Effect of Constant Predation in a Food Chain Model with Delay

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Abstract

In this paper effect of constant predation in food chain model is studied. Here the prey is structured as mature and immature and interaction of mature prey with predator is taken as Holling type II functional response. In the set of parameters, there are passages from instability to stability, which are called bifurcation points. Further, we discuss the dynamical behaviour of this model. It has found that crossing the bifurcation point, the stability of the system changed but it can be controlled by a particular range of the predation.

Subject class [2010]:49K05, 49K15, 49S05.
Keywords: Stage structured prey- predator, Functional response, Predation on predator, Bifurcation

1 Introduction

Predator-prey systems have been studied and the analysis of the food chain is an active research area in the mathematical biological science. In the food chain, a number of species is linked to each other. Here, we consider a simple food chain model with prey is delayed and top predator like predation on the middle predator. There are several interesting cases of simple food chains of three interacting species categorized by the types of functional response.

The functional response is the feeding rate describes the transfer of biomass between tropic levels. Holling type II predator's functional response describes the average feeding rate of the predator when the predator spends some time for searching prey.

Mathematical models of prey-predator systems create a major interest during the last few decades. Most of stage structured models considered two stages of the species one immature and other mature, the age of maturity is represented by a time delay. A stage structured prey predator models have been studied by many authors[10, 11, 12, 9].

Xu and Liao[2] investigated the local stability of positive equilibrium point and local Hopf bifurcation in delayed three species food chain model with Holling type II functional response. Agarwal and Devi [12] studied with ratio dependent the prey - predator model

where prey population is stage - structured and the predator population influenced by resource biomass. Agarwal and Kumar [7] studied the behaviour of resource biomass on the prey - predator stage - structure model with Holling type III functional response. Muratori and Rinaldi [8] studied the second order nonlinear dynamical system (predator-prey), the interactions of the tree and damaging insects in the forest. They discussed the influence of acidic deposition, an increase of which can cause sudden insect infestation and the collapse of the forest ecosystem.
The above studied, authors have considered the stage - structured prey predator model with different functional responses. In this paper, we consider the interaction of one predator species depending on mature prey species. We assume that the predators do not need any handling time or searching time for the immature prey species.

2 Mathematical models

In this paper, the effect of constant predation on the predator population in the prey-predator model is studied by considering Holling type II functional response. The underlying system consists of the state variables $x_i(t)$, the density of immature prey; $x_m(t)$, the density of mature prey population; $y(t)$, the density of predator population. The mathematical model is describe under following assumption;

1. Prey populations have two stages immature, mature and maturity period is taken as τ.
2. Only mature prey population interact with predator and interaction of prey and predator considered by Holling type II functional response.
3. Predator growth controlled by constant predation(ρ) under Holling type II functional response.

Considering above point we describe following mathematical model;

$$\frac{dx_i(t)}{dt} = \alpha x_m(t) - \alpha_0 e^{-\gamma\tau} x_m(t-\tau) - d_1 x_i(t), \tag{2.1}$$

$$\frac{dx_m(t)}{dt} = \alpha_0 e^{-\gamma\tau} x_m(t-\tau) - \frac{k_1 x_m(t) y(t)}{a_1 + x_m(t)} - d_2 x_m(t)^2, \tag{2.2}$$

$$\frac{dy(t)}{dt} = \frac{c_1 x_m(t) y(t)}{a_1 + x_m(t)} - d_3 y(t) - \frac{a_0 \rho y(t)}{a_2 + y(t)}, \tag{2.3}$$

With $x_m(t) = \phi_m(t) \geq 0$, $\tau \leq t < 0$, $x_i(0) \geq 0$ and $y(0) > 0$.
The meaning of the parameters of the model is given as follow:

$\mathbf{H_1}$. At any time $t > 0$, birth into the immature prey population is proportional to the existing mature prey population with proportionality constant α_0.

H$_2$. k_1 is the maximum value which per capita reduction rate of prey can attain, c_1 is the conversion factor for prey population to predator and a_1 is the half saturation constant.

H$_3$. Parameters a_0, a_2 have similar biological meaning as k_1 and a_1 respectively.

H$_4$. $d_i, i = 1, 2, 3$ are natural death rate of immature, mature prey population and predator population respectively.

Then by using of equation of continuity of initial conditions, we require

$$x_i(t) = \int_{-\tau}^{0} \alpha e^{-\gamma(t-s)} x_m(s) ds \tag{2.4}$$

then the system of equation become as follow as:

$$\frac{dx_m}{dt} = \alpha_0 e^{-\gamma\tau} x_m(t-\tau) - \frac{k_1 x_m y}{a_1 + x_m} - d_2 x_m^2, \tag{2.5}$$

$$\frac{dy}{dt} = \frac{c_1 x_m y}{a_1 + x_m} - d_3 y - \frac{a_0 \rho y}{a_2 + y}, \tag{2.6}$$

$x_m(t) = \phi_m(t) \geq 0$, $-\tau \leq t < 0$ and $y(0) > 0$

3 Boundeness of the system

In theoretical biology, boundeness of the system implies that the system is biologically well behaved. The following lemma ensures the boundeness of the system.

Lemma 3.1. *The region of attraction of the system is given as*

$$R = \left\{ (x_m(t), y(t)) : 0 \leq x_m(t) \leq \frac{\alpha_0 e^{-\gamma\tau}}{d_2}, 0 \leq \frac{x_m(t)}{k_1} + \frac{y(t)}{c_1} \leq \frac{(\alpha_0 e^{-\gamma\tau} + d_3)^2}{4 d_2 k d_3} \right\} \tag{3.1}$$

Proof. From the equation (2.5)

$$\frac{dx_m}{dt} = \alpha_0 e^{-\gamma\tau} x_m(t-\tau) - k_1 \frac{x_m y}{a_1 + x_m} - d_2 x_m^2,$$

$$\frac{dx_m}{dt} \leq \alpha_0 e^{-\gamma\tau} x_m(t-\tau) - d_2 x_m^2, \tag{3.2}$$

then by mathematical manipulation , we get

$$\lim_{t\to\infty} \sup x_m(t) \le \frac{\alpha_0 e^{-\gamma\tau}}{d_2}. \tag{3.3}$$

Now from equation (2.5) and (2.6) of model,

$$\frac{d}{dt}\left(\frac{x_m(t)}{k_1} + \frac{y(t)}{c_1}\right) \le \alpha_0 e^{-\gamma\tau} x_m(t-\tau) + \frac{d_3 x_m(t)}{k} - \frac{d_2 x_m(t)^2}{k} - \frac{d_3 x_m(t)}{k} - \frac{d_3 y(t)}{c_1}, \tag{3.4}$$

$$\frac{d}{dt}\left(\frac{x_m(t)}{k_1} + \frac{y(t)}{c_1}\right) \le F(x_m(t)) - \frac{d_3 x_m(t)}{k} - \frac{d_3 y(t)}{c_1}, \tag{3.5}$$

where $F(x_m) = \alpha_0 e^{-\gamma\tau} x_m(t-\tau) + \frac{d_3 x_m(t)}{k} - \frac{d_2 x_m^2(t)}{k}$ then maximum value of $F(x_m)$ is given as, $F_{max}(x_m(t)) = \frac{(\alpha_0 e^{-\gamma\tau} + d_3)^2}{4 d_2 k}$

then after mathematical simulation, we obtained

$$\lim_{t\to\infty} \sup \left(\frac{x_m(t)}{k_1} + \frac{y(t)}{c_1}\right) \le \frac{(\alpha_0 e^{-\gamma\tau} + d_3)^2}{4 d_2 k d_3} \tag{3.6}$$

This completes the proof of lemma. □

4 Equilibrium points and stability analysis

In this section we analyze the existence of the equilibrium points. The system has only three equilibrium points.

1. Trivial equilibrium point $E_0(0,0)$.
2. Axial equilibrium point $E_1(x_{m1}, 0)$, where $x_{m1} = \frac{\alpha_0 e^{-\gamma\tau}}{d_2}$, and
3. Interior equilibrium point $E^*(x_m^*, y^*)$.

Existence of equilibrium point $E^*(x_m^*, y^*)$
Here x_m^* and y^* are positive solution of the following equations

$$\alpha_0 e^{-\gamma\tau} x_m(t-\tau) - \frac{k_1 x_m y}{a_1 + x_m} - d_2 x_m^2 = 0 \tag{4.1}$$

$$\frac{c_1 x_m y}{a_1 + x_m} - d_3 y - \frac{a_0 \rho y}{a_2 + y} = 0 \tag{4.2}$$

then from equation 4.1, we get

$$y = \frac{(\alpha_0 e^{-\gamma\tau} - d_2 x_m)(a_1 + x_m)}{k_1} = w(x_m)(say) \tag{4.3}$$

using this value in equation 4.2, we obtained

$$\begin{aligned} g(x_m) &= (c_1 - d_3)x_m w(x_m) + [(c_1 - d_3)a_2 - a_0\rho]\, x_m \\ &\quad - d_3 a_1 w(x_m) - d_3 a_1 a_2 - a_0 a\rho = 0 \end{aligned} \tag{4.4}$$

then

(i). $g(0) = -d_3 a_1 w(0) - d_3 a_1 a_2 - a_0 a\rho < 0$

(ii). $g\left(\frac{\alpha_0 e^{-\gamma\tau}}{d_2}\right) = [(c_1 - d_3)a_2 - a_0 z]\, \frac{\alpha_0 e^{-\gamma\tau}}{d_2} - d_3 a_1 a_2 - a_0 a\rho > 0$, if only if

$[(c_1 - d_3)a_2 - a_0\rho]\, \alpha_0 e^{-\gamma\tau} > (d_3 a_1 a_2 + a_0 a\rho)\, d_2$

then there exist a positive root $x_m = x_m^*$ of equation (4.4), this root must be unique for $g'(x_m) > 0$ in $\left(0, \frac{\alpha_0 e^{-\gamma\tau}}{d_2}\right)$.

After evaluating the value of x_m^*, value of $y^* = \frac{(\alpha_0 e^{-\gamma\tau} - d_2 x_m^*)(a_1 + x_m^*)}{k_1}$ may be determined.

The stability analysis of the equilibrium point is given by the variation matrices, characteristic matrix obtain by Jacobian of system of differential equations,

(4.5)

$$V(E) = \begin{bmatrix} \alpha_0 e^{-\gamma\tau - \lambda\tau} - k_1 \frac{y a_1}{(a_1 + x_m)^2} - 2d_2 x_m - \lambda & \frac{-x_m k_1}{(a_1 + x_m)} \\ \frac{y a_1 c_1}{(a_1 + x_m)^2} & \frac{x_m c_1}{(a_1 + x_m)} - d_3 - \frac{\rho a_2 a_0}{(a_2 + y)^2} - \lambda \end{bmatrix}$$

The stability analysis of the equilibrium points E_0, E_1 and E^* is given as;

1. For trivial equilibrium point E_0, the eigenvalues of this point are given by following characteristic equation,

$$(\alpha_0 e^{-\gamma\tau - \lambda\tau} - \lambda)\left(-\frac{a_0\rho}{a_2} - d_3 - \lambda\right) = 0 \tag{4.6}$$

Eigen value corresponding x_m direction is given by the equation $\lambda_1 = \alpha_0 e^{-\gamma\tau - \lambda_1\tau} > 0$ and in direction y , $\lambda_2 = -\frac{a_0\rho}{a_2} - d_3 < 0$. So this point saddle point.

2. For equilibrium point E_{m1} characteristic equation is give as,

$$\left(\alpha_0 e^{-\gamma\tau-\lambda\tau} - 2d_2 x_{m1} - \lambda\right)\left(\frac{x_{m1}c_1}{(a_1+x_{m1})} - d_3 - \frac{\rho a_0}{a_2} - \lambda\right) = 0 \tag{4.7}$$

Eigenvalue corresponding x_m direction is given by the equation
$\lambda_1 = \alpha_0 e^{-\gamma\tau-\lambda_1\tau} - 2d_2 x_{m1}$.
Consider that $\Re\lambda_1 \geq 0$, then we compute the real part of λ_1 and obtained that, $\Re\lambda_1 = \alpha_0 e^{-\gamma\tau}\left(e^{-\tau\Re\lambda_1}\cos(\tau Im\lambda_1)\right) - 2d_2x_{m1} = \alpha_0 e^{-\gamma\tau-\tau\Re\lambda_1} - 2d_2x_{m1} < 0$ for $x_{m1} = \dfrac{\alpha_0 e^{-\gamma\tau}}{d_2}$, a contradiction, hence $\Re\lambda_1 \leq 0$.
From this characteristics equation, we note that the eigenvalue, namely, $\lambda_2 = \dfrac{x_{m1}c_1}{(a_1+x_{m1})} - d_3 - \dfrac{za_0}{a_2} > 0$ iff, $x_{m1}\left[(c_1-d_3)a_2 - \rho a_0\right] > (d_3a_2+\rho a_0)a_1$ if $c_1 > d_3$ corresponding to y direction.
Therefore, the equilibrium E_{m1} is a saddle point which is unstable in the y direction and stable in the x_m-direction.

3. Now stability analysis of interior equilibrium point (x_m^*, y^*) is given as, characteristic equation for the equilibrium point (x_m^*, y^*) is given as follow as

$$\Phi(\lambda,\tau) = \left(\lambda^2 + A_1\lambda + A_2\right) - e^{-\lambda\tau}\left(\lambda A_3 + A_4\right) = 0 \tag{4.8}$$

where

$$A_1 = \left(\frac{k_1y^*a_1}{(a_1+x_m^*)^2} - \frac{x_m^*c_1}{(a_1+x_m^*)} + \frac{\rho a_2a_0}{(a_2+y^*)^2} + d_3 + 2d_2x_m^*\right),$$

$$A_2 = \left(\frac{k_1y^*a_1}{(a_1+x_m^*)^2} + 2d_2x_m^*\right)\left(d_3 + \frac{\rho a_2a_0}{(a_2+y^*)^2} - \frac{x_m^*c_1}{(a_1+x_m^*)}\right) + \frac{k_1a_1c_1x_my}{(a_1+x_m)^3},$$

$$A_3 = \alpha_0 e^{-\gamma\tau},$$

$$A_4 = \left(d_3 + \frac{\rho a_2a_0}{(a_2+y^*)^2} - \frac{x_m^*c_1}{(a_1+x_m^*)}\right)\alpha e^{-\gamma\tau},$$

The stability analysis of E^* can be seen by following theorem;

Theorem 4.1. *A necessary and sufficient condition for (x_m^*, y^*) is locally asymptotically stable for $\tau \geq 0$ is,*

a. *The real part of all roots of $\Phi(\lambda,\tau) = 0$ are negative.*

b. *For all b and $\tau > 0$, $\Phi(ib,\tau) \neq 0$ where $i = \sqrt{-1}$*

Theorem 4.2. *The positive equilibrium point (x_m^*, y^*) for the system (2.5)-(2.6), is locally asymptotically stable providing condition*

1. *$d_2x_m^* > \dfrac{\alpha_0 e^{-\gamma\tau}}{2}$ and*

2. $d_3a_1 > x_m^*(c_1 - d_3)$ *if* $c_1 > d_3$.

Proof. Now for $\tau = 0$, the equation (4.5) become

$$\lambda^2 + \beta_1\lambda + \beta_2 = 0 \tag{4.9}$$

Where

$\beta_1 = \frac{k_1y^*a_1}{(a_1+x_m^*)^2} - \frac{x_m^*c_1}{(a_1+x_m^*)} + \frac{\rho a_2a_0}{(a_2+y^*)^2} + d_3 + 2d_2x_m^* - \alpha_0$

$\beta_2 = \left(\frac{k_1y^*a_1}{(a_1+x_m^*)^2} + 2d_2x_m^* - \alpha_0\right)\left(d_3 + \frac{\rho a_2a_0}{(a_2+y^*)^2} - \frac{x_m^*c_1}{(a_1+x_m^*)}\right) + \frac{k_1a_1c_1x_my}{(a_1+x_m)^3}$

By the Routh-Hurwitz criteria, the system is locally asymptotically stable around the interior point $E^*(x_m^*, y^*)$ if $\beta_1, \beta_2 > 0$.

Now for delay is non zero $\tau \neq 0$, let $\tau = a(t) + ib(t)$ then the equation (4.8), equating real and imaginary part of this equation, we get

$$\cos b\tau(aA_3 + A_4) + bA_3 \sin b\tau = e^{a\tau}\{a^2 - b^2 + aA_1 + A_2\} \tag{4.10}$$

$$bA_3 \cos b\tau - \sin b\tau(aA_3 + A_4) = e^{a\tau}(2ab + A_1b) \tag{4.11}$$

Now when root are purely imaginary $\lambda = ib(t)$ then from equations (4.10) - (4.11), we get

$$A_4 cos b\tau + bA_3 sin b\tau = -b^2 + A_2 \tag{4.12}$$

$$bA_3 cos b\tau - A_4 sin b\tau = A_1 b \tag{4.13}$$

Squaring and adding (4.12) and (4.13), we obtained

$$b^4 + b^2\left(A_1^2 - 2A_2 - A_3^2\right) + \left(A_2^2 - A_4^2\right) = 0 \tag{4.14}$$

Now

$$A_2^2 - A_4^2 = (A_2 - A_4)(A_2 + A_4)$$

as $A_2 > 0$ $A_4 > 0$ then $(A_2 + A_4) > 0$ and

$$A_2 - A_4 = \left(d_3 + \frac{\rho a_2 a0}{(a_2+y^*)^2} - \frac{x_m^*c_1}{a_1+x_m^*}\right)\left(\frac{k_1y^*a_1}{(a_1+x_m^*)^2} + 2d_2x_m^* - \alpha_0e^{-\gamma\tau}\right) + \frac{k_1a_1c_1y^*x_m^*}{(a_1+x_m^*)^3} > 0$$

it is providing

1. $d_2x_m^* > \frac{\alpha_0e^{-\gamma\tau}}{2}$ and

2. $d_3a_1 > x_m^*(c_1 - d_3)$ if $c_1 > d_3$.

as $A_2 - A_1 > 0$ this implies that $A_2^2 - A_1^2 > 0$

Now $A_1^2 - 2A_2 - A_3^2 = (A_1 - A_3)(A_1 + A_3) - 2A_2$ as $A_1 > A_3$ and $A_1 + A_3 > 2A_2$ this implies that $A_1^2 - 2A_2 - A_3^2 > 0$

$$b^4 + b^2\left(A_1^2 - 2A_2 - A_3^2\right) + \left(A_2^2 - A_4^2\right) > 0 \tag{4.15}$$

above equation is contradicts with (4.14), hence $\phi(ib, \tau) \neq 0$. Then it satisfied (b) condition of theorem (4.1). Therefore equilibrium point $E^*(x_m^*, y^*)$ is locally asymptotically stable for all $\tau \geq 0$. □

5 Bifurcation analysis

Lemma 5.1. *The following transverse conditions are satisfied:*
$\Re\left[\frac{d\lambda}{d\tau}\right] \neq 0$, *thus the system of equation (2.5)-(2.6) undergoes Hopf bifurcations at positive equilibrium point* $E^*(x_m^*, y^*)$ *for* $\tau = \tau_c$.

Proof. Let $\lambda = a(\tau) + ib(\tau)$ be the root of equation (4.8) near $\tau = \tau_c$ satisfying $a(\tau_c) = 0$, $b(\tau_c) = b_0$,

Differentiating the equation (4.8) with respect to τ, we get

$$\left[\frac{d\lambda}{d\tau}\right]^{-1} = \frac{(2\lambda - A_1) - e^{-\lambda\tau}A_3}{\lambda(\lambda A_3 + A_4)e^{-\lambda\tau}} - \frac{\tau}{\lambda},$$

Now for $\tau = \tau_c$, $\lambda(\tau_c) = ib(\tau_c) = b_0$ we obtained that

$$\Re\left[\frac{d\lambda}{d\tau}\right]^{-1}_{\tau=\tau_0} = \frac{(A_3A_2 + A_3b_0^2 - A_1A_4)A_1b_0^2 - 2b_0A_4(A_2b_0 - b_0^3)}{A_1^2b_0^4 - (A_2b_0 - b_0^3)^2} \neq 0, \tag{5.1}$$

so equation (5.1) shows the transversally condition holds hence Hopf bifurcation occurs at $\tau = \tau_0$. □

The value of delay obtained by the equations (4.12)-(4.13)

$$\cot b\tau_c = \frac{bA_3bA_1 + A_4(-b^2 + A_2)}{bA_3(-b^2 + A_2) - A_4A_1b}. \tag{5.2}$$

6 Persistence

Biologically, persistence means the survival of all populations in future time. Mathematically, persistence of a system means that strictly positive solutions do not have omega limit points on boundary of nonnegative cone, then persistence of the system is given by following theorem.

Theorem 6.1. *Assume that* $\frac{\alpha_0 e^{-\gamma\tau}a_1}{k_1} > y_{max}$ *and* $w_0 = \left(\frac{cx_my}{a_1 + x_m}\right)_{min}$. *The system is permanent.*

Proof. From equation(2.5), we have

$$\frac{dx_m}{dt} \geq \alpha_0 e^{-\gamma\tau}x_m(t-\tau) - \frac{k_1x_my_{max}}{a_1} - d_2x_m^2, \tag{6.1}$$

then by comparing principle, it follows that

$$\liminf_{t\to\infty} x_m(t) \geq \frac{1}{d_2}\left(\alpha_0 e^{-\gamma\tau} - \frac{k_1 y_{max}}{a_1}\right), \tag{6.2}$$

and from equation (2.6), we have

$$\frac{dy}{dt} = \frac{c_1 x_m y}{a_1 + x_m} - d_3 y - \frac{a_0 \rho y}{a_2 + y}, \tag{6.3}$$

Consider that $w_0 = \left(\frac{c x_m y}{a_1 + x_m}\right)_{min}$, then the above equation become,

$$\frac{dy}{dt} \geq w_0 - d_3 y - \frac{a_0 \rho y}{a_2}, \tag{6.4}$$

then by comparing principle, it follows that,

$$\liminf_{t\to\infty} y(t) \geq \frac{w_0 a_2}{(d_2 a_2 + a_0 \rho)}, \tag{6.5}$$

This is completes the proof of theorem. □

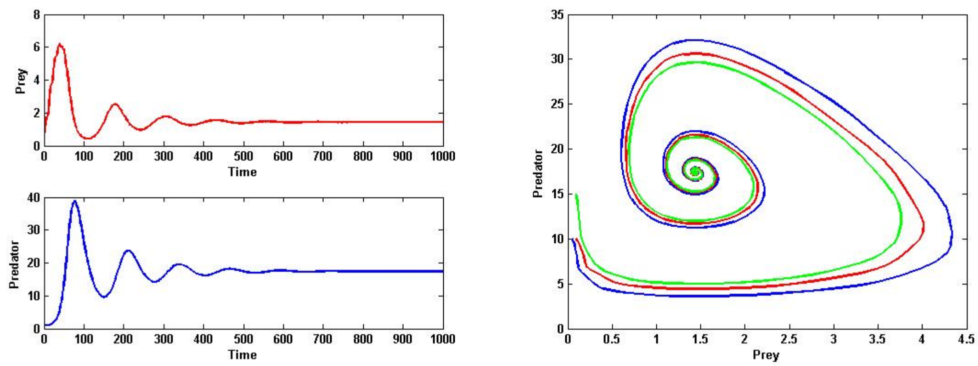
7 Numerical Simulations

In this section, we present some numerical simulations for the supporting theoretical prediction with the set of parameters values,
$\alpha_0 = 0.9, \gamma = 0.01, \tau = 10, k_1 = 0.45, a_1 = 10, c_1 = 0.32, d_3 = 0.04, a_0 = 0.09,$
$a_2 = 20, \rho = 0.05, d_2 = 0.09.$
Then the system of differential equation become as,

$$\frac{dx_m}{dt} = 0.9e^{-0.01*10} x_m(t-10) - \frac{0.45 x_m y}{10 + x_m} - 0.09 x_m^2, \tag{7.1}$$

$$\frac{dy}{dt} = \frac{0.32 x_m y}{10 + x_m} - 0.04y - \frac{0.09 * 0.05y}{20 + y}, \tag{7.2}$$

and $(1.433, 17.42)$ be the interior equilibrium point corresponding the above set of parameter. Behavior of the population for these parameters seen by the figures.
Figures (1(a))-(1(b)), time series graph for x_m and y and global stability for the positive equilibrium E^*. From figures we observed that both populations converges to their equilibrium point $E^*(1.433, 17.42)$.
Figures (2(a))-(2(b)) describe that behavior of prey and predator population at variation of constant predation ρ, we observed that the value of ρ increases prey population has positive effect that means prey population also increases but predator has very less negative effect.
From figure (3), we seen that half saturation constant a_1 of prey functional response has positive effect on both populations. In figure (4(a)), we observed the behavior of predator population at value of delay τ, here predator population decreases as the value of τ increases. From figure (4(b)) we observed that for value of $d_2 \leq 0.001$ system loss their global stability.



(a) Time series of prey and predator (b) Global stability, $E^*(1.433, 17.42)$

Fig. 1: Local and global stabilities of system

8 Conclusion

In this paper, we conclude the effect of delay on the system (Prey-Predator model), also see the behaviour of the system at a value of death rate of prey and effect of constant predation. We obtain the result when plants (prey) are consumed by animals (predator) and growth of animals are controlled by constant predation on prey it may be disease effect or may be harvesting or may be migration, by graphical study we conclude that as death rate of plants is very low(less than 0.001) then the system of food chain is not permanent stable. The effect of constant predation on system obtained by numerical simulation. We saw that value of constant predation ρ has positive effect on prey population and very less negative effect on the predator.

9 acknowledgment

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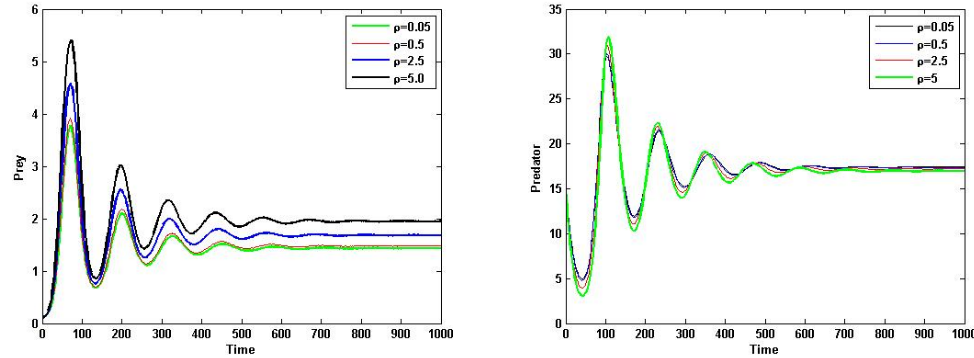
References

[1] B. Dubey and R.K.Upadhyaa, 'Persistence and extinction of one prey and two predator system', Nonlinear Analysis, Vol. 9(4), 307-329, 2004.

[2] C. Xu and M. Liao, 'Bifurcation behaviors in delayed three species food chain model with Holling type II functional response', Applicable Analysis, Vol. 92(12), 2468-2486, 2013.



(a) Variation of the prey population with time for different ρ

(b) Variation of the predator population with time for different ρ

Fig. 2: Behavior of prey and predator population at different value of ρ

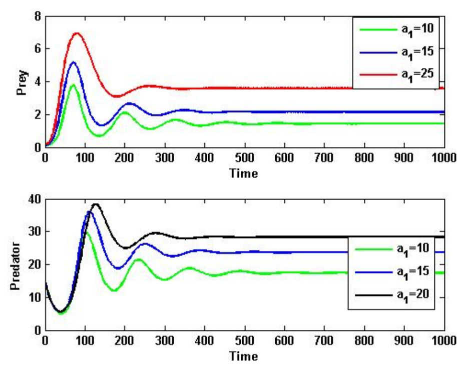


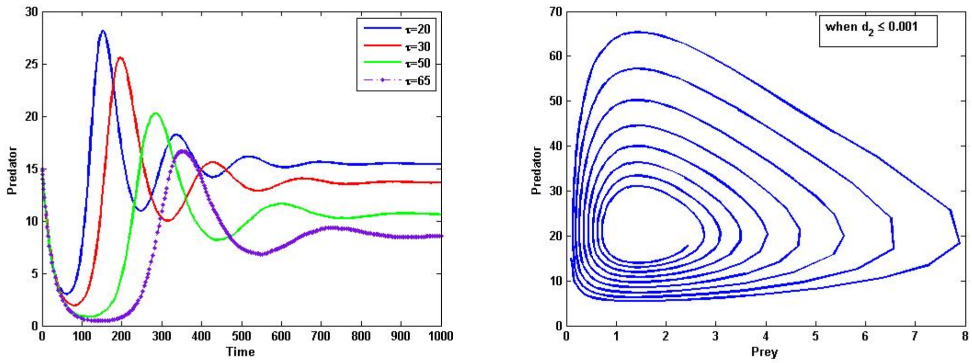
Fig. 3: Variation of the prey, predator population with time for different a_1

[3] D. Mukherjee, P. Das and D. Kesh, 'Dynamic of plant herbivore model with Holling type II functional response' Computational and Mathematical Biology, Vol. 2(1), 1-9, 2011.

[4] G.T. Skalski and J.F.Gilliam, 'Functional responses with predator interference: viable alternative to the Holling type II model', Ecological Society of America, Vol. 82(11), 3083-3092, 2001.

[5] H.I.freedman and P. Waltman, ' mathematical analysis of some three species food chain models', Math. Bio Sci. Vol. 33, 257-276, 1977.

[6] M. Agarwal and V. Gupta, 'Stability and persistence in ratio dependent food chain model with stage structured predator', Our Earth, Vol. 8(3),1-14, 2011.



(a) Variation of the predator population with time for different values of delay

(b) Behavior of prey and predator population when $d_2 \leq 0.001$

Fig. 4

[7] M. Agarwal and A. Kumar, 'Effect of resource biomass on stage structured predator prey system having Holling type III functional response', International Journal of Applied Mathematics and Statistical Sciences, Vol. 5(5),15-32, 2016.

[8] S. Muratori and S. Rinaldi, 'Catastrophic bifurcation in a second order dynamical system with application to acid rain and forest collapse', Appl. Math. Modelling, Vol. 13, 674-681, 1989.

[9] S. Boonrangsiman and K. Bunwong, 'Hopf bifurcation and dynamical behavior of a stage structured predator sharing a prey', Int. J. of Math. Model and Method in app. sci., Vol. 6(8), 893-900, 2012.

[10] W. Wang and L.Chen, 'A predator prey system with stage structure for predator', Computers math. Applic. Vol. 33(8), 83-91, 1997.

[11] W. Wang, G. Mulone, F. Salemi, V. Salone, 'Permanence and stability of stage structured predator prey model', JMAA, 262, 499-528, 2001.

[12] M. Agarwal and S. Devi, Persistence in a ratio dependent predator-prey-resource model with stage structure for prey, Int. J. of Biomathematics, vol. 3(3), 313-336, 2010.

[13] W.S. M. Sanjaya, M. Momat, Z. Salleh, I. Mohd and N. M. Mohamad Noor, 'Numerical simulation dynamical model of three species food chain with Holling type II functional response', Malaysian J. Math. Sci., Vol. 5(1) 1-12, 2011.