Ricci solitons on Kenmotsu Manifolds with respect to quarter symmetric non-metric ϕ-connection

S. K. Hui [1], R. Prasad [2] & D. Chakraborty [3]

[1] *Department of Mathematics, University of Burdwan, Golapbag, Burdwan, West Bengal, India*

[2] *Department of Mathematics and Astronomy, University of Lucknow, India*

[3] *Department of Mathematics, Sidho Kanho Birsha University Purulia, West Bengal, India*

skhui@math.buruniv.ac.in, rp.manpur@rediffmail.com & debabratamath@gmail.com

Abstract

The object of the present paper is to study Ricci solitons on Kenmotsu manifolds with respect to quarter symmetric non-metric ϕ-connection and obtain a necessary and sufficient condition of a Ricci soliton on a Kenmotsu manifold with respect to quarter symmetric non-metric ϕ-connection to be a Ricci soliton on a Kenmotsu manifold with respect to Levi-Civita connection.

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1 Introduction

In [36] Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. He proved that they could be divided into three classes: (i) homogeneous normal contact Riemannian manifolds with $c > 0$, (ii) global Riemannian products of a line or a circle with a Kähler manifold of constant holomorphic sectional curvature if $c = 0$ and (iii) a warped product space $\mathbb{R} \times_f \mathbb{C}^n$ if $c < 0$. It is known that the manifolds of class (i) are characterized by admitting a Sasakian structure. The manifolds of class (ii) are characterized by a tensorial relation admitting a cosymplectic structure. Kenmotsu [28] characterized the differential geometric properties of the manifolds of class (iii) which are nowadays called Kenmotsu manifolds and later studied by several authors.

As a generalization of both Sasakian and Kenmotsu manifolds, Oubiña [30] introduced the notion of trans-Sasakian manifolds, which are closely related to the locally conformal Kähler manifolds. A trans-Sasakian manifold of type (0, 0), $(\alpha, 0)$ and $(0, \beta)$ are called the cosympletic, α-Sasakian and β-Kenmotsu manifolds respectively, α, β being scalar functions. In particular, if $\alpha = 0, \beta = 1$; and $\alpha = 1, \beta = 0$ then a trans-Sasakian manifold will be a Kenmotsu and Sasakian manifold respectively.

In [11] Friedmann and Schouten introduced the notion of semisymmetric linear connection on a differentiable manifold. Then in 1932 Hayden [13] introduced the idea of metric connection with torsion on a Riemannian manifold. A systematic study of the semisymmetric metric connection on a Riemannian manifold has been given by Yano in 1970 [38]. In 1975, Golab [12] introduced the idea of a quarter symmetric linear connection in differentiable manifolds.

A linear connection $\overline{\nabla}$ in an n-dimensional differentiable manifold M is said to be a quarter symmetric connection [12] if its torsion tensor τ of the connection $\overline{\nabla}$ is of the form

$$\begin{aligned} \tau(X,Y) &= \overline{\nabla}_X Y - \overline{\nabla}_Y X - [X,Y] \\ &= \eta(Y)\phi X - \eta(X)\phi Y, \end{aligned} \tag{1.1}$$

where η is a 1-form and ϕ is a tensor of type (1,1). In particular, if $\phi X = X$ then the quarter symmetric connection reduces to the semi-symmetric connection. Thus the notion of quarter symmetric connection generalizes the notion of the semi-symmetric connection. Again if the quarter symmetric connection $\overline{\nabla}$ satisfies the condition

$$(\overline{\nabla}_X g)(Y,Z) \neq 0 \tag{1.2}$$

for all X, Y, $Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields on the manifold M, then $\overline{\nabla}$ is said to be a quarter symmetric non-metric connection. Furthermore, a quarter symmetric non-metric connection is said to be a quarter symmetric non-metric ϕ-connection [4] if

$$(\overline{\nabla}_X \phi)(Y) = 0 \tag{1.3}$$

for all $X, Y \in \chi(M)$.

In 1982, Hamilton [14] introduced the notion of Ricci flow to find a canonical metric on a smooth manifold. Then Ricci flow has become a powerful tool for the study of Riemannian manifolds, especially for manifolds with positive curvature. Perelman ([31], [32]) used Ricci flow and its surgery to prove Poincare conjecture. The Ricci flow is an evolution equation for metrics on a Riemannian manifold defined as follows:

$$\frac{\partial}{\partial t} g_{ij}(t) = -2R_{ij}.$$

A Ricci soliton emerges as the limit of the solutions of the Ricci flow. A solution to the Ricci flow is called Ricci soliton if it moves only by a one parameter group of diffeomorphism and scaling. A Ricci soliton (g, V, λ) on a Riemannian manifold (M, g) is a generalization of an Einstein metric such that [15]

$$\pounds_V g + 2S + 2\lambda g = 0, \tag{1.4}$$

where S is the Ricci tensor, $\pounds_V$ is the Lie derivative operator along the vector field V on M and λ is a real number. The Ricci soliton is said to be shrinking, steady and expanding according as λ is negative, zero and positive respectively.

During the last two decades, the geometry of Ricci solitons has been the focus of attention of many mathematicians. In particular, it has become more important after Perelman applied Ricci solitons to solve the long standing Poincare conjecture posed in 1904. In [34] Sharma studied the Ricci solitons in contact geometry. Thereafter, Ricci solitons in contact metric manifolds have been studied by various authors such as Bagewadi et. al ([1], [2], [3], [27]), Bejan and Crasmareanu [5], Blaga [7], Hui et. al ([8],[18]-[26]), Chen and Deshmukh [9], Deshmukh et. al [10], He and Zhu [16], Nagaraja and Premalatha [29], Tripathi [37] and many others.

Motivated by the above studies the present paper deals with the study of Ricci solitons on Kenmotsu manifolds with respect to quarter symmetric non-metric ϕ-connection. The paper is organized as follows. Section 2 is concerned with preliminaries. Section 3 is devoted to the study of Ricci solitons on Kenmotsu manifolds with respect to quarter symmetric non-metric ϕ-connection and obtain a necessary and sufficient condition of a Ricci soliton on Kenmotsu manifold with respect to quarter symmetric non-metric ϕ-connection to be a Ricci soliton on Kenmotsu manifold with respect to Levi-Civita connection.

2 Preliminaries

A smooth manifold (M^n, g) $(n = 2m + 1 > 3)$ is said to be an almost contact metric manifold [6] if it admits a (1,1) tensor field ϕ, a vector field ξ, an 1-form η and a Riemannian metric g which satisfy

$$\phi\xi = 0, \qquad \eta(\phi X) = 0, \qquad \phi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, Y) = -g(X, \phi Y), \qquad \eta(X) = g(X, \xi), \qquad \eta(\xi) = 1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.3}$$

for all vector fields X, Y on M.

An almost contact metric manifold $M^n(\phi, \xi, \eta, g)$ is said to be Kenmotsu manifold if the following condition holds [28]:

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.4}$$

$$(\nabla_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.5}$$

where ∇ denotes the Riemannian connection of g.

In a Kenmotsu manifold $M^n(\phi, \xi, \eta, g)$, the following relations hold ([17],[28],[33]):

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.6}$$

$$S(X, \xi) = -(n - 1)\eta(X) \tag{2.7}$$

for any vector field X, Y, Z on M and R is the Riemannian curvature tensor and S is the Ricci tensor of type (0,2) such that $g(QX,Y) = S(X,Y)$.

Let M be an n-dimensional Kenmotsu manifold and ∇ be the Levi-Civita connection on M. Then a quarter symmetric non-metric ϕ-connection $\overline{\nabla}$ in a Kenmotsu manifold is defined by [4]

$$\overline{\nabla}_X Y = \nabla_X Y + \eta(X)\phi Y + g(X,Y)\xi - \eta(Y)X - \eta(X)Y. \tag{2.8}$$

If R and $\overline{R}$ are respectively the curvature tensor of Levi-Civita connection ∇ and the quarter symmetric non-metric ϕ-connection $\overline{\nabla}$ in a Kenmotsu manifold then we have [4]

$$\begin{aligned}\overline{R}(X,Y)Z &= R(X,Y)Z + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y \\ &+ g(Y,Z)X - g(X,Z)Y.\end{aligned} \tag{2.9}$$

From (2.9) we have

$$\overline{S}(Y,Z) = S(Y,Z) + (n-1)\eta(Y)\eta(Z) + (n-1)g(Y,Z), \tag{2.10}$$

where $\overline{S}$ and S are the Ricci tensor of a Kenmotsu manifold with respect to the quarter symmetric non-metric ϕ-connection and Levi-Civita connection respectively. Also contracting (2.10) over Y and Z, it follows that

$$\overline{r} = r + (n+1)(n-1), \tag{2.11}$$

where $\overline{r}$ and r are the scalar curvature of a Kenmotsu manifold with respect to the quarter symmetric non-metric ϕ-connection and Levi-Civita connection respectively.

3 Ricci solitons on Kenmotsu manifolds with respect to quarter symmetric non-metric ϕ-connection

Consider (g, ξ, λ) is a Ricci soliton on a Kenmotsu manifold with respect to quarter symmetric non-metric ϕ-connection. Then we have

$$(\overline{\pounds}_\xi g)(Y,Z) + 2\overline{S}(Y,Z) + 2\lambda g(Y,Z) = 0. \tag{3.1}$$

From (2.1), (2.2), (2.4) and (2.11) we have

$$\begin{aligned}(\overline{\pounds}_\xi g)(Y,Z) &= g(\overline{\nabla}_Y \xi, Z) + g(Y, \overline{\nabla}_Z \xi) \\ &= g(\nabla_Y \xi, Z) + g(Y, \nabla_Z \xi) \\ &= g(-\eta(Y)\xi, Z) + g(Y, -\eta(Z)\xi) \\ &= -2\eta(Y)\eta(Z).\end{aligned} \tag{3.2}$$

Using (2.10) and (3.2) in (3.1), we get

$$S(Y,Z) = -2(\lambda + n - 1)g(Y,Z) - 2(n-2)\eta(Y)\eta(Z). \tag{3.3}$$

This leads to the following:

Theorem 3.1. *If (g, ξ, λ) is a Ricci soliton on a Kenmotsu manifold M with respect to quarter symmetric non-metric ϕ-connection then M is an η-Einstein manifold.*

Putting $Z = \xi$ in (3.3) and using (2.1) and (2.2), we get

$$S(Y, \xi) = -2(\lambda + 2n - 3)\eta(Y). \tag{3.4}$$

From (2.7) and (3.4), we obtain

$$\lambda = -\frac{1}{2}(3n - 5) < 0. \tag{3.5}$$

This leads to the following:

Theorem 3.2. *A Ricci soliton (g, ξ, λ) on a Kenmotsu manifold $M^n(\phi, \xi, \eta, g)$ with respect to quarter symmetric non-metric ϕ-connection is always shrinking.*

In [35] Singh and Srivastava studied quasi-projectively flat and ϕ-projectively flat Kenmotsu manifolds with respect to quarter symmetric non-metric ϕ-connection. An n-dimensional Kenmotsu manifold M is said to be quasi-projectively flat with respect to quarter symmetric non-metric ϕ-connection if

$$g(\overline{P}(\phi X, Y)Z, \phi W) = 0$$

for all X, Y, Z, W on M and $\overline{P}$ is the projective curvature tensor with respect to quarter symmetric non-metric ϕ-connection given by

$$\overline{P}(X, Y)Z = \overline{R}(X, Y)Z - (n - 1)[\overline{S}(Y, Z)X - \overline{S}(X, Z)Y]. \tag{3.6}$$

Also an n-dimensional Kenmotsu manifold M is said to be ϕ-projectively flat with respect to quarter symmetric non-metric ϕ-connection if $(\overline{P}(\phi X, \phi Y)\phi Z) = 0$ for all X, Y, Z on M and $\overline{P}$ is defined in (3.6).
We have the following:

Theorem 3.3. *[35] A quasi-projectively flat (or ϕ-projectively flat) Kenmotsu manifold M with respect to quarter symmetric non-metric ϕ-connection is Einstein manifold with respect to Levi-Civita connection and its Ricci tensor is of the form $S(Y, Z) = -(n-1)g(Y, Z)$ for all Y, Z on M.*

We now prove the following:

Theorem 3.4. *If (g, ξ, λ) is a Ricci soliton on a quasi-projectively flat (or ϕ-projectively flat) Kenmotsu manifold M with respect to quarter symmetric non-metric ϕ-connection then*

1. $\lambda + n - 2 = 0,$
2. *ξ is a geodesic vector field.*

Proof. Let us take a quasi-projectively flat (or ϕ-projectively flat) Kenmotsu manifold M with respect to quarter symmetric non-metric ϕ-connection. Then by Theorem 3.3, M is Einstein manifold with respect to Levi-Civita connection and its Ricci tensor is of the form

$$S(Y,Z) = -(n-1)g(Y,Z) \tag{3.7}$$

and hence we get

$$\overline{S}(Y,Z) = (n-1)\eta(Y)\eta(Z). \tag{3.8}$$

So, by virtue of (3.8) we have from (3.1) that

$$g(\overline{\nabla}_Y\xi, Z) + g(Y, \overline{\nabla}_Z\xi) + 2[\lambda g(Y,Z) + (n-1)\eta(Y)\eta(Z)] = 0. \tag{3.9}$$

Putting $Y = Z = \xi$ in (3.9) and using (2.1) and (2.2), we obtain $g(\nabla_\xi\xi, \xi) = -(\lambda + n - 2)$, but $g(\nabla_X\xi, \xi) = 0$ for any vector field X on M, since ξ has a constant term. Hence we get (i).
Consequently (3.9) becomes

$$g(\overline{\nabla}_Y\xi, Z) + g(Y, \overline{\nabla}_Z\xi) + 2[(n-1)\eta(Y)\eta(Z) - (n-2)g(Y,Z)] = 0. \tag{3.10}$$

Setting $Y = \xi$ in (3.10) and using (2.1)–(2.4) we get $g(\nabla_\xi\xi, Z) = 0$ for any vector field Z on M and hence we have $\nabla_\xi\xi = 0$, i.e., ξ is a geodesic vector field. Thus we get (ii). □

Let (g, V, λ) be a Ricci soliton on a Kenmotsu manifold with respect to quarter symmetric non-metric ϕ-connection. Then we have

$$(\overline{\pounds}_V g)(Y,Z) + 2\overline{S}(Y,Z) + 2\lambda g(Y,Z) = 0, \tag{3.11}$$

where $\overline{\pounds}_V$ is the Lie derivative along the vector field V on M with respect to quarter symmetric non-metric ϕ-connection.
By virtue of (2.8), we have

$$\begin{aligned} (\overline{\pounds}_V g)(Y,Z) &= g(\overline{\nabla}_Y V, Z) + g(Y, \overline{\nabla}_Z V) \\ &= (\nabla_Y V - \eta(Y)\phi V + g(Y,V)\xi - \eta(V)Y - \eta(Y)V, Z) \\ &+ g(Y, \nabla_Z V - \eta(Z)\phi V + g(Z,V)\xi - \eta(V)Z - \eta(Z)V) \\ &= (\pounds_V g)(Y,Z) - [\eta(Y)g(\phi V, Z) + \eta(Z)g(Y, \phi V)] \\ &- 2\eta(V)g(Y,Z). \end{aligned} \tag{3.12}$$

In view of (2.10) and (3.12), (3.11) yields

$$\begin{aligned} &(\pounds_V g)(Y,Z) + 2S(Y,Z) + 2\lambda g(Y,Z) \\ -\ &[\eta(Y)g(\phi V, Z) + \eta(Z)g(\phi V, Y)] \\ +\ &2\{n - 1 - \eta(V)\}g(Y,Z) + 2(n-1)\eta(Y)\eta(Z) = 0. \end{aligned} \tag{3.13}$$

If (g, V, λ) is a Ricci soliton on a Kenmotsu manifold with respect to Levi-Civita connection then (1.4) holds. Thus from (1.4) and (3.13) we can state the following:

Theorem 3.5. *A Ricci soliton (g, V, λ) on a Kenmotsu manifold is invariant under quarter symmetric non-metric ϕ-connection if and only if the relation*

$$\begin{aligned} & [\eta(Y)g(\phi V, Z) + \eta(Z)g(\phi V, Y)] \\ - \quad & 2\{n-1-\eta(V)\}g(Y,Z) - 2(n-1)\eta(Y)\eta(Z) = 0 \end{aligned}$$

holds for arbitrary vector fields Y, Z and V.

Let(g, V, λ) be a Ricci soliton on a Kenmotsu manifold M with respect to quarter symmetric non-metric ϕ-connection such that V is pointwise collinear with ξ, i.e., $V = b\xi$, where b is a function. Then (3.11) holds, which implies that

$$\begin{aligned} & bg(\overline{\nabla}_X\xi, Y) + (Xb)\eta(Y) + bg(X, \overline{\nabla}_Y\xi) \\ + \quad & (Yb)\eta(X) + 2\overline{S}(X,Y) + 2\lambda g(X,Y) = 0. \end{aligned} \tag{3.14}$$

Putting $Y = \xi$ in (3.14) and using (2.1)–(2.4), (2.7), (2.8) and (2.10) we get

$$(Xb) + (\xi b)\eta(X) + 2(\lambda + n - 1 - b)\eta(X) = 0. \tag{3.15}$$

Again setting $X = \xi$ in (3.15) and using (2.8), we get

$$(\xi b) + \lambda + n - 1 - b = 0. \tag{3.16}$$

In view of (3.16) it follows from (3.15) that

$$db = -[\lambda + n - 1 - b]\eta. \tag{3.17}$$

Applying d on (3.17) we get

$$[\lambda + n - 1 - b]d\eta = 0. \tag{3.18}$$

Since $d\eta \neq 0$, we have from (3.18) that

$$\lambda + n - 1 - b = 0$$

and hence from (3.17) it follows that $db = 0$, which implies that 'b' is constant.
This leads to the following:

Theorem 3.6. *If (g, V, λ) is a Ricci soliton on a Kenmotsu manifold M with respect to quarter symmetric non-metric ϕ-connection such that V is pointwise collinear with ξ, then V is a constant multiple of ξ and the Ricci soliton is shrinking, steady and expanding as $n > b + 1$, $n = b + 1$ and $n < b + 1$, respectively.*

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