Logarithmic Variation of Load Carrying Capacity for the Rotation of Porous Journal Bearing

Dr. Mohammad Miyan

Associate Professor, Department of Mathematics,
Shia P.G.College, University of Lucknow, Lucknow, India

Abstract

The second order rotatory theory of hydrodynamic lubrication was supported on the expression obtained by holding the terms containing up to second powers of rotation number in the extended generalized Reynolds equation. In this paper, there are some new superb basic solutions for the porous bearings at regarding the second order rotatory theory of hydrodynamic lubrication. The expressions for logarithmic variation of the load carrying capacity with reference to rotation number M and viscosity μ are obtained by taking the arbitrary values of the quantitative relation L/D and e. The analysis of equation for load capacity, tables and graphs showed that the load carrying capacity of the bearing varies with the viscosity of the fluid and increases slightly with viscosity. The load capacity increases with increasing values of rotation number. Within the absence of rotation, the equation of load capacity reduces to the classical solutions of the classical theory of hydrodynamic lubrication.

Subject class [2010]:
Keywords: Permeability, Porous bearing, Load capacity, Reynolds equation, Rotatory theory.

1 Introduction

Within the idea of hydrodynamic lubrication, two-dimensional classical theories were first given by O. Sir Reynolds [1]. In 1886, with relevancy a classical experiment by B. Tower [2], he developed an awfully necessary equation, which was spoken as Reynolds Equation [2]. The formation and basic mechanism of the fluid film was analyzed by that experiment on taking some important assumptions [3] i.e., the fluid film thickness is implausibly very little as compare to the axial and longitudinal dimensions of fluid film and if the things layer is to transmit pressure between the shaft and so the bearing, the layer ought to have varied thickness [4]. Later O. Reynolds himself derived associate improved version of Reynolds Equation legendary as: Generalized Reynolds Equation that depends on body, film thickness, density and surface or thwart wise velocities [5], [6], [7]. The rotation of the fluid film concerning associate axis that lays at intervals the film offers some new finishes up in lubrication problems with hydrodynamics. The origin of rotation is derived by bound general theorems associated with vorticity within the rotating fluid dynamics [7]. The rotation induces an element of vorticity within the direction of rotation of fluid film and therefore the effects arising from it are predominant, for giant Taylors number, it ends up in the streamlines changing into confined to plane transverse to the direction of rotation of the film [8], [9]. The new extended version of Generalized Reynolds Equation is alleged to be Extended Generalized Reynolds Equation that takes into consideration of the consequences of the uniform rotation regarding an axis that lies within the fluid film and depends on the rotation number M, [10], [11] i.e. the root of the standard Taylors number. The generalization of the classical theory of hydrodynamic lubrication is understood because the Rotatory Theory of hydrodynamic Lubrication [12], [13]. The Second Order Rotatory Theory of hydrodynamic Lubrication was given by retentive the terms containing up to second powers of M by neglecting higher powers of M [12], [14].

In the case of porous bearing, the bearing, is infinitely short, so the pressure gradient in x-direction is too smaller than in y-direction i.e., in y-direction the gradient $\frac{\partial P}{\partial y}$ is of order of $\left(\frac{P}{L}\right)$ and in the x-direction, is order of $\left(\frac{P}{B}\right)$ i.e, $L << B$ then $\frac{P}{L} >> \frac{P}{B}$, so $\frac{\partial P}{\partial x} << \frac{\partial P}{\partial y}$. Hence, the terms of $\frac{\partial P}{\partial x}$ can neglected with respect to $\frac{\partial P}{\partial y}$ in the expanded version of Generalized Reynolds Equation.

2 BOUNDARY CONDITIONS AND SOLUTION OF DIFFERENTIAL EQUATION

The Extended Generalized Reynolds Equation visible of second order rotatory theory of fluid mechanics lubrication, in ascending powers of rotation no. M and by holding the terms containing up to second powers of M and neglecting higher powers of M, are written as:

$$\frac{\partial}{\partial x}\left[F_1\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[F_1\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[F_2\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[F_2\frac{\partial P}{\partial x}\right] \tag{2.1}$$

$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}(h-MF_2)\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2 U}{2}F_1\right]-\rho W^* \tag{2.2}$$

Where,

$$F_1=\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right],\ F_2=-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)$$

In the expression: P is the pressure, ρ is the fluid density, x, y and z are coordinates, U is the sliding velocity, μ is the viscosity and W^* is fluid velocity in z-direction. Let we choose $U=-U$, $h=h(x)$, $P=P(y)$ and $W^*=-\frac{\partial P}{\partial z}|_{z=0}\frac{\phi}{\mu}$, Where $\frac{\partial P}{\partial z}$ shows the pressure gradient at the surface of bearing and ϕ is the property called permeability, which with the porosity and pore sizes. As the requirements of continuity, we have for the porous matrix $\frac{\phi}{\mu}\nabla W^*=\nabla^2 P=0$ i.e.,$\nabla^2 P=0$. Now the problem is to solve the governing equation (2.2) for the pressures in oil film simultaneously with that of Laplace for the porous matrix with a common pressure gradient $\frac{\partial P}{\partial z}$ at boundary, we have the equation of continuity i.e.,$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}=0$. We have use the assumption that bearing is infinitely short and $\frac{\partial P}{\partial z}$ is linear across the matrix and is zero at the outer surface of the porous bearing shell to solve the equation (2.2), we have $\frac{\partial^2 P}{\partial x^2} = 0, \frac{\partial^2 P}{\partial z^2} = K(\text{constant}), \frac{\partial^2 P}{\partial y^2} = -K$ i.e., $\frac{\partial P}{\partial z}|_{z=0} = KH = \frac{\partial^2 P}{\partial y^2}|_{z=0}H$. The H is the wall thickness of porous bearing. Now the equation (2.2) becomes

$$[F_1\rho]\frac{d^2P}{dy^2} - \left(\frac{d}{dx}F_2\right)\frac{dP}{dy} = \frac{d}{dx}\left[\frac{\rho U}{2}H - MF_2\right] - \rho\left(-\frac{dP}{dz}|_{z=0}\frac{\phi}{\mu}\right) \tag{2.3}$$

The film thickness $'h'$ and $'y'$ can be taken as; $h = C(1 + e\cos\theta), y = R\theta$, where e is the eccentricity, θ is the angular coordinates measured from x-direction and R is the radius of bearing. For the determination of pressure the boundary conditions are taken as: $P = 0, y = \pm\frac{L}{2}$.

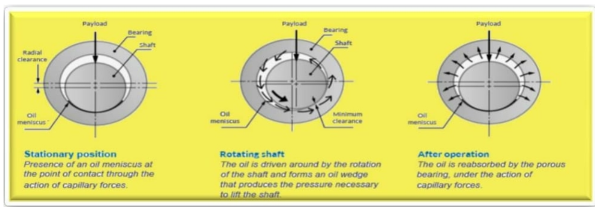

Figure-1. The three stages of the shaft of porous bearing under operation

The rotation of shaft of the porous bearing [15] is shown in the figure-1. The solution of the differential equation (4.1) under the doundary conditions gives the pressure for porous bearing as follows:

$$\begin{aligned} P = {} & \frac{(3\mu CUe\sin\theta + 12KH\phi R)(L^2 - 4y^2)}{4(1 + e\cos\theta)^3 R} + \frac{\rho Ce\sin\theta(U\mu + 4KH\phi)(L^2y - 4y^3)}{8\mu R(1 + e\cos\theta)^2}M \\ & + \frac{53U\mu\rho^2 Ce\sin\theta(1 + e\cos\theta) - 68RKH\phi\rho^2(1 + e\cos\theta)(L^2 - 4y^2)}{2240\mu^2 R}M^2 \end{aligned} \tag{2.4}$$

The load capacity for porous bearing is given by

$$W = \sqrt{W_x^2 + W_y^2}$$

where W_x and W_y are the components of the load capacity in x-direction and y-direction respectively.

$$W_x = -2\int_0^{\pi}\int_0^{L/2} P\cos\theta R d\theta dy$$

$$W_x = 2\int_0^{\pi}\int_0^{L/2} P\sin\theta R d\theta dy$$

The W_x and W_y in the increasing values of M are given by

$$(2.5) \quad W_x = -\left(\frac{\mu U e^2}{C^2(1-e^2)^2} + \frac{KH\phi R\pi e}{(1-e^2)^{5/2}}\right) L^3 + \frac{\rho C(\mu U + 4KH\phi)}{64\mu}\left(\frac{1}{e}\log\frac{1+e}{1-e} - \frac{2}{1-e^2}\right) L^4 M + \left(\frac{106\mu U e^2\rho^2 C^2 - 204RCKH\phi\rho^2\pi e}{13440\mu^2}\right) L^3 M^2$$

$$(2.6) \quad W_y = -\left(\frac{\mu U\pi e}{4C^2(1-e^2)^{3/2}} + \frac{4KH\phi R}{(1-e^2)^2}\right) L^3 + \frac{\pi e\mu C(\mu U + 4KH\phi)}{128\mu(1-e^2)^{3/2}} L^4 M + \left(\frac{53\mu U\rho^2 C^2\pi e - 272RCKH\phi\rho^2}{4480\mu^2}\right) L^3 M^2$$

3 NUMERICAL SIMULATIONS AND GRAPHICAL ANALYSIS

By taking the values of different mathematical terms in C.G.S. system as follows: $\theta = 30^o, \mu = 0.0002, C = 0.0067, \rho = 0.9, U = 10^2, h = 0.00786, y = 1, H = 0.05, \phi = 0.0025, R = 3.35$; the calculated values of load capacity with respect to M, by taking $\mu = 0.0002$, are given by table 1.

e↓	L/D ↓	M→	0.2	0.4	0.6	0.8	1.0
0.2	0.5	W	318359.2200	318360.4200	318362.3400	318364.9997	318368.3960
0.2	1.0	W	2546874.560	2546884.912	2546901.120	2546923.118	2546951.093
0.9	0.5	W	402079.4716	402082.259	402085.7945	402090.0741	402095.0935
0.9	1.0	W	3216635.899	3216658.291	3216686.69	3216721.014	3216761.365

Table 1

Also by taking the values of different mathematical terms in C.G.S. system as follows: $\theta = 30^o, e = 0.9, C = 0.0067, \rho = 0.9, U = 10^2, h = 0.00786, H = 0.05, \phi = 0.0025, R = 3.35, L/D = 1$; the calculated values of load capacity with respect to μ by taking $M = constant = 1.0$, are given by Table 2.

μ	0.0002	0.0003	0.0004	0.0005	0.0006
W	3217061.181	4825306.167	6433586.076	8041213.813	9650176.837

Table 2

4 CONCLUSIONS

The exponential variation of the load capacity with respect to ratation number M and viscosity μ are obtained by taking the constant values of the ratio L/D and e, i.e., are as follows:

$$W = 7.297\log_e M + 40209(L/D = 0.5, e = 0.9);$$
$$W = 74\log_e M + 3E + 06(L/D = 1.0, e = 0.9);$$
$$W = 6E + 06\log_e \mu + 5E + 07(L/D = 1.0, e = 0.9)$$

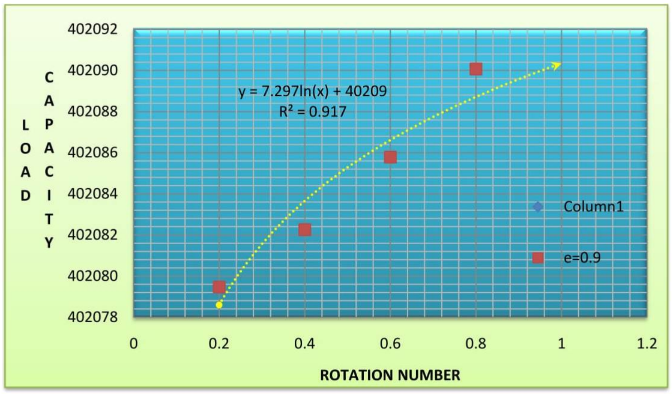

Figure-2. Variation of load capacity with respect to *M* for *e=0.9; L/D=0.5*.

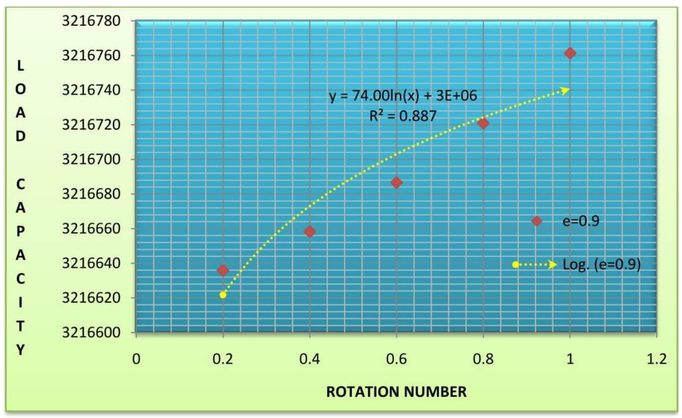

Figure-3. Variation of load capacity with respect to *M* for *e=0.9; L/D=1.0*.

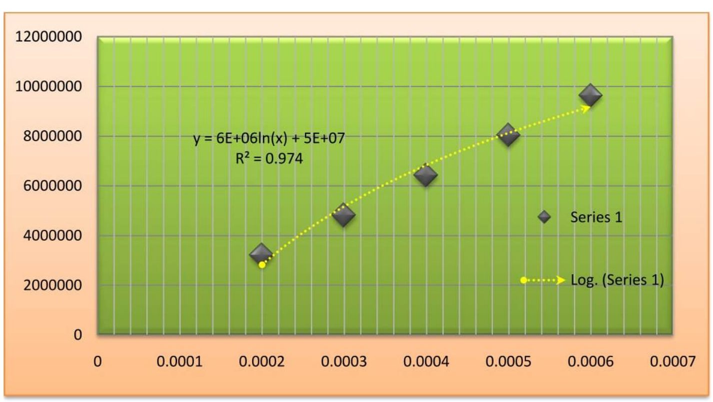

Figure-4. Variation of load capacity with respect to viscosity for *e=0.9; L/D=1.0*.

Hence, $W \propto \alpha log_e M + \beta + \gamma$, where α, β and γ are arbitrary constants.
The expression of load capacity, tables and graphs show that in second order rotatory theory, the load capacity W varies directly with the rotation number M and does not changes linearly with μ, it slightly increases with μ due to presence of the permeability factor in the numerator and denominator. The load capacity W also varies with the viscosity of the fluid medium. On taking ($M = 0$) i.e., in the absence of rotation, we get the classical solutions.

References

[1] Reynolds, O., 1886, Phil. Trans. Roy. Soc. London, Part I, 177.

[2] Reynolds, O., 1886, On the Theory of Lubrication and its Application to Mr. Beauchamp Tower's Experiment-Phil. Trans. Roy. Soc. London, 177 (I), 157.

[3] Shaw, M. C., and Macks, E. F., 1949, *Analysis and Lubrication of Bearings* – Mc. Graw Hill Book Company, Inc., New York.

[4] Cameron, A., 1958, *The Viscous Wedge Trans.* ASME, 1, 248.

[5] Pinkus, O., and Sternlicht, B. 1961, *TTheory of Hydrodynamic Lubrication* Mc. Graw Hill Book Company, Inc. New York, 5-64.

[6] Dowson, D., 1962, *A Generalized Reynolds Equations for Fluid Film Lubrication,*-Int. J. Mech. Sci., 4, 159.

[7] Chandrasekhar, S., 1970, *Hydrodynamic and Hydro magnetic Stability* Oxford University Press, London, 83.

[8] Saibel, E. A. and Macken, N.A., 1973, *Annual Review of Fluid Mechanics* Inc. Palo. Alto, Cal. Vol.5.

[9] Halling, J., 1975, *Principles of Tribology* The Macmillan Press Ltd., London, 369.

[10] Banerjee, M. B., Gupta, R. S. and Dwivedi, A. P., 1981, *The Effects of Rotation in Lubrication Problems*- WEAR, 69, 205.

[11] Banerjee, M. B., Chandra, P. and Dube, G. S., 1981 , *Effects of Small Rotation in Short Journal Bearings* – Nat. Acad. Sci. Letters, Vol. 4, No.9.

[12] Banerjee, M. B., Dube, G. S. and Banerjee, K., 1982, *The Effects of Rotation in Lubrication Problems: A New Fundamental Solutions*-WEAR, 79, pp. 311-323.

[13] Cameron, A., 1981, *Basic Lubrication Theory*-Ellis Harwood Limited, Coll. House, Watergate, Chicester, p. 45-162.

[14] Dube, G. S. and Chatterjee, A., 1988, *Proc. Nat. Acad. Sci. India*-58(58), I: 79.

[15] http://sintertech.org/portfolio/metafram-self-lubricating-bearings.