# Generalization of Mittag - Leffler Function and It's Properties

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**Abstract**

The main objective of this paper is to provide a new generalization of Mittag - Leffler function using extended Beta function [10]. Some integral representations, recursion formulas, Mellin transform and derivative formula are obtained for new extended Mittag - Lefler function.

## 1 Introduction

Gösta Mittag - Leffler [6, 7] in the year 1905 introduced the function

$$E_{\alpha}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)} \tag{1.1}$$

where $\alpha \in$ C; $\Re(\alpha)>0$. It is the generalization of exponential series, since, for $\alpha=1$, we have exponential series. In the year 1905, Wiman [16, 17] introduced a generalization of (1.1) in the form

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)} \tag{1.2}$$

Prabhakar [11] introduced the generalized Mittag - Leffler function by

[1] corresponding author

$$E^{\gamma}_{\alpha,\beta}\left(z\right)=\sum_{k=0}^{\infty}\frac{\left(\gamma\right)_k z^k}{\Gamma(\alpha k+\beta)k!} \tag{1.3}$$

where $\alpha,\beta,\gamma\in C$, $\Re(\alpha)>0$, $\Re(\beta)>0$, $(\gamma)_0=1$ and $\left(\gamma\right)_n=\gamma(\gamma+1)\ldots\ldots\ldots(\gamma+n-1)=\frac{\Gamma(\gamma+n)}{\Gamma(\gamma)}$, $\gamma\neq 0$. For $\gamma=1$, it reduces to the Mittag - Leffler function given in equation (1.2). Özarslan and Yilmaz [9] extended the Mittag - Leffler as follows

$$E^{(\gamma;c)}_{\alpha,\beta}\left(z;p\right)=\sum_{k=0}^{\infty}\frac{B_p(\gamma+k,c-\gamma)\left(c\right)_k z^k}{B\left(\gamma,c-\gamma\right)\Gamma(\alpha k+\beta)k!} \tag{1.4}$$

$\left(p\geq 0;\Re(c)>\Re(\gamma)>0\right)$,
where for $B_p(x,y)$ we have

$$B_p(x,y)=\int_0^1 t^{x-1}\left(1-t\right)^{y-1}e^{\frac{-p}{t(1-t)}}\,dt \tag{1.5}$$

$(\Re(p)>0,\Re(x)>0,\Re(y)>0)$, the extended Euler's Beta function defined in [1].

In this paper, we extended the Mittag - Leffler function defined in (1.4) as follows:

$$E^{(\gamma,c)}_{\alpha,\beta}\left(z;p,\delta_1,\delta_2\right)=\sum_{k=0}^{\infty}\frac{B_p^{(\delta_1,\delta_2)}(\gamma+k,c-\gamma)\left(c\right)_k z^k}{B\left(\gamma,c-\gamma\right)\Gamma(\alpha k+\beta)k!} \tag{1.6}$$

$\left(p\geq 0;\Re(c)>\Re(\gamma)>0\right)$, using the extended Euler's Beta function

$$B_p^{(\delta_1,\delta_2)}(x,y)=\int_0^1 t^{x-1}\left(1-t\right)^{y-1}\ {}_1F_1\left(\delta_1;\delta_2;\frac{-p}{t\left(1-t\right)}\right)dt \tag{1.7}$$

$(\Re(p)>0,\Re(x)>0,\Re(y)>0,\Re(\delta_1)>0,\Re(\delta_2)>0)$, defined in [10].

This paper is organized as follows: In Section 2, different integral representations of the extended Mittag - Leffler function in terms of Prabhakar's Mittag-Leffler function are obtained. Further, relations between the extended Mittag - Leffler function and elementary functions are discussed. Some recursion formulas are also given for different parameters. The Mellin transform of the extended Mittag - Leffler function is obtained by means of the generalized Wright hypergeometric function [14]. In Section 3, we have obtained fractional derivative representations of the extended Mittag - Leffler function and some derivative formulas are given.

## 2 Properties of the extended Mittag - Leffler function

In this section, we discuss the integral representations, recursion formulas and Mellin transform of the extended Mittag - Leffler function.

**Theorem 2.1.** *For the extended Mittag - Leffler function, we have*

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2) = \frac{1}{B(\gamma,c-\gamma)}\int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1} \times {}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right)E_{\alpha,\beta}^{c}(tz)\,dt \tag{2.1}$$

*where* $p \geq 0, \Re(c) > \Re(\gamma) > 0, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\delta_1) > 0, \Re(\delta_2) > 0.$

*Proof.* Using equation (1.7) in equation (1.6), we get

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p;\delta_1;\delta_2) = \sum_{k=0}^{\infty}\left\{\int_0^1 t^{\gamma+k-1}(1-t)^{c-\gamma-1}\ {}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right)dt\right\} \times \frac{(c)_k\, z^k}{B(\gamma,c-\gamma)\Gamma(\alpha k+\beta)k!}$$

Interchanging the order of integration, which is permissible under conditions stated in Theorem 2.1, we find that

$$\begin{aligned} E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2) &= \int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1}\ {}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right)dt \\ &\quad \times \sum_{k=0}^{\infty}\frac{(c)_k\,(tz)^k}{B(\gamma,c-\gamma)\Gamma(\alpha k+\beta)k!} \\ &= \frac{1}{B(\gamma,c-\gamma)}\int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1} \\ &\quad \times {}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right)E_{\alpha,\beta}^{c}(tz)\,dt. \end{aligned}$$

□

**Corollary 2.1.** *Using the identity* ${}_1F_1(a;b;z) = e^z\ {}_1F_1(b-a,b,-z)$ *in the Theorem 2.1, we get*

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2) = E_{\alpha,\beta}^{(\gamma;c)}(z;p)_1\,F_1\left(\alpha;\beta;\frac{p}{t(1-t)}\right). \tag{2.2}$$

**Corollary 2.2.** *Taking* $t = \frac{u}{1+u}$ *in the Theorem 2.1, we get*

$$\begin{aligned} E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2) &= \frac{1}{B(\gamma,c-\gamma)} \int_0^\infty \frac{u^{\gamma-1}}{(u+1)^c} \, {}_1F_1\left(\alpha;\beta;\frac{-p(1+u)^2}{u}\right) \\ &\quad \times E_{\alpha,\beta}^{c}\left(\frac{uz}{1+u}\right) du. \end{aligned} \tag{2.3}$$

**Corollary 2.3.** *Taking* $t = \sin^2\theta$ *in the Theorem 2.1, we get*

$$\begin{aligned} E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2) &= \frac{2}{B(\gamma,c-\gamma)} \int_0^{\frac{\pi}{2}} \sin^{2\gamma-1}\theta \cos^{2c-2\gamma-1}\theta \\ &\quad \times {}_1F_1\left(\alpha;\beta;\frac{-p}{\sin^2\theta\cos^2\theta}\right) E_{\alpha,\beta}^{c}\left(z\sin^2\theta\right) d\theta. \end{aligned} \tag{2.4}$$

Using the recurrence formula

$$E_{\alpha,\beta}^{c}(tz) = \beta E_{\alpha,\beta+1}^{c}(tz) + \alpha z \frac{d}{dz} E_{\alpha,\beta+1}^{c}(tz)$$

of Bayram and Kurulay [5] in equation (2.1) we get the following recurrence formula for the new extended Mittag - Leffler function.

**Corollary 2.4.** *For the extended Mittag - Leffler function, we have*

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2) = \beta E_{\alpha,\beta+1}^{(\gamma,c)}(z;p,\delta_1,\delta_2) + \alpha z \frac{d}{dz} E_{\alpha,\beta+1}^{(\gamma,c)}(z;p,\delta_1,\delta_2)$$

*where* $p \geq 0, Re(c) > Re(\gamma) > 0, Re(\alpha) > 0, Re(\beta) > 0, Re(\delta_1) > 0, Re(\delta_2) > 0.$

**Theorem 2.2.** *The extended Mittag - Leffler function* $E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)$ *satisfies the fol-*

*lowing relations:*

$$(\delta_2-\delta_1)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1-1,\delta_2)+(2\delta_1-\delta_2+z)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)$$
$$=\delta_1 E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+1,\delta_2); \tag{2.5}$$

$$\delta_2(\delta_2-1)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2-1)+z(\delta_2-\delta_1)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2+1)$$
$$=\delta_2(\delta_2-1+z)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2); \tag{2.6}$$

$$(\delta_1-\delta_2+1)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)+(\delta_2-1)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2-1)$$
$$=\delta_1 E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+1,\delta_2); \tag{2.7}$$

$$\delta_2 E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)-\delta_2 E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1-1,\delta_2)-zE_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2+1)=0; \tag{2.8}$$

$$\delta_2(\delta_1+z)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)+z(\delta_1-\delta_2)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2+1)$$
$$=\delta_1\delta_2 E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+1,\delta_2); \tag{2.9}$$

$$(\delta_1-1+z)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)+(\delta_2-\delta_1)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1-1,\delta_2)$$
$$=(\delta_1-1)E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2-1). \tag{2.10}$$

**Theorem 2.3.** *The following recursion formulas hold true for the parameter $\delta_1$ of the extended Mitteg - Leffer function $E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)$ :*

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+n,\delta_2)$$
$$=E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)-\frac{p}{\delta_2 t(1-t)}\sum_{n_1=1}^{n}E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+n_1,\delta_2+1), \tag{2.11}$$

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1-n,\delta_2)$$
$$=E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)+\frac{p}{\delta_2 t(1-t)}\sum_{n_1=0}^{n-1}E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1-n_1,\delta_2+1). \tag{2.12}$$

**Theorem 2.4.** *The following recursion formulas hold true for the parameter $\delta_1$ of the extended Mitteg - Leffer function $E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)$ :*

(2.13)
$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+n,\delta_2)=\sum_{n_1=0}^{n}\binom{n}{n_1}\left(\frac{-p}{t(1-t)}\right)^{n_1}\frac{1}{(\delta_2)_{n_1}}E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+n_1,\delta_2+n_1),$$

(2.14)
$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1-n,\delta_2)=\sum_{n_1=0}^{n}\binom{n}{n_1}\left(\frac{p}{t(1-t)}\right)^{n_1}\frac{1}{(\delta_2)_{n_1}}E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2+n_1).$$

**Theorem 2.5.** *The following recursion formulas hold true for the parameter* $\delta_2$ *of the extended Mitteg - Leffer function* $E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)$ :

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2-n) = E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2) - \frac{\delta_1 p}{t(1-t)} \sum_{n_1=1}^{n} \frac{E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+1,\delta_2+2-n_1)}{(\delta_2-n_1)(\delta_2-n_1+1)}, \tag{2.15}$$

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2-n) = \sum_{n_1=0}^{n} \binom{n}{n_1} \frac{(\delta_1)_{n_1}}{(\delta_2-n)_{n_1}(\delta_2)_{n_1}} \left(\frac{-p}{t(1-t)}\right)^{n_1} \times E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1+n_1,\delta_2+n_1). \tag{2.16}$$

**Definition 2.1.** *The generalized Wright function* ${}_p\psi_q$ $(p,q \in N_0)$ *is defined as [3, p.183]*

$$ {}_p\psi_q\left[(a_1,A_1),.....(a_1,A_2);(b_1,B_1),.....(b_1,B_2);z\right] = \sum_{n=0}^{\infty}\prod_{n=0}^{\infty} \frac{\Gamma(a_1+nA_1)...\Gamma(a_p+nA_p)}{(b_1+nB_1)...\Gamma(b_p+nB_p)}\frac{z^n}{n!} \tag{2.17}$$

*where*

$$A_j \in R(j=1,...,p), B_j \in R(j=1,...,q); 1+\sum_{n=0}^{q} B_j - \sum_{j=0}^{p} A_j \geq 0. \tag{2.18}$$

In the next theorem, we give the Millin transform of the extended Mittag - Leffler function in terms of the Wright generalized hypergeometric function.

**Theorem 2.6.** *The Millin transform of the new extended Mittag - Leffler function is given by*

$$M\{E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2);s\} = \frac{\Gamma^{(\alpha,\beta)}(s)\Gamma(c+s-\gamma)}{\Gamma(\gamma)\Gamma(c-\gamma)} \times {}_2\psi_2\left[(c,1),(\gamma+s,1)(\beta,\alpha),(c+2s,1);z\right] \tag{2.19}$$

*where* $p \geq 0$, $\Re(c) > \Re(\gamma) > 0, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\delta_1) > 0, \Re(\delta_2) > 0, \Re(s) > 0$ *and* ${}_2\psi_2$ *is the Wright generalized hypergeometric function.*

*Proof.* Taking the Mellin transform of the extended Mittag - Leffler function we have

$$\begin{aligned} M\{E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2);s\} &= \int_0^\infty p^{s-1} E_{\alpha,\beta}^{(\gamma,c)}(z;p;\delta_1;\delta_2)\,dp \\ &= \frac{1}{B(\gamma,c-\gamma)} \int_0^\infty p^{s-1} \int_0^1 t^{\gamma-1}(1-t)^{c-\gamma-1} \\ &\quad \times {}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right) E_{\alpha,\beta}^{c}(tz)\,dtdp \end{aligned}$$

Interchanging the order of integration, which is valid because of the conditions stated in Theorem 2.6, we find that

$$M\{E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2);s\} = \frac{1}{B(\gamma,c-\gamma)}\int_0^1 \left[t^{\gamma-1}(1-t)^{c-\gamma-1}\ E_{\alpha,\beta}^{c}(tz)\,dt\right] \times \int_0^\infty p^{s-1}\ {}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right)dp$$

Now taking $u=\frac{p}{t(1-t)}$ in the above equation and using the fact that $\Gamma^{(\alpha,\beta)}(s)=\int_0^\infty u^{s-1}$ ${}_1F_1(\alpha;\beta;-u)\,du$, we get

$$M\{E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2);s\} = \frac{\Gamma^{(\alpha,\beta)}(s)}{B(\gamma,c-\gamma)}\int_0^1 t^{\gamma+s-1}(1-t)^{c+s-\gamma-1}\ E_{\alpha,\beta}^{c}(tz)\,dt$$

Now proceeding as in Özarslan and Yilmaz [9] we get the result. □

**Corollary 2.5.** *Taking $s=1$ in the Theorem 2.6 and using $\Gamma^{(\alpha,\beta)}(1)=\frac{\beta}{\alpha}$ (see [10, p. 4603] ), we get*

$$\int_0^\infty E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2)\,dp = \frac{\beta}{\alpha\ B(\gamma,c-\gamma)}\int_0^1 t^{\gamma+s-1}(1-t)^{c+s-\gamma-1}\ E_{\alpha,\beta}^{c}(tz)\,dt \tag{2.20}$$

**Corollary 2.6.** *Taking the inverse Mellin transform on both sides of equation (2.19), we get*

$$E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2) = \frac{1}{2\pi i\Gamma(\gamma)\Gamma(c-\gamma)}\int_{v-i\infty}^{v+i\infty}\Gamma^{(\alpha,\beta)}(s)\,\Gamma(c+s-\gamma) \times\ {}_2\psi_2\left[(c,1),(\gamma+s,1)(\beta,\alpha),(c+2s,1);z\right]p^{-s}ds \tag{2.21}$$

*where $v>0$.*

## 3 Derivative properties of the new exteded Mittag-Leffler function

**Definition 3.1.** *The Riemann-Liouville fractional derivative operator of order $\mu$ is defined as [2, 4, 12, 13, 15]*

$$D_z^{\mu}(\{f(z)\} = \frac{1}{\Gamma(-\mu)}\int_0^z f(t)(z-t)^{-\mu-1}\,dt,\ Re(\mu)<0, \tag{3.1}$$

*where the integration path is a line from 0 to $z$ in complex plane. When $m-1<Re(\mu)<m$ $(m=1,2,3...)$ it is defined as*

$$D_z^{\mu}(\{f(z)\} = \frac{d^m}{dz^m}D_z^{\mu-m}(\{f(z)\}\,.$$

**Definition 3.2.** *The extended Riiemann - Liouville fractional derivative is defined as [8]*

$$(3.2)\quad D_z^{\mu,p}(\{f(z)\} = \frac{1}{\Gamma(-\mu)}\int_0^z f(t)(z-t)^{-\mu-1}\exp\left(\frac{-pz^2}{t(z-t)}\right)dt,\ \ Re(\mu)<0, \Re(p)>0$$

*and for $m-1<Re(\mu)<m$ $(m=1,2,3...)$ it is defined as*

$$D_z^{\mu,p}(\{f(z)\} = \frac{d^m}{dz^m}D_z^{\mu-m}(\{f(z)\}.$$

*when $p=0$, we get Riemann-Liouville fractional derivative operator of order $\mu$.*

**Theorem 3.1.** *If $p\geq 0, Re(\mu)>Re(\lambda)>0, Re(\alpha)>0, Re(\beta)>0,$ then*

$$(3.3)\qquad D_z^{\lambda-\mu,p}\left\{z^{\lambda-1}E_{\alpha,\beta}^{c}(z)\right\} = \frac{z^{\mu-1}B(\lambda,c-\lambda)}{\Gamma(\mu-\lambda)_1F_1\left(\alpha;\beta;\frac{p}{t(1-t)}\right)}E_{\alpha,\beta}^{(\gamma,c)}(z;p,\delta_1,\delta_2).$$

*Proof.* From equation (3.2) we have

$$\begin{aligned}&D_z^{\lambda-\mu,p}(\left\{z^{\lambda-1}E_{\alpha,\beta}^{c}(z)\right\}\\ =\ &\frac{1}{\Gamma(\mu-\lambda)}\int_0^z t^{\lambda-1}E_{\alpha,\beta}^{c}(t)(z-t)^{-\lambda+\mu-1}\exp\left(\frac{-pz^2}{t(z-t)}\right)dt\\ =\ &\frac{z^{-\lambda+\mu-1}}{\Gamma(\mu-\lambda)}\int_0^z t^{\lambda-1}E_{\alpha,\beta}^{c}(t)\left(1-\frac{t}{z}\right)^{-\lambda+\mu-1}\exp\left(\frac{-pz^2}{t(z-t)}\right)dt.\end{aligned}$$

If $u=\frac{t}{z}$ in the above equation then we get

(3.4)
$$D_z^{\lambda-\mu,p}(\left\{z^{\lambda-1}E_{\alpha,\beta}^{c}(z)\right\} = \frac{z^{\mu-1}}{\Gamma(\mu-\lambda)}\int_0^1 u^{\lambda-1}E_{\alpha,\beta}^{c}(uz)(1-u)^{-\lambda+\mu-1}\exp\left(\frac{-p}{u(1-u)}\right)du$$

With the help of Corollary (2.1) and equation (2.1), equation (3.4) reduces to

$$D_z^{\lambda-\mu,p}\left\{z^{\lambda-1}E_{\alpha,\beta}^{c}(z)\right\} = \frac{z^{\mu-1}B(\lambda,c-\lambda)}{\Gamma(\mu-\lambda)_1F_1\left(\alpha;\beta;\frac{p}{t(1-t)}\right)}E_{\alpha,\beta}^{(\gamma,c)}(z;p;\delta_1;\delta_2).$$

□

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