Pressure Distribution in Porous Journal Bearing with Nanolubricant for Second Order Lubrication Theory

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Abstract

The analysis of porous bearing i.e., infinitely short is bestowed supported the idea of journal bearings and takes into consideration of fluid film layers thickness and viscosity of nanoparticles additive fluid. In the present paper, the results of additives on the nano lubricants has been investigated and reported, aiming for improvement and also the improvement of fluid mechanics bearing performance. The additive with 5% volume fractions with viscosity was thought about as blended with the base binder. A dimensionless pressure distribution with an improved viscosity over the fluid film has been calculated. The results show a rise in the pressure distribution in response to extending within the viscosity of the additives with relation to the assorted values of the rotation number. The non-dimensional pressure distribution is obtained from integration of Extended Generalized Reynolds Equation under the effects of second order rotation by using the Reynolds boundary conditions. The nano lubricant Aerosil is employed for the analysis with 5% volume fraction of nanoparticles. The fluid film properties and structure considerably influence the film fluid hydrodynamic lubrication for the nano lubricants under the effects of second order rotation.

Subject class [2010]Mathematics Subject Classification :76D08
Keywords: Bearing pressure distribution; Nanolubricants; Porous journal bearing, Second order rotation.

1 Introduction

In the thin film lubrication (TFL), effective viscosity of adsorbed layer under relative motion is much bigger than the bulk Newtonian viscosity. The adsorbed molecular layer thickness and viscosity are the foremost essential factors in thin film lubrication. The analysis on the viscosity of nanofluids urged by Einstein [1] supported dilute suspended spherical particles in viscous fluids. Einsteins pioneering formula is valid for spherical particles with low nanoparticles volume fraction. Brinkman [2] contributed to increase the

Einstein equation considering the impact of moderate spherical nanoparticles suspensions in viscous fluids. Batchelor [3] planned the second order formula for the body of nanofluids considering Brownian movement impact of spherical nanoparticles additives and their interactions. Each these models [2-3] predict viscosity of nanoparticles suspensions from the base fluid viscosity and volume fraction of nanoparticles. Krieger and Dougherty [4] derived the shear body equation considering particle concentrations. Tichy [5] and Qingwen et al. [6] studied the impact of lubricator molecular structure and developed thin film model of solid surfaces adhered high body layer. Supported thin film lubrication analysis, as body and thickness of adsorbed layer at solid surfaces will increase; results indicate increase in pressure distribution and reduce in coefficient of friction. Nabhan et al. [7] conjointly investigated binary fluid film (oil-in-water) lubricated hydrodynamic journal bearing. Nanoparticles additive lubricators increase the pressure distribution of fluid film bearings as a result of increase in lubricant viscosity. Chen et al. [8] derived changed Krieger and Dougherty equation to predict high shear viscosity of nanofluids supported mixture nanoparticles structure considering the consequences of variable packing fraction. They bestowed classification of nanofluids supported the volume fraction of nanoparticles V_ϕ depending on nanoparticles concentration and structure into the followings:
(i) dilute ($0 < V_\phi \leq 0.001$),
(ii) semi-dilute ($0.001 < V_\phi \leq 0.05$),
(iii) semi-concentrated ($0.05 < V_\phi \leq 0.1$), and
(iv) concentrated ($V_\phi \geq 0.1$).
Meurisse and Espejel [9] conjointly have given a Generalized Reynolds Equation for a three-layered film model. Analysis of bearing load capability and friction coefficient in three-layered film is influenced by coefficient of fluid film. Nair et al. [10] given characteristics of statically loaded bearing operational lubricants with nanoparticles additives. Duangthongsuk and Wongwises [11] planned correlations for thermal conductivity and viscosity of nanofluids supported experimental results. The viscosity of nanofluids decreases considerably with increasing temperature and will increase with increasing particle volume concentration. Szeri [12] investigated composite film configuration for increase in load capability (high viscosity lubricant) and reduce in friction (low viscosity lubricant). The composite film bearing consists of upper and lower viscosity fluid layers adjacent to bearing surface and journal surface, severally. Hosseini et al. [13-14] given an empirical model of viscosity of nanoparticles suspensions i.e., derived from viscosity of the base liquid, particle volume fraction, particle size, properties of the wetter layer, and temperature. Shenoy et al. [15] have given the influence of API-SF engine oil with nanoparticles additives on the characteristics of an outwardly adjustable statically loaded fluid film bearing. Babu et al. [16] gave the impact lubricants with nanoparticles additives on static and dynamic characteristics of bearing. The lubricants with oxide, metal chemical compound, and corundom nanoparticles are employed within the analysis. Consistency models for nanoparticles additive lubricants were developed victimization the on the suitable experimental analysis. Mahbubul et al. [17] have given review of theoretical models of viscosity for suspensions and represented viscosity correlations for volume concentrations, temperature, and particle diameter of nanofluids.
The theory of hydrodynamic lubrication, two-dimensional classical theories [18] was initially given by Reynolds. In 1886, within the wake of a classical experiment by Beauchamp

Tower [18], he developed a vital equation that was familiar as: Reynolds Equation. The formation and basic mechanism of fluid film was analyzed by that experiment on taking some necessary assumptions given as:
(i) The fluid film thickness is incredibly thin as compare to the axial and longitudinal dimensions of fluid film.
(ii) If the lubricator layer is to transmit pressure between the shaft and therefore the bearing, the layer should have variable thickness.
Later Reynolds himself derived an improved version of Reynolds Equation familiar as: Generalized Reynolds Equation [19-20] that depends on density, viscosity, film thickness, surface and transversal velocities. The rotation of fluid film regarding associated axis that lies across the film offers some new leads to lubrication issues of hydrodynamics. The origin of rotation is often derived by some general theorems associated with vorticity within the rotating fluid dynamics. The rotation induces a part of vorticity within the direction of rotation of fluid film and therefore the effects arising from it are predominant, for giant Taylors number, it leads to the streamlines changing into confined to plane transversal to the direction of rotation of the film.
The new extended version of Generalized Equation is claimed to be Extended Generalized Reynolds Equation [19-20] that takes into consideration of the consequences of the uniform rotation regarding associated axis that lies across the fluid film and depends on the rotation number (M), i.e. the root of the standard Taylors number. The generalization of the classical theory of fluid dynamics lubrication is understood as the Rotatory Theory of fluid dynamics Lubrication [21-23].
The First Order Rotatory Theory of Hydrodynamics Lubrication and therefore the Second Order Rotatory Theory of Hydrodynamics Lubrication [21-23] was given by retentive the terms containing up to first and second powers of (M) severally by neglecting higher powers of (M).
The present paper analyzes regarding the pressure within the porous bearings with respect to the impact of second order rotation. These bearings are made by porous material and therefore the lubricator flows out of the bearing surface with a certain speed. These bearings are typically employed in several helpful devices, like vacuum cleaners, extractor fans, motor automobile starters, hair dryers etc. These bearings are infinitely short. The geometrical description of the bearings is delineating in figure-1 [24], of solid shaft in mold metal bush and of shaft and bush spread out.

1.1 Additive Aerosil

The Aerosil i.e., fumed silica; also called pyrogenic oxide as a result of it's made during a flame, consists of microscopic droplets of amorphous oxide coalesced into branched, chain-like, three- dimensional secondary particles that then agglomerate into tertiary particles. The ensuing powder has an especially low bulk density and high surface area. Its three dimensional structure ends up in viscosity-increasing, thixotropic behavior once used as a meterail or reinforcing filler. Aerosil contains a terribly robust thickening result. Primary particle size is 5-50 nm. The particles are non-porous and have an area of $50 - 600m^2/g$. The density is $160 - 190kg/m^3$. The Aerosil nonolubricant is empolyed for this analysis. The viscosity of the nanofluids will increase with reference to solid content and reduces

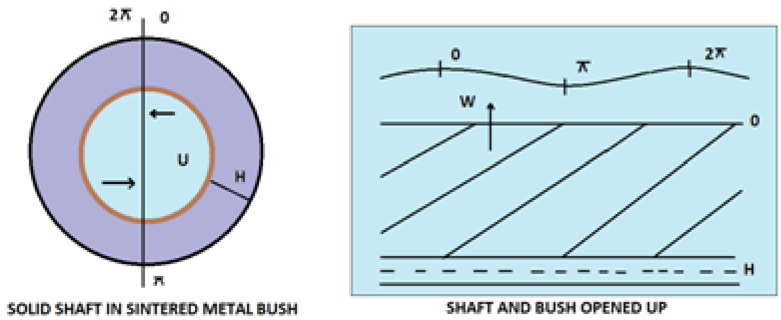

Fig. 1

with temperature. The Aerosil suspensions give higher viscosities over others, being the Aerosil at 5% of volume fraction the foremost viscous [25].

2 Governing Equations and Boundary Conditions

If the bearing is infinitely short, then the pressure gradient in $x-$direction is way smaller than the pressure gradient in $y-$direction. In $y-$direction the gradient $\partial P/\partial y$ is of the order of (P/L) and within the $x-$direction, and is of order (P/B). If $L << B$ then $P/L >> P/B$, therefore $\partial P/\partial x << \partial P/\partial y$ Then the terms containing $\partial P/\partial x$ may be neglected as compared to the terms $\partial P/\partial y$ containing within the expanded form of Generalized Reynolds Equations.
Where;
B: Total breadth of bearing parallel to the direction of motion; L: Bearing length normal to the direction of motion; P: Pressure; x: Co-ordinate along span of the bearing system; y:Co-ordinate along length of the bearing system.
The "Extended Generalized Reynolds Equation" [21-24] in the view of the second order Rotatory theory of hydrodynamic lubrication is shown as follows:

$$\frac{\partial}{\partial x}[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sinh h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}})\frac{\partial P}{\partial x}]+$$
$$\frac{\partial}{\partial y}[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sinh h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}})\frac{\partial P}{\partial y}]+$$
$$\frac{\partial}{\partial x}[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sinh h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}})\frac{\partial P}{\partial y}]-$$
$$\frac{\partial}{\partial y}[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sinh h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}})\frac{\partial P}{\partial x}]$$
$$=-\frac{U}{2}\frac{\partial}{\partial x}[\rho\sqrt{\frac{2\mu}{M\rho}}(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sinh h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}})]$$
$$-\frac{U}{2}\frac{\partial}{\partial y}[\rho\sqrt{\frac{2\mu}{M\rho}}(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sinh h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}-\cos h\sqrt{\frac{M\rho}{2\mu}}})]-\rho W^* \tag{2.1}$$

Where;
C: Radial clearance ; D: Diameter of the bearing; h: Film thickness; M: Rotation number; P: Pressure ; R:Radius of the bearing; U: Sliding velocity; W^*: Fluid velocity in z-direction; μ: Absolute viscosity of fluid; ρ: Density of fluid.

The "Extended Generalized Reynolds Equation" with respect to second order rotatory theory of hydrodynamic lubrication [21-24] can be written as:

$$\frac{\partial}{\partial x}[-\frac{h^3}{12\mu}(1-\frac{17M^2\rho^2h^4}{1680\mu^2})\rho\frac{\partial P}{\partial x}]+\frac{\partial}{\partial y}[-\frac{h^3}{12\mu}(1-\frac{17M^2\rho^2h^4}{1680\mu^2})\rho\frac{\partial P}{\partial y}]+$$
$$\frac{\partial}{\partial x}[-\frac{M\rho^2h^5}{120\mu^2}(1-\frac{31M^2\rho^2h^4}{3024\mu^2})\frac{\partial P}{\partial y}]-\frac{\partial}{\partial y}[-\frac{M\rho^2h^5}{120\mu^2}(1-\frac{31M^2\rho^2h^4}{3024\mu^2})\frac{\partial P}{\partial x}]$$
$$=-\frac{\partial}{\partial x}[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}(1-\frac{31M^2\rho^2h^4}{3024\mu^2})\}]-\frac{\partial}{\partial y}[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}(1-\frac{17M^2\rho^2h^4}{1680\mu^2})\}]-\rho W^* \tag{2.2}$$

Taking

$$h = h(x), U = -U, P = P(y) \;\; and \;\; W^* = -\partial P/\partial z|_{z=0}\phi/\mu \tag{2.3}$$

Where; z: Co-ordinate across the fluid film; $\partial P/\partial z$ is the pressure gradient at the bearing surface;; and ϕ is the property called permeability, which varies with porosity and size of pores. From the reqrirments of contininuity, we have for the porous matrix;

$$\phi/\mu \nabla W^* = \nabla^2 P = 0 \; i.e., \; \nabla^2 P = 0 \tag{2.4}$$

The problem then is to solve the governing equation (2.1) for the pressures in lubricant film simultaneously with that of Laplace for the porous matrix with a common pressure gradient $\partial P/\partial z$ at the boundary, we have

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = 0 \tag{2.5}$$

We have two assumptions to solving the equations (2.1) and (2.5) as follows:
(2.5.1) the bearing is infinitely short.
(2.5.2) $\partial P/\partial z$ is linear across the matrix and is zero at the outer surface of the porous bearing shell.
From (2.4), (2.5), we have

$$\frac{\partial^2 P}{\partial x^2} = 0, \frac{\partial^2 P}{\partial z^2} = K \;\; (constant), \;\; \frac{\partial^2 P}{\partial y^2} = -K \tag{2.6}$$

From (2.3), we have

$$\frac{\partial P}{\partial z}|_{z=0} = KH = \frac{\partial^2 P}{\partial y^2}|_{z=0} H \tag{2.7}$$

Where; H: Wall thickness of porous bearing.

Now the equation (2.2) becomes

$$\begin{aligned}[-\frac{h^3}{12\mu}(1 - \frac{17M^2\rho^2h^4}{1680\mu^2})\rho]\frac{d^2P}{dy^2} + [\frac{M\rho^2}{120\mu^2}\frac{d}{dx}(h^5 - \frac{31M^2\rho^2h^9}{3024\mu^2})] \\ \frac{dP}{dy} = \frac{d}{dx}[\frac{\rho U}{2}\{h - \frac{M^2\rho^2h^5}{120\mu^2}(1 - \frac{31M^2\rho^2h^4}{3024\mu^2})\}] \\ -\rho(-\frac{dP}{dz}|_{z=0}\frac{\phi}{\mu})\end{aligned} \tag{2.8}$$

The film thickness $'h'$ and $'y'$ can be taken as:

$$h = C(1 + e\cos\theta), y = R\theta \tag{2.9}$$

Where; θ: Angular co-ordinates (bearing angle), θ being measured from x-direction; e: Eccentricity ratio.
For the determination of pressure the boundary conditions are as follows:

$$P = 0, y = \pm L/2 \tag{2.10}$$

3 Determination of Pressure

The solution of the differential equation (2.8) under the boundary condition (2.10) gives the pressure for porous bearing as follows:

$$P = \left[\begin{array}{c} (3\frac{CU\alpha}{4Rh^3})\mu + (\frac{3KH\phi}{h^3} + \rho C\alpha U\frac{yM}{8Rh^2}) + (\frac{KH\phi yM}{2Rh^2} + 53U\frac{\rho^2 C\alpha h}{2240R}M^2)\frac{1}{\mu} \\ -\frac{17KH\phi\rho^2 hM^2}{560}\frac{1}{\mu^2} \end{array} \right](L^2 - 4y^2) \tag{3.1}$$

4 Result and Discussion

The Aerosil nanolubricants have been used with viscosity 0.1 gm/cm. sec. at 40^0C, 0.077 gm/cm. sec. at 60^0C and 0.055 gm/cm. sec. at 80^0C with a 5% of volume fraction i.e., semi diluted lubricant [8] of nanolubricants [25]. The values of different mathematical terms are taken in C.G.S. system and are as follows:
$\mu_{Aerosil} = 0.1$P (at 40^0C) [25], $\rho_{Aerosil} = 1.7gm./cm^3(1.6gm./cm^3 \leq \rho_{Aerosil} \leq 1.9gm./cm^3)$ [25], $\phi = 0.0025$ cm., C=0.0067cm., h=0.00786 cm., H=0.05 cm., M=0.1, R= 3.35 cm., $U = 10^2$ cm./sec., y=1 cm.
The nondimensional pressure is calculated using eq. (3.1). The variation of pressure (P) of nanoparticles additive couple stress fluid lubricated for porous journal bearing with respect to rotation number (M) are shown in the table (4.1) and with respect to viscosity in table (4.2).

The graphs (4.1), (4.2) and (4.3) show the variation of pressure distribution with bearing angle θ for (M) = 0.1, 0.5 and 1.0 respectively; $\mu_{Aerosil} = 0.1$P and $\rho_{Aerosil} = 1.7gm./cm^3$. The sixth degree approximation curves; $y = -4.005\ x^6 + 125.4\ x^5 - 1565\ x^4 + 9700\ x^3 - 30696\ x^2 + 47809\ x - 25389$; $y = -4.028\ x^6 + 126.2\ x^5 - 1574\ x^4 + 9754\ x^3 - 30863\ x^2 + 48075\ x - 25534$; $y = -4.058\ x^6 + 127.1\ x^5 - 1585\ x^4 + 9821\ x^3 - 31072\ x^2 + 48408\ x -$

$\theta \rightarrow$	M $\downarrow$	0^0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0
$\mu \rightarrow$		0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
P —	0.1	0.81665	3631.18679	4754.85900	6504.403982	9994.7422	11870.84631	11165.91976	8839.568764	1.8375
	0.5	0.97217	3658.866337	4797.68958	6558.796412	10070.7062	11952.0992	11238.6888	8895.294478	2.1875
	1.0	1.16657	3693.4591	4851.227802	6626.786949	10165.6612	12053.66531	11329.65011	8964.951621	2.625

Table (4.1):Pressure Distribution with respect to bearing angle (θ)

μ	0.055	0.077	0.1
M	0.1	0.1	0.1
P for $\theta=30^0$	2000.665811	2797.804537	3631.186792
P for $\theta=45^0$	2620.729393	3664.030293	4754.859004
P for $\theta=60^0$	2962.816677	4142.34521	5362.454041
P for $\theta=90^0$	5506.203978	7700.593751	9994.742200

Table (4.2): Pressure Distribution with respect to viscosity

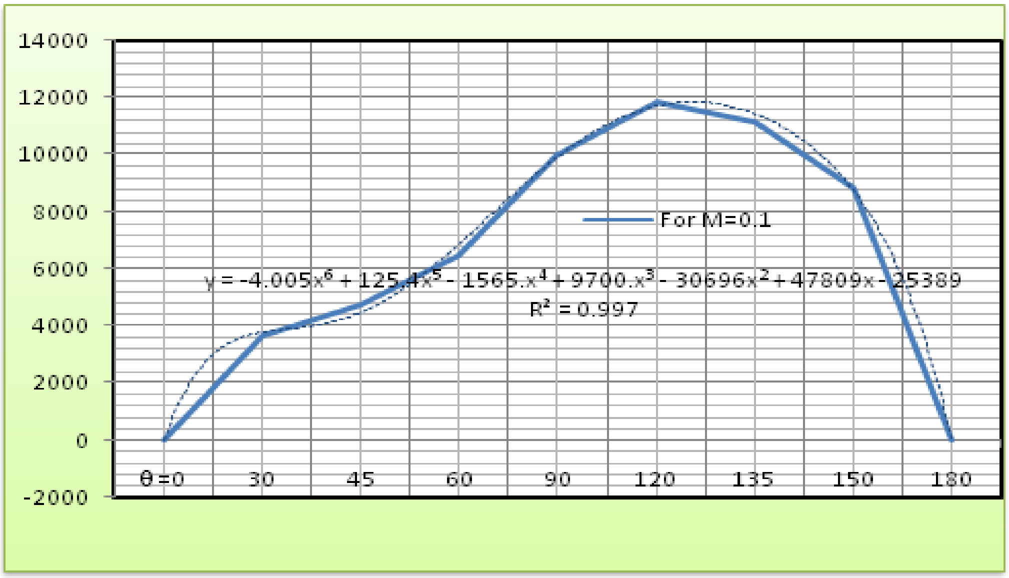

Fig. 2: Pressure Distribution for various values of Bearing angle $'\theta'$ for (M) = 0.1.

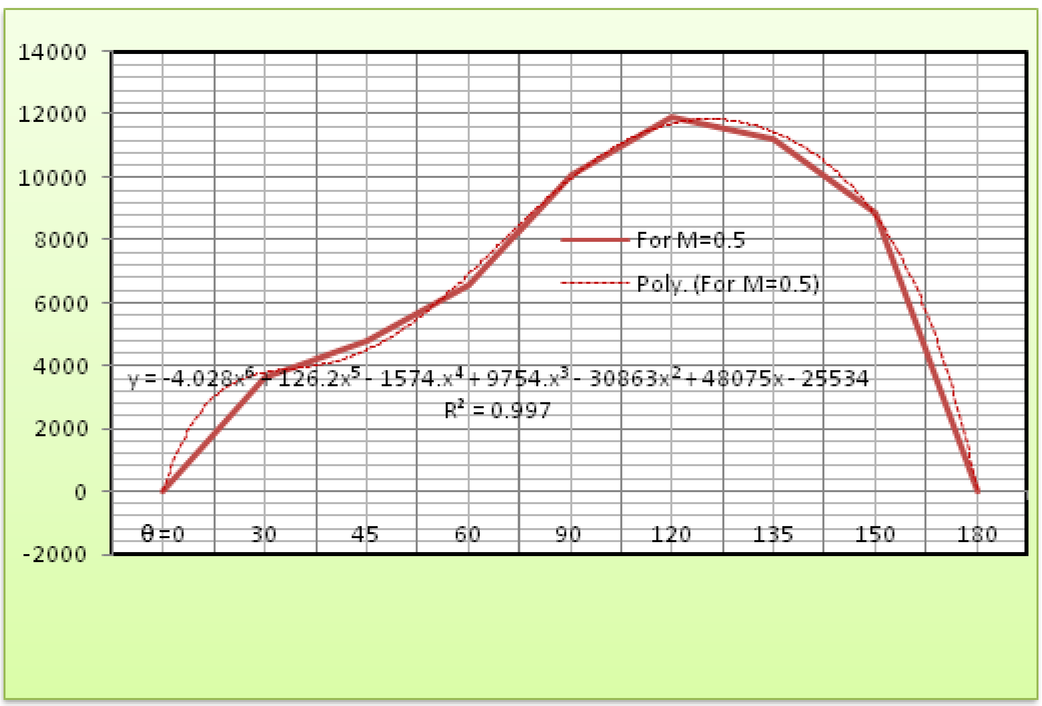

Fig. 3: Pressure Distribution for various values of Bearing angle $'\theta'$ for (M) = 0.5.

Fig. 4: Pressure Distribution for various values of Bearing angle $'\theta'$ for (M) = 1.0.

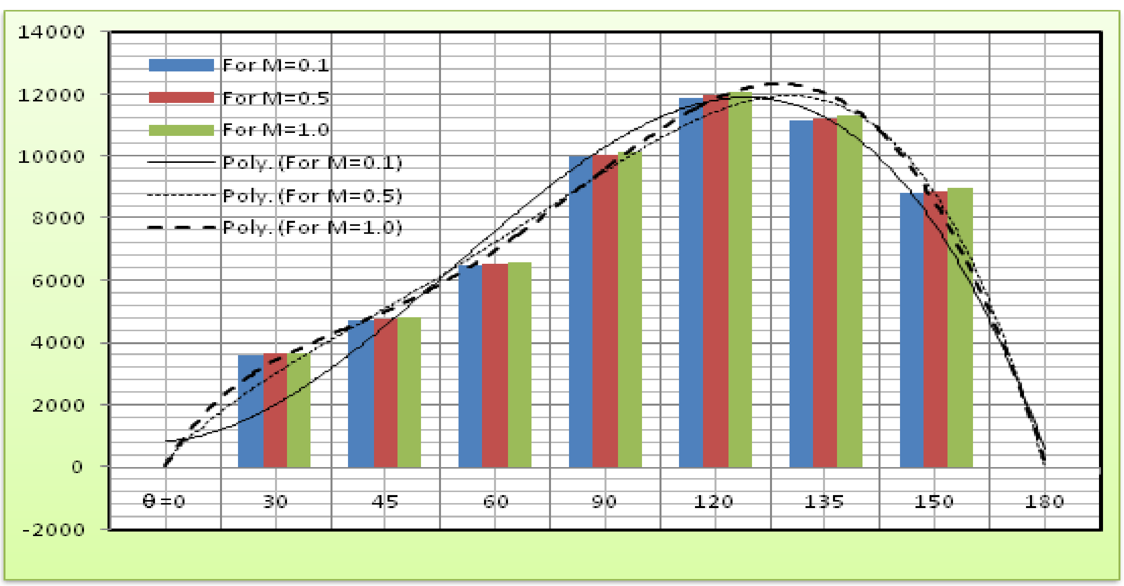

Fig. 5: Comparative Pressure Distribution for various values of Bearing angle $'\theta'$ for (M) = 0.1; 0.5; 1.0 with polynomial trends.

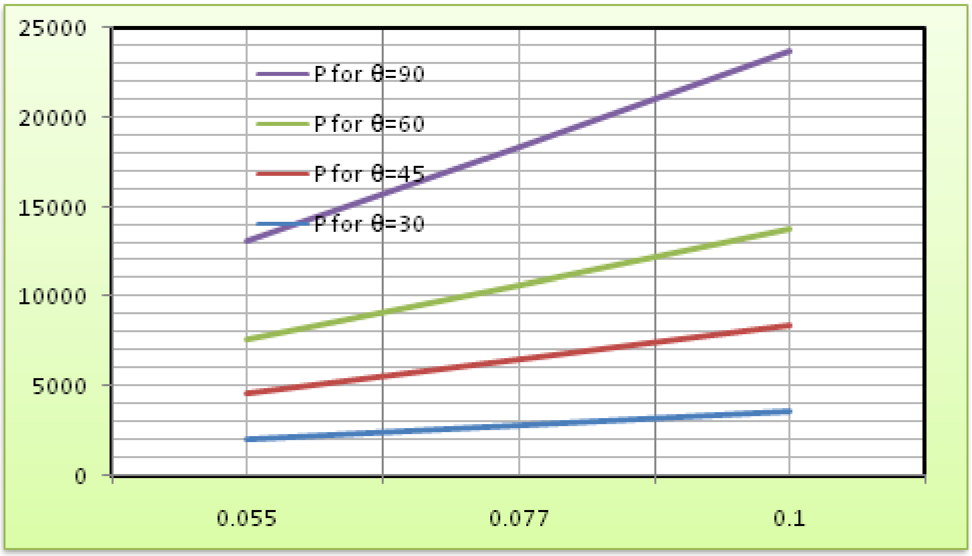

Fig. 6: Pressure Distribution for with respect to Viscosity μ; $(M) = 0.1$ and $(\theta) = 30^0, 45^0, 60^0, 90^0$.

25714; $R^2 = 0.997$ have been drawn for $(M) = 0.1$, 0.5 and 1.0 respectively. The graph shown the increasing trend up to bearing angle $\theta = 130^0$ then decreasing trend up to $\theta = 180^0$. The graphs show that the maximum and minimum pressure distributions occur at $\theta = 130^0$ and $\theta = 0$, 180^0 respectively. the graph (4.4) shows the comparative variation of pressure distribution with bearing angle θ for $M = 0.1,0.5,1.0$ and $\mu_{Aerosil} = 0.1$ P with the sixth degree approximation curves. The graph (4.5) shows the pressure distribution with respect to Viscosity $'\mu'; (M) = 0.1; \rho_{Aerosil} = 1.7gm./cm^3$ and $(\theta) = 30^0, 45^0, 60^0, 90^0$. The graph shows that the pressure distribution increases for increasing the viscosity of nanolubricants and also the rotation number (M). The results are excellent as compared to the analysis of bearing with normal lubricants.

5 Conclusions

The dynamic analysis of the hydrodynamic porous bearing operative underneath nanolubricants is analyzed during this analysis. The pressure distribution of the bearing primarily depends upon the viscosity of the material getting used. The addition of the nanoparticles on industrial materials might enhance the viscosity of lubricant and successively changes the performance characteristics. The analysis of porous bearing i.e., infinitely short is bestowed supported the speculation of journal bearings and takes into consideration of fluid film layers thickness and viscosity of nanoparticles additive fluid and also the rotation number. The fluid film layers in porous bearing are assumed to be Newtonian. The non-dimensional pressure distribution is obtained from integration of Extended Generalized Reynolds Equation underneath the impact of second order rotation by victimization the

Reynolds boundary conditions. The nanolubricant Aerosil is employed for the analysis with 5% volume fraction of nanoparticles. The fluid film properties and structure considerably influence the thin film fluid mechanics lubrication for the nanolubricants underneath the impact of second order rotation. Results reveal a rise in pressure distribution of bearings operative on nanoparticles dispersions as compared to plain lubricants. The pressure are minimum and negligible at $\theta = 0^0$ and $\theta = 180^0$. The pressure distribution will increase with the viscosity of the nanofluids and with increasing the values of rotation number (M). The results obtained are excellent as compared to plain lubricants.

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