35882	0.82131	0.58645	0.45766
	UP	3.13420	2.65487	2.41760	2.30612	2.24451
$\omega t = \frac{\pi}{2}$	LP	2.04589	1.05973	0.63499	0.45220	0.35254
	UP	2.49968	2.09418	1.89853	1.80766	1.75769

Tab. 2: Variation of τ with M ($R_1 = 2, \beta = 1.2, \alpha = 1.4, K_1 = 0.5, K_2 = 3, P_s = 2.5, P_O = 2, \omega = \frac{\pi}{8}$).

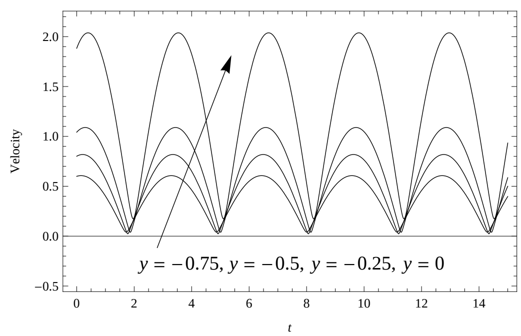


(a) Variation of $u_1(y,t)$ for M at $\omega = 6$.

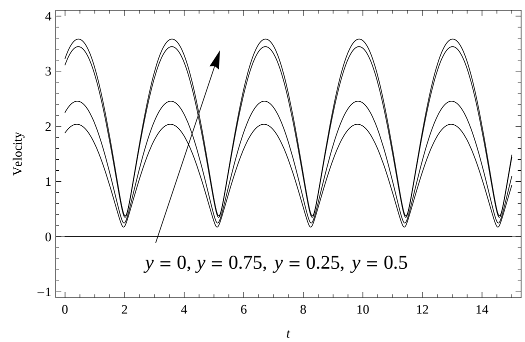


(b) Variation of $u_2(y,t)$ for M at $\omega = 6$.

Fig. 3

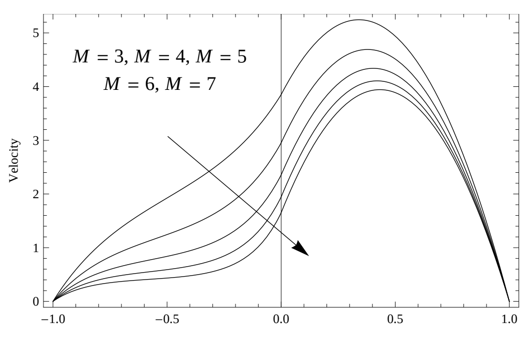


(a) Variation of $u_1(y,t)$ for y at $\omega = 2$.

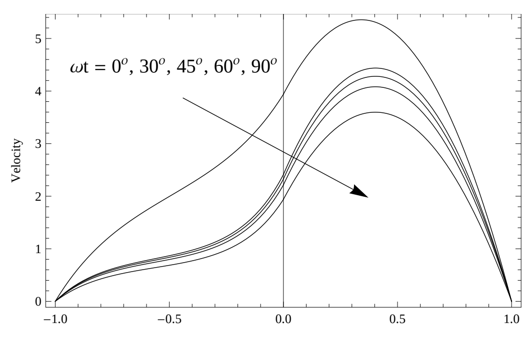


(b) Variation of $u_2(y,t)$ for y at $\omega = 2$.

Fig. 4



(a) Variation of $u(y,t)$ for y at $\omega t = \pi/4$.



(b) Variation of $u(y,t)$ for y at $M = 5, t = 1$

Fig. 5

	mod τ	$\alpha = 1$	$\alpha = 1.1$	$\alpha = 1.2$	$\alpha = 1.3$	$\alpha = 1.4$
$\omega t = 0$	LP	2.72399	2.72469	2.72524	2.72568	2.72605
	UP	3.31773	3.31951	3.32089	3.32198	3.32287
$\omega t = \frac{\pi}{4}$	LP	2.55591	2.55556	2.55524	2.55495	2.55469
	UP	3.12397	3.12149	3.11932	3.11741	3.11572
$\omega t = \frac{\pi}{2}$	LP	2.00506	2.00376	2.00264	2.00168	2.0084
	UP	2.46220	2.45593	2.45061	2.44604	2.44208

Tab. 3: Variation of τ with α ($R_1 = 1, \beta = 1.2, M = 1, K_1 = 0.5, K_2 = 3, P_s = 2.5, P_O = 2, \omega = \frac{\pi}{8}$).

	mod τ	$\beta = 1$	$\beta = 1.1$	$\beta = 1.2$	$\beta = 1.3$	$\beta = 1.4$
$\omega t = 0$	LP	2.66969	2.69955	2.72605	2.74973	2.77101
	UP	3.46637	3.39042	3.32287	3.26238	3.20790
$\omega t = \frac{\pi}{4}$	LP	2.49994	2.52891	2.55469	2.57777	2.59855
	UP	3.24676	3.17739	3.11372	3.06057	3.01094
$\omega t = \frac{\pi}{2}$	LP	1.95595	1.97968	2.00084	2.01984	2.03700
	UP	2.54111	2.48865	2.44208	2.40047	2.36311

Tab. 4: Variation of τ with β ($R_1 = 1, \alpha = 1.4, M = 1, K_1 = 0.5, K_2 = 3, P_s = 2.5, P_O = 2, \omega = \frac{\pi}{8}$).

6 Conclusions

We have studied the pulsating MHD flow of two immiscible viscous fluids in porous medium between two parallel pates. The fluid in region-I was taken to be conducting while of region-II to be non-conducting. We used no slip conditions at the plates and matching conditions at the interface of two porous mediums. The analytical results are presented separately for velocities mass flux and shear stress in both regions. The results reveal that the pulsating velocities in both regions is reduced by increasing Hartmann number M and frequency parameter ωt.The shear stress at both plates are reduced by decreasing M and ωt. Shear stress at both plate show mix trends with Reynolds number R_1. As β is increasing, the shear stress at lower plate is increasing while the shear stress at upper plate is decreasing. As α is increasing for $\omega t = 0$, the shear stress at both plates are increasing while for $\omega t = \frac{\pi}{4}, \frac{\pi}{2}$, the shear stress follows reverse trend.

References

[1] Hafeez Y. Hafeez and Chifu E. Ndikilar : *Flow of viscous fluid between two parallel porous plates with botom injection and top suction*-Progress in Physics vol.10, Issue,1 2014

[2] J.C.Umavathi and Abdul Mateen : *Oscillatory Hartmann two fluid flow and heat transfer in a horizontal channel*-Int.J.of Applied Mechanics and Engineering, 2006, vol.11, No.1, pp.155-178,2006

[3] Chamkha A.J. : *Flow of two-immiscible fluids in porous and non-porous channels* – ASME Journal of Fluids Engineering, vol.122,2000

[4] Z. Q. Chen et.al : *An experimental study of two phase flow and boiling heat transfer in bi-dispersed porous channels*- Int. Comm. Heat Mass Transfer, Vol. 27, No. 3, pp. 293-302 pp.117-124,2000

[5] K.D. Singh and S.K. Rana (1992) :*Three dim flow and heat transfer through a porous medium*-Indian J.pure appl.math.,23(7):905-914,December 1992

[6] Chen, S. C., and Vafai, K. *Analysis of Free Surface Momentum and Energy Transport in Porous Media*-Numer. Heat Transfer, Part A, 29, pp.281–296,1996.

[7] Vafai, K., and Kim, S.: *Forced Convection in a Channel Filled with a Porous Medium: An Exact Solution*- ASME J. Heat Transfer, 111, pp. 1103–1106,1989.

[8] R. Hilfer:*Macroscopic Equations of Motion for Two Phase Flow in Porous Media*- Physical Review E ,1998.

[9] Muhammad Sahimi(1995) book *Flow and Transport in Porous Media and Fractured Rock* WILEY-VCH Verlag GmbH & Co. KGaA.

[10] Srinivasan, V., and Vafai, K.: *Analysis of Linear Encroachment in Two-Immiscible Fluid Systems*- ASME J. of Fluids Engineering, 116, pp. 135–139,1994.

[11] Shail R. : *On laminar tow-phase flow in magnetohydrodynamics.* – Int. J. Eng. Sci., vol.11, p.1103,1973.

[12] Berman A S.: *Laminar flow in channels with porous walls*-Journal of applied physics,v.24,1232,1953.