Estimation of the value of Π using Minimum and Maximum Circumference of a Circle[1]

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Abstract

In this paper we are providing another relation for finding the more accurate and precise method for the value of pi by using Minimum and Maximum circumference of a circle. The relationship can be expressed as $\pi = 90(10^n)(\sin(10^{(-n)}) + \tan(10^{(-n)}))$ where $n = 0, 1, 2, 3, 4, \cdots, \infty$ (any positive integers including zero.). Greater the value on n, the more precise and accurate would be the value of π. We will be showing the comparison as well to describe the given formula is more useful in modern mathematics.

1 Introduction

This paper is dedicated to finding the more accurate and precise value of Pi using Minimum and Maximum Circumference of a circle. The value of Pi and its estimation has been done by many authors since last century. Here we are again exploring its value and methods to achieve more accurate and highly precise value of pi. Many authors and mathematician have applied their own methods and explained this irrational number. After a long time yet there is no accurate or 100 percent valid approach to find out its value. In this paper we will explain another apporach for calculating the value of Pi by using the basics of Co-Oridnate geometry and Trigonometry. We will be applying the Minimum and Maximum Circumference for calculating the value of Pi. We will explain the given relationship and would compare them with one of the earliest method given by 'Archimedes'. We will show how the given formula is better in finding the value of the value of Pi. By using our given relationship we would be to compute the value of Pi more quickly and exact value in very less time and iterations on modern computers.

2 Derivation

2.1 Minimum Circumference of the Circle

Here we are showing a circle with unit radius where AC and CD are the two radii of the circle. In the Figure {1} In $\triangle ABC$, let,

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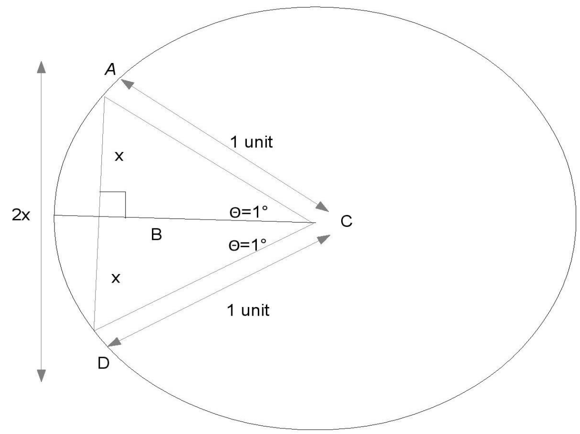

Fig. 1

$$AC = CD = radius = 1 unit$$
$$AB = BD = x$$
$$\angle ACB = 1^\circ$$
$$\angle BCD = 1^\circ$$
$$\angle ACD = 2^\circ$$

Draw $\perp AB$ on base BC

$$\angle ABC = 90^\circ, \quad \{Since\ radius = 1\ unit,\ \theta = 1^\circ\}$$
$$\sin\theta = \tfrac{x}{1}$$
$$\sin\theta = x$$
$$therefore\ 2x = 2\sin\theta$$

We know that an angle around any circle or arbitary point is 360.
So, the Circumference of the circle will be:

$$2\sin\theta * 180 = 2\pi r \ \ \{since\ r = 1\ unit\}$$
$$\pi = 180\sin\theta$$

If $\theta < 1^\circ$

$$Then\ \pi = 10 * 180\sin(10^{(-1)} \ \ \{Where\ \theta = 1/10 = 10^{(-1)}\}$$
$$And,\ \pi = (10^2) * 180\sin(10^{(-2)}) \ \ \{Where\ \theta = 1/100 = 10^{(-2)}\}$$
$$And,\ \pi = (10^3) * 180\sin(10^{(-3)}) \ \ \{Where\ \theta = 1/1000 = 10^{(-3)}\}$$
$$\vdots$$

Also,

$$\pi = (10^n) * 180\sin(10^{(-n)}) \quad \{where\theta = 10^{(-n)}\} \tag{2.1}$$

the equation (2.1) is about minimum circumference of the circle. Now we will be finding another relationship by using maximum circumference of the circle.

2.2 Maximum Circumference of the Circle

From figure 2 we are going to find the maximum circumference of the circle: In $\triangle ABC$, Let,

$$AC = radius = 1unit$$
$$AB = BD = x$$
$$\angle ACB = 1^\circ$$
$$\angle BCD = 1^\circ$$
$$\angle ACD = 2^\circ$$

Draw $\perp AB$ on base BC

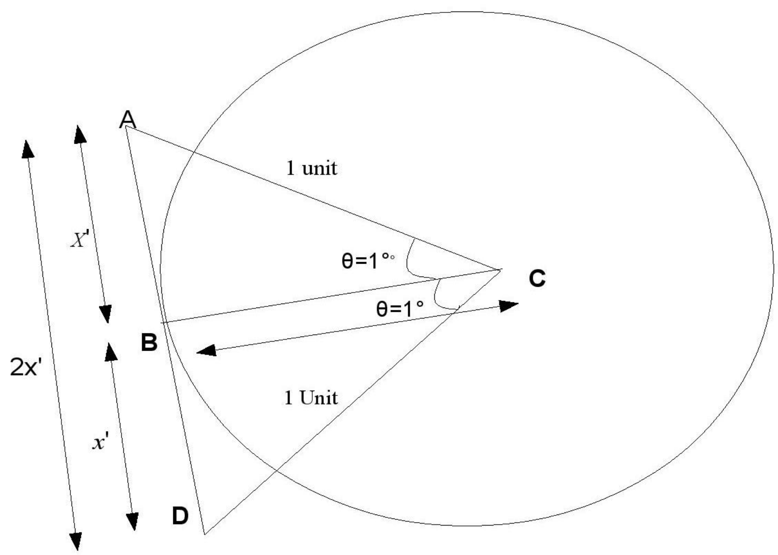

Fig. 2

$$\angle ABC = 90^\circ$$
$$tan\theta = x'/1$$

$$tan\theta = x' \quad \{radius = 1unit \ and \ \theta = 1^\circ\}$$

So,

$$2x' = 2\tan\theta$$

We Know that angle around any circle or arbitrary point is 360°. So, Circumference of Circle:

$$2\tan\theta * 180 = 2\pi r \quad \{since \ r = 1\}$$

$$\pi = 180 \tan\theta$$

If $\theta < 1$, Then,

$$\pi = 10 * 180 \tan(10^{(-1)} \quad \{Where\ \theta = 1/10 = 10^{(-1)}\}$$

And,

$$\pi = (10^2) * 180 \tan(10^{(-2)}) \quad \{Where\ \theta = 1/100 = 10^{(-2)}\}$$

And,

$$\pi = (10^3) * 180 \tan(10^{(-3)}) \quad \{Where\ \theta = 1/1000 = 10^{(-3)}\}$$

(2.2)
$$\pi = (10^n) * 180 \tan(10^{(-n)}) \quad \{Where\ \theta = 10^{(-n)}\}$$

The equation (2.2) is about maximum circumference of the circle. Now that we have found maximum and minimum circumference of the circle so for nearest value and maximum precision, we will find π by finding out arithmetic mean of equations (2.1) and (2.2).

$$\pi = ((10^n) * 180 \sin(10^{(-n)}) + (10^n) * 180 \tan(10^{(-n)}))/2$$

$$\pi = ((10^n) * 180 * \sin(10^{(-n)}) + \tan(10^{(-n)}))/2$$

$$\pi = (10^n) * 90 * \{\sin(10^{(-n)}) + \tan(10^{(-n)}))\}$$

(2.3)
$$\pi = 90 * (10^n)\{\sin(10^{(-n)}) + \tan(10^{(-n)}))\}$$

The equation (2.3) would give the best and precise value of π when the $n = 0, 1, 2, \cdots$ (positive integer)
we will be showing next the comparison and the estimation by putting various values of n

3 Validation

We are going to compare the given formula with one of the earliest **Archimedes** formula which is given below
According to **Archimedes** the value of π could be given as:

$$\pi = n \sin(180/n)$$

Where n is any positive integer.
Iteration 1: Putting $n = 0$

$$\pi = 0 \sin(180/0) = 0 \qquad (a)$$

Iteration 2: Putting $n = 1$

$$\pi = 1 \sin(180/1) = 0 \qquad (b)$$

Iteration 3: Putting $n = 12$

$$\pi = 12\sin(180/12) = 3.105828541 \qquad (c)$$

Iteration 4: Putting $n = 24$

$$\pi = 24\sin(180/24) = 3.132628613 \qquad (d)$$

Iteration 5: Putting $n = 96$

$$\pi = 96\sin(180/96) = 3.141031951 \qquad (e)$$

Iteration 6: Putting $n = 576$

$$\pi = 576\sin(180/576) = 3.141577078$$

Iteration 7: Putting $n = 100000$

$$\pi = 100000\sin(180/100000) = 3.141592653$$

Using our 'Minimum Circumference' relation for calculating the value of π

$$\pi = (10^n) * 180\sin(10^{(-n)}) \quad \{Where\ \theta = 10^{(-n)}\}$$

Iteration 1: Putting $n = 0$

$$\pi = (10) * 180\sin(1) = 3.1414331587110 \qquad (A)$$

Iteration 2: Putting $n = 1$

$$\pi = (10) * 180\sin(1/10) = 3.141591058616955904400 \qquad (B)$$

Iteration 3: Putting $n = 12$

$$\pi = (10^{12}) * 180\sin(1/10^{12})$$
$$\pi = 3.14159265358979323846 \qquad (C)$$

It is clear from equations (a), (b),(c) and (A),(B), (C) that we are getting more accurate and precise value of π when using the following equation:

$$\pi = (10^n) * 180\sin(10^{(-n)})$$

Now, using the relation given for **'Maximum Circumference'** for estimating the value of π

$$\pi = (10^n) * 180\tan(10^{(-n)}) \quad \{Where\ \theta = 10^{(-n)}\}$$

Iteration 1: Putting $n = 0$

$$\pi = 180 * (10^0)\{\tan(1)\} = 3.1419116870 \qquad (A1)$$

Iteration 2: Putting $n = 1$

$$\pi = 180 * (10^0)\{\tan(1/10)\} = 3.141595843589 \qquad (B1)$$

Iteration 3: Putting $n = 12$

$$\pi = 180 * (10^{12})\{\tan(1/10^{12})\}$$
$$\pi = 3.1415926535897932384626433835985 \qquad (C1)$$

Again, we can get the more accurate value of π after combining the relations given for 'Minimum' and 'Maximum' circumference. In this case the estimated value could be given as:

$$\pi = 90 * (10^n)\{(\sin(10^{(-n)}) + \tan(10^{(-n)}))\} \;\; \{Where\ \theta = 10^{(-n)}\}\ from\ equationQ$$

Iteration 1: Putting $n = 0$

$$\pi = 90 * (10^0)\{\sin(1) + \tan(1)\} = 3.1416724228 \qquad (A2)$$

Iteration 2: Putting $n = 1$

$$\pi = 90 * (10^1)\{\sin(1/10) + \tan(1/10)\}\pi = 3.1415934510 \qquad (B2)$$

Iteration 3: Putting $n = 12$

$$\pi = 90 * (10^{12})\{\sin(1/10^{12}) + \tan(1/10^{12})\}$$
$$\pi = 3.1415926535897932384626433835985 \qquad (C2)$$

We can estimate the more appropriate value of *pi* in first two iterations. However the Archimedes suggested that his formula gives more accurate value when the value of n is higher (equation (c), (d) and (e)). In this case our given relation is more accurate and precise for finding the value of the *pi* .

4 Conclusion and Future Work

It can be concluded here that the value of *pi* can be estimated more quickly and precisely by using our suggested formula in comparison to Archimedes'.
It is very clear from the equation C, C1 and C2 that the value of *pi* is very correct to the first 31st place from the decimal on a normal calculator that can display number till 31st place after decimal. However we assume that the given formula can give more accurate digits on a system made for calculations of higher degree such as Mainframe or Super computer. However, *pi* is irrational number having infinite length so none of the formula can give all those values without any error. Our formula can be used on higher degree computer mainly made of complex computation to achieve the higher degree of accuracy. In future context we would be giving the more ease and possible solutions for calculating the value of *pi* that would influence every student and mathematician throughout the world.

References

[1] Internet and Wikipedia.