Hydrodynamic Permeability of a Membrane of Porous Cylindrical Particle with Varying Permeability

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Abstract

The present paper concern the flow through random assemblage of porous cylindrical particles of radially varying permeability. The Brinkman equation is used for formulation of flow through porous medium and Stokes equation is used for flow through clear fluid region. To model flow through assemblage of particles, cell model technique has been used i.e. the porous cylindrical shell is assumed to be confined within a hypothetical cell of same geometry. Effect of various parameters on the permeability of swarm is being discussed analytically as well as graphically.

Keywords: Brinkman equation, Modified Bessel's Functions, Hydrodynamic permeability, Cell model, Variable permeability.

1 Introduction:

Flow through porous media occurs in a wide range of industrial and natural processes. Flow through random assemblage of particles has been area of interest from last few decades due application in membrane filtration process. In order to study the flow with high porosity Brinkman (1947) suggested a modification to Darcy's (1856) model by adding a Laplacian term in velocity, which is commonly known as Brinkman equation.

The ground flows of water and oil, the filtration of water through the sand, the filtration of various solutions through porous membranes are few examples of flow through porous structures. We can characterized a complex porous structure by swarm of impermeable particles. The shape of these particles are usually taken as spherical and cylindrical. The problem of flow through a swarm of particles can be easily solved by using cell model technique. In the cell model we study slow flow past a swarm of concentrated particles. This technique is used to replace a system of chaotically distributed particles in to a periodic array of particles. We assume every particle in array enclosed in an envelope (cell) and the interaction effect of the multitude of particles being accounted by suitable boundary conditions at the enveloping surface. Happel (1959) and Kuwabara (1959) proposed cell model with different boundary conditions on the cell surface. Happel (1959) proposed cell

models in which the particle and outer cell, both are spherical/cylindrical. The Happel model assumes uniform velocity condition and no tangential stress at the cell surface. This condition leads to an axially symmetric flow. In Kuwabara model he used the zero vorticity condition on the cell surface. Cunningham (1910) proposed another boundary condition on the cell surface which is also known as Mehta and Morse (1975) condition. Cunningham assumed that the tangential velocity as a component of the fluid velocity. This approach signifying the homogeneity of the flow on the cell boundary. Later, Kvashnin (1979) proposed the condition that the tangential component of velocity reaches a minimum at the cell surface. Uchida (1954) gave the concept of cell model by considering enveloping surface of cubical shape. He singled out a particle from the swarm and assumed it to be confined within a cubic cell acting as a fluid envelope. The advantage of the model is that a cubic envelope is space filling however, a major drawback of the model is that of difference in outer and inner geometry. Thus, the entire disturbance due to each particle is confined within the cell of the fluid with which it is associated.
Number of authors have been investigated the problems of flow through porous medium by using different cell models. Vasin at al.(2008) investigated flow around a spherical particle with a porous shell and calculated a hydrodynamic permeability of the porous media build up by such particles. They used different boundary conditions on the outer surface of cells and compare the results obtained. Yadav at al. (2010) consider similar problem as Vasin at al.(2008) and found the effect of stress jump condition on the flow. Vasin and Filippov (2009) calculated hydrodynamic permeability of a membrane simulated by a set of identical impenetrable cylinders covered with a porous layer by the HappelBrenner cell method. They studied both transverse and longitudinal flows of filtering liquid with respect to the cylindrical fibers that compose the membrane. Boundary conditions on the cell surface that correspond to the Happel, Kuwabara, Kvashnin, and Cunningham models are considered and Brinkman equations (1947) are used to describe the flow of liquid in the porous layer. Yadav at al. (2013) studied the hydrodynamic permeability of biporous membrane built up by porous cylindrical particles located in another porous medium by using cell model technique. They considered four known boundary conditions, namely, Happels, Kuwabaras, Kvashnins and Cunningham/Mehta Morses on the outer surface of the cell and comparison of the resulting hydrodynamic permeability has been undertaken. Srivastava and Deo(2013) studied a fully developed flow of an electrically conducting viscous fluid through a porous medium of variable permeability under the transverse applied uniform magnetic field between two parallel plates. The variation of permeability is taken quadratic on the transverse direction. They used Brinkman equation for flow through porous medium and obtained numerical solution for the velocity and volumetric flow rate for the two cases, Poiseuille and Couette flow. Verma and Datta (2012) found analytical solution for fully developed laminar flow of a viscous incompressible fluid in an annular region between two coaxial cylindrical tubes filled with a porous medium of variable permeability when the permeability of the porous medium varies with the radial distance. In the present article we investigate similar problem as considered by Verma and Datta (2012) and find analytical solution by using cell model. We use Brinkman equation to analyze the flow in porous region.

2 Mathematical formulation:

We have considered an axis symmetric Stokes flow of an incompressible fluid through a membrane build up by swarm of porous cylindrical particles of radius $\tilde{b}$ each enclosing an impermeable core of radius $\tilde{a}$ ($\tilde{b} > \tilde{a}$). We assume that axes of all cylindrical particles are parallel to each other. We use cell model to investigate the flow following Happel and Brenner (1983). By virtue of cell model, each porous cylindrical shell is assumed to be enveloped by a concentric cylinder of radius $\tilde{c}$ ($\tilde{c} > \tilde{b}$) named as cell surface. The Stokes flow of a viscous incompressible fluid is assumed to approach towards cell surface as well as partially passing through the composite cylindrical particle along the axis of cylinder ($\tilde{Z}$ - axis) with constant velocity $\tilde{U}$ from left to right as shown in Fig.1. The radius $\tilde{c}$ of hypothetical cell is chosen in such a way that the particle volume fraction m of the swarm is equal to the particle volume fraction of the cell (More precisely, the volume fraction of the partially porous particles to the volume of a cell is equal to the volume fraction of particles in the membrane.), i.e.

$$m^2 = \frac{\pi\tilde{c}^2}{\pi\tilde{a}^2}. \tag{2.1}$$

The governing equation of motion in the cell region outside the porous cylindrical shell is Stokes equation, which is

$$\nabla^2\tilde{u}_1 = \frac{1}{\tilde{\mu}}\nabla\tilde{p} \tag{2.2}$$

Here $\tilde{u}_1$ is the fluid velocity, $\tilde{\mu}$ is the fluid viscosity and $\tilde{p}$ is the pressure in the cell region (region- I). In cylindrical polar coordinates $(\tilde{r}, \theta, \tilde{z})$ the above stokes equation for present flow can be written as

$$\frac{d^2\tilde{u}_1}{d\tilde{r}^2} + \frac{1}{\tilde{r}}\frac{d\tilde{u}_1}{d\tilde{r}} = \frac{1}{\tilde{\mu}}\frac{\partial\tilde{p}}{\partial\tilde{z}} \tag{2.3}$$

In the porous region (region II) the governing equation is Brinkman equation given by

$$\tilde{\mu}_e\nabla^2\tilde{u}_2 - \frac{\tilde{\mu}}{k}\tilde{u}_2 = \nabla\tilde{p} \quad ; \qquad \tilde{b} \geq \tilde{r} \geq \tilde{a}. \tag{2.4}$$

Here $\tilde{u}_2$ is the fluid velocity in the porous region, k is the permeability of the porous region -II and $\tilde{\mu}_e$ is the effective viscosity in the porous region ($\tilde{b} \geq \tilde{r} \geq \tilde{a}$). Authors have different opinions about the role of effective viscosity. Liu and Masaliyah (2005) say that, depending on the type of porous medium, the effective viscosity may be either smaller or greater than the fluid viscosity. Many authors, for example, Brinkman (1947) and Chikh et al. (1995), assume that $\tilde{\mu}_e = \tilde{\mu}$. This assumption is valid for a high-porosity medium. In the present work, following Brinkman and Chikh, we assume that $\tilde{\mu}_e = \tilde{\mu}$. With this assumption, Brinkman momentum Eq. (2.4) becomes

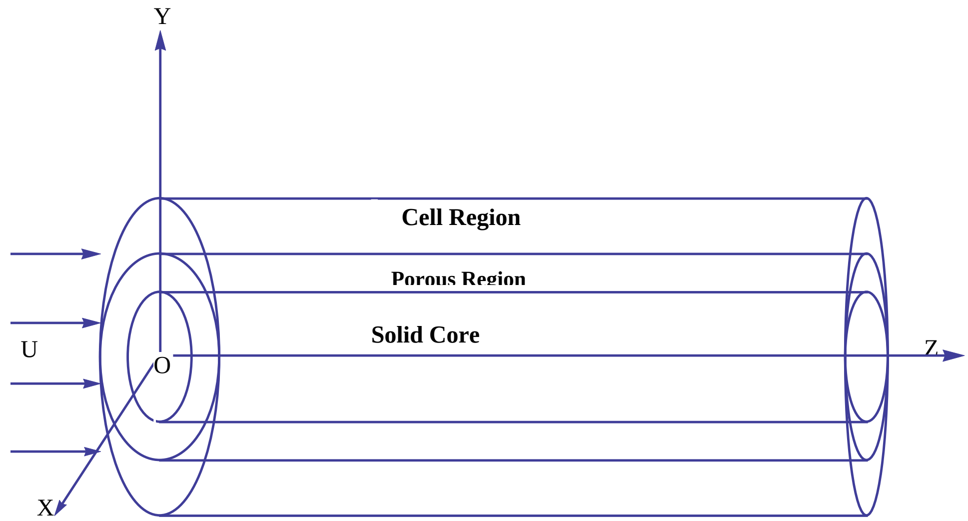

Fig. 1: Porous cylindrical particle with cell surface

$$\nabla^2 \tilde{u}_2 - \frac{\tilde{u}_2}{k} = \frac{1}{\tilde{\mu}} \nabla \tilde{p} \tag{2.5}$$

In cylindrical polar coordinates $(\tilde{r}, \theta, \tilde{z})$ above Eq.(2.5) for the present flow can be written as

$$\frac{d^2 \tilde{u}_2}{d\tilde{r}^2} + \frac{1}{\tilde{r}} \frac{d\tilde{u}_2}{d\tilde{r}} - \frac{\tilde{u}_2}{k} = \frac{1}{\tilde{\mu}} \frac{\partial \tilde{p}}{\partial \tilde{z}} \; ; \qquad \tilde{b} \geq \tilde{r} \geq \tilde{a}. \tag{2.6}$$

Now, we introduce non dimensional quantities as follows

$$r = \frac{\tilde{r}}{\tilde{a}}, \;\; u_1 = \frac{\tilde{u_1}}{\tilde{U}}, \;\; u_2 = \frac{\tilde{u_2}}{\tilde{U}}, \;\; z = \frac{\tilde{z}}{\tilde{a}}, \;\; n = \frac{\tilde{b}}{\tilde{a}}, \;\; m = \frac{\tilde{c}}{\tilde{a}}. \tag{2.7}$$

Using these non dimensional variables Stokes Eq.(2.3) and Brinkman Eq. (2.6) takes the following form

$$\frac{d^2 u_1}{dr^2} + \frac{1}{r} \frac{du_1}{dr} = -P \tag{2.8}$$

and

$$\frac{d^2u_2}{dr^2} + \frac{1}{r}\frac{du_2}{dr} - \frac{a^2u_2}{k} = -P. \tag{2.9}$$

respectively, where $P = -(a^2/\tilde{U}\tilde{\mu})(\partial\tilde{p}/\partial\tilde{z})$ is non dimensional pressure gradient.

3 Boundary Conditions

In our present case, the Happel, Kuwabara, Kvashnin, and Cunningham boundary conditions lead to the following single condition on the cell surface, which is in non dimensional variables can be expressed as

$$\frac{du_1}{dr} = 0, \quad \text{at} \quad r = m. \tag{3.1}$$

On the fluid porous interface at $r = n$, we assume following matching conditions -
(i) Continuity of velocity i.e.

$$u_1 = u_2, \quad \text{at} \quad r = n \tag{3.2}$$

and (ii) Continuity of tangential stress gives us

$$\frac{du_1}{dr} = \frac{du_2}{dr}, \quad \text{at} \quad r = n. \tag{3.3}$$

On the surface of impermeable inner cylindrical core we have no slip condition, which is

$$u_2 = 0 \quad \text{at} \quad r = 1. \tag{3.4}$$

4 Solution and Results

Now we consider the case when permeability of the porous region - II vary with radial distance according to the law $k(r) = k_o r^N$, where N is a real number. With this permeability Eq.(1.9) become

$$\frac{d^2u_2}{dr^2} + \frac{1}{r}\frac{du_2}{dr} - \frac{\alpha^2u_2}{r^N} = -P. \tag{4.1}$$

where $\alpha^2 = a^2/k_o$ is the permeability parameter. Since it is very cumbersome to deal with the general value of N, therefore we will consider three particular cases of permeability variation : case I, when $k = k_o$; case II, when $k = k_o r$ and case III, when $k = k_o r^2$. Here k_o is characteristic permeability, which may be taken as permeability on the surface of an inner cylinder or as mean permeability of the porous region-II.

4.1 Case I

When permeability of the porous region is constant. Let $k = k_0$, i.e. $N = 0$ in Eq.(4.1). With this permeability Brinkman Eq.(4.1) becomes

$$\frac{d^2u_2}{dr^2} + \frac{1}{r}\frac{du_2}{dr} - \alpha^2 u_2 = -P \tag{4.2}$$

Equation (4.2) is a modified Bessels equation of order zero. Its general solution is given by

$$u_2(r) = C_1 I_0(\alpha r) + D_1 K_0(\alpha r) + \frac{P}{\alpha^2} \tag{4.3}$$

Where I_0 and K_0 are modified Bessel' s functions of order zero of first and second kind, respectively. C_1 and D_1 are constants of integration. Solution of Stokes equation (2.8) for cell region-I is given by

$$u_1(r) = A_1 + B_1 \log r - \frac{Pr^2}{4} \tag{4.4}$$

Here A_1 and B_1 are constants of integration. Using boundary conditions (3.1), (3.2), (3.3) and (3.4) to determine constants A_1, B_1, C_1 and D_1. We get

$$\begin{aligned}
A_1 &= \frac{P\left[\alpha n K_0(\alpha) + 2K_1(\alpha n)\right]\left[I_0(\alpha)K_0(\alpha n) - K_0(\alpha)I_0(\alpha n)\right]}{2\alpha^2 K_0(\alpha)\left[K_0(\alpha)I_1(\alpha n) + I_0(\alpha)K_1(\alpha n)\right]} - \frac{PK_0(\alpha n)}{\alpha^2 K_0(\alpha)} \\
&\quad + \frac{n^2 P}{4} + \frac{P}{\alpha^2} - \frac{Pm^2}{2}\left[\log(n) + \frac{(I_0(\alpha)K_0(\alpha n) - K_0(\alpha)I_0(\alpha n))}{n\alpha\left(K_0(\alpha)I_1(\alpha n) + I_0(\alpha)K_1(\alpha n)\right)}\right] \\
B_1 &= \frac{m^2 P}{2} \\
C_1 &= \frac{P\alpha(m^2 - n^2)K_0(\alpha) - 2nPK_1(\alpha n)}{2\alpha^2 n\left(K_0(\alpha)I_1(\alpha n) + I_0(\alpha)K_1(\alpha n)\right)} \\
D_1 &= -\frac{P\alpha(m^2 - n^2)I_0(\alpha) + 2nPI_1(\alpha n)}{2\alpha^2 n\left(K_0(\alpha)I_1(\alpha n) + I_0(\alpha)K_1(\alpha n)\right)}
\end{aligned} \tag{4.5}$$

The fluid velocity u_1 in the cell region and u_2 in the porous region is given by Eq.(4.4) and Eq.(4.3), respectively when permeability of the porous region is constant k_0. The constants A_1, B_1, C_1 and D_1 are given by Eq.(4.5). The velocity profile of the flow within the cell region and porous region for different values of permeability variation parameter $\alpha = 3, 6, 9$ and 12, when permeability of the porous region is $k = k_o$ and $P = 1, n = 2, m = 2.5$ is shown in Fig(2).

4.1.1 Rate of volume Flow

The dimensionless rate of volume flow through the cross - section of cylindrical particle enveloped by hypothetical cell is given by

$$Q = 2\pi\left[\int_n^m r\, u_1(r)dr + \int_1^n r\, u_2(r)dr\right] \tag{4.6}$$

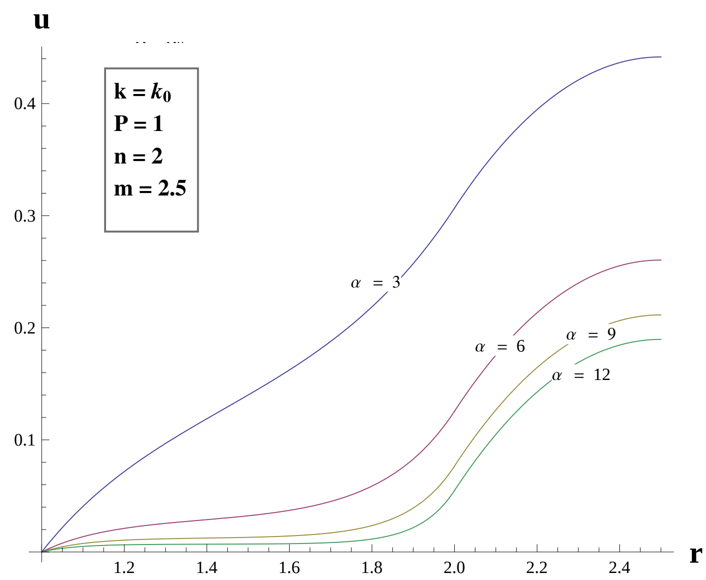

Fig. 2: Velocity profile of the flow within the cell region and porous region for different values of α when permeability of the porous region is $k = k_o$.

$$
\begin{aligned}
Q \quad = \quad & 2\pi \left[\frac{-2\alpha C_1 \left[I_1(\alpha) - nI_1(\alpha n)\right] + 2\alpha D_1 \left[K_1(\alpha) - nK_1(\alpha n)\right] + \left(n^2 - 1\right) P}{2\alpha^2} \right. \\
& \left. + \left(\frac{A_1 - 8P}{16}\right) \left(m^2 - n^2\right) + \frac{1}{4} B_1 \left(-m^2 + 2m^2 \log m + n^2 - 2n^2 \log n\right) \right] \qquad (4.7)
\end{aligned}
$$

Where I_1 and K_1 are modified Bessel's functions of first order. The variation for rate of volume flow through the cross section with α is shown in Fig.(3).

4.1.2 Hydrodynamic Permeability

Hydrodynamic permeability of the membrane when permeability of the porous region is constant $k = k_o$ is defined as

$$
L_1 = \frac{Q}{\pi m^2 P}. \qquad (4.8)
$$

Where volume flow rate$Q$ is given by Eq.(4.7). Variation of hydrodynamic permeability with permeability variation parameter α is shown in Fig(4).

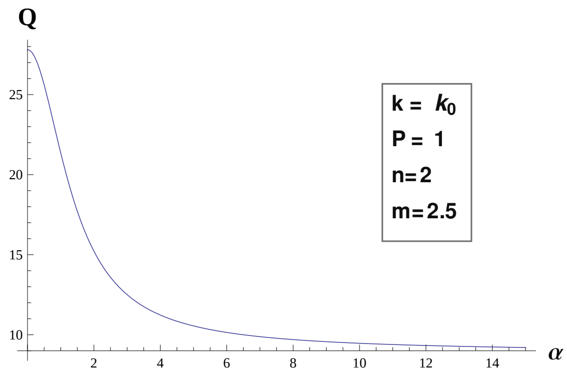

Fig. 3: Variation of rate volume flow Q with α when permeability of the porous region is vary as $k = k_o$.

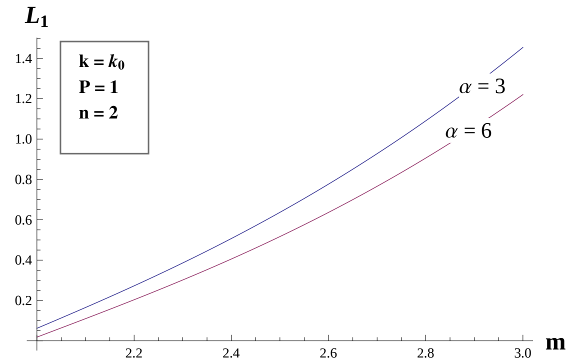

Fig. 4: Variation for Hydrodynamic permeability for different values of α when permeability of the porous region is vary as $k = k_o$.

4.2 Case II

When the permeability k of porous region of the particle is varying according to the law $k = k_0 r$, i.e. $N = 1$ in Eq.(4.1). With this permeability Brinkman Eq.(4.1) becomes

$$r\frac{d^2u_2}{dr^2} + \frac{1}{r}\frac{du_2}{dr} - \alpha^2 u_2 = -rP \tag{4.9}$$

General solution of this differential equation is

$$u_2(r) = C_2 I_o\left(2\sqrt{r}\alpha\right) + D_2 K_o\left(2\sqrt{r}\alpha\right) + \frac{P\left(\alpha^2 r + 1\right)}{\alpha^4} \tag{4.10}$$

where I_o and K_o are modified Bessel' s functions of order zero of first and second kind, respectively and C_2, D_2 are constant of integration.Solution of Stokes equation (2.8) for cell region-I is given by

$$u_1(r) = A_2 + B_2 \log r - \frac{Pr^2}{4}. \tag{4.11}$$

Here A_2 and B_2 are constants of integration. Using boundary conditions (3.1), (3.2), (3.3) and (3.4) to determine constants A_2, B_2, C_2 and D_2. We get

$$
\begin{aligned}
A_2 &= \frac{P}{4\alpha^4 K_0(2\alpha)}\Big[(2+n\alpha)^2 K_0(2\alpha) - 4\left(\alpha^2+1\right) K_0\left(2\alpha\sqrt{n}\right) \\
&\quad - \frac{2\left[K_0(2\alpha) I_0\left(2\alpha\sqrt{n}\right) - I_0(2\alpha) K_0\left(2\alpha\sqrt{n}\right)\right]\left[\alpha\sqrt{n}\left(\alpha^2 n + 2\right) K_0(2\alpha) + 2\left(\alpha^2+1\right) K_1\left(2\alpha\sqrt{n}\right)\right]}{\left[K_0(2\alpha) I_1\left(2\alpha\sqrt{n}\right) + I_0(2\alpha) K_1\left(2\alpha\sqrt{n}\right)\right]} \\
&\quad - 8m^2\alpha^4\left(K_0(2\alpha)\log(n) + \frac{I_0(2\alpha) K_0\left(2\alpha\sqrt{n}\right) - K_0(2\alpha) I_0\left(2\alpha\sqrt{n}\right)}{\alpha\sqrt{n}\left(K_1\left(2\alpha\sqrt{n}\right) + I_1\left(2\alpha\sqrt{n}\right)\right)}\right)\Big] \\
B_2 &= \frac{m^2 P}{2} \\
C_2 &= -\frac{P\alpha[\alpha^2(n^2-m^2)+2n] K_0(2\alpha) + 2P(\alpha^2+1)\sqrt{n} K_1\left(2\alpha\sqrt{n}\right)}{2\alpha^4\sqrt{n}\left(K_0(2\alpha) I_1\left(2\alpha\sqrt{n}\right) + I_0(2\alpha) K_1\left(2\alpha\sqrt{n}\right)\right)}
\end{aligned}
$$

$$
D_2 = -\frac{P\alpha[\alpha^2(m^2-n^2)-2n] I_0(2\alpha) + 2P(\alpha^2+1)\sqrt{n} I_1\left(2\alpha\sqrt{n}\right)}{4\alpha^4\sqrt{n}\left(K_0(2\alpha) I_1\left(2\alpha\sqrt{n}\right) + I_0(2\alpha) K_1\left(2\alpha\sqrt{n}\right)\right)} \tag{4.12}
$$

The fluid velocity u_1 in the cell region and u_2 in the porous region is given by Eq.(4.11) and Eq.(4.12), respectively when permeability of the porous region is $k = k_0 r$. The constants A_2, B_2, C_2 and D_2 are given by Eq.(4.12). The velocity profile of the flow within the cell region and porous region for different values of permeability variation parameter $\alpha = 3, 6, 9$ and 12, when permeability of the porous region is $k = k_o r$ and $P = 1, n = 2, m = 2.5$ is shown in Fig(5).

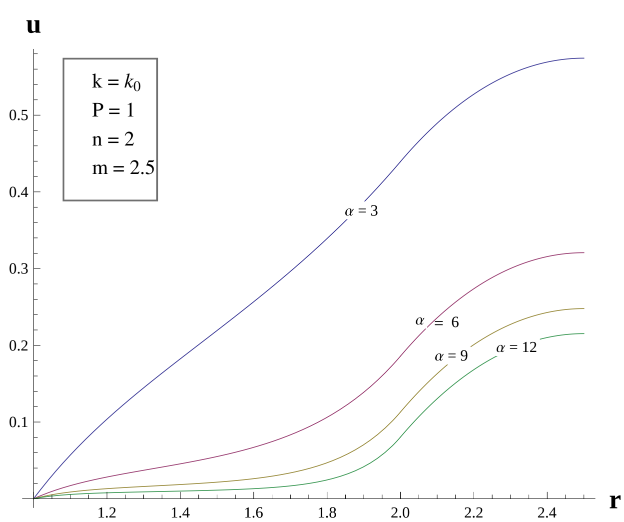

Fig. 5: Velocity profile of the flow within the cell region and porous region for $\alpha = 3, 6, 9$ and 12 when permeability of the porous region is $k = k_o r$.

4.2.1 Rate of Volume Flow:

The dimensionless rate of volume flow through the cross - section of cylindrical particle enveloped by hypothetical cell when permeability of the porous region vary $k = k_o r$ is given by

$$Q = 2\pi \left[\int_n^m r\, u_1(r) dr + \int_1^n r\, u_2(r) dr \right]$$

$$\begin{aligned} Q &= C_2 \frac{2\pi}{\alpha^2} \{ \alpha n \sqrt{n} I_1(2\sqrt{n}\alpha) - \alpha I_1(2\alpha) - n I_2(2\sqrt{n}\alpha) + I_2(2\alpha) \} \\ &+ \frac{\pi}{3\alpha^4} \{ 3(q^2 - 1) + 2\alpha^2(q^3 - 1) \} + D_2 \frac{2\pi}{\alpha^2} \big\{ -\alpha n \sqrt{n} K_1(2\sqrt{n}\alpha) + \alpha K_1(2\alpha) \\ &- n K_2(2\sqrt{n}\alpha) + K_2(2\alpha) \big\} + \frac{1}{16} \big(8A_2 \left(m^2 - n^2 \right) + 4B_2 \big(-m^2 \\ &+ 2m^2 \log m + n^2 - 2n^2 \log n \big) + P \left(n^4 - m^4 \right) \big) \end{aligned} \tag{4.13}$$

The variation for rate of volume flow through the cross section with α is shown in Fig.(6).

4.2.2 Hydrodynamic Permeability

Hydrodynamic permeability of the membrane when permeability of the porous region is constant $k = k_o r$ is defined as

$$L_2 = \frac{Q}{\pi m^2 P} \tag{4.14}$$

Where Q is given by Eq.(4.13). Variation of hydrodynamic permeability with permeability variation parameter α is shown in Fig(7).

4.3 Case III

When the permeability of Porous medium is $k = k_0 r^2$ In this case Brinkman's equation takes form

$$r^2 \frac{d^2 u_2}{dr^2} + r \frac{du_2}{dr} - \frac{u_2}{k_0} = -P r^2 \tag{4.15}$$

where $P = (a^2 p_0 / u \tilde{\mu}_1)(\partial p / \partial z)$.

$$r^2 \frac{d^2 u_2}{dr^2} + r \frac{du_2}{dr} - \alpha^2 u_2 = -P r^2. \tag{4.16}$$

The corresponding solution of Brinkman's equation becomes

$$u_2(r) = C_3 \cosh(\alpha \log r) + D_3 \sinh(\alpha \log r) + \frac{P r^2}{\alpha^2 - 4}, \quad \text{for} \quad \alpha \neq 2 \tag{4.17}$$

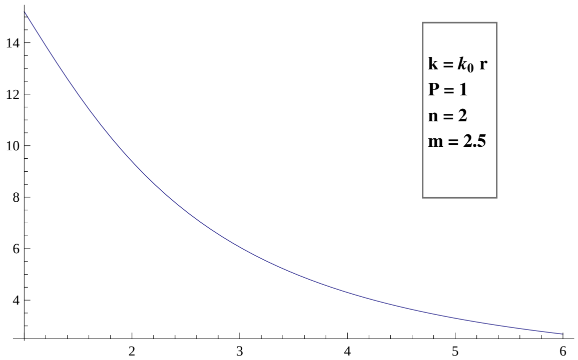

Fig. 6: Variation of rate volume flow Q with α when permeability of the porous region is vary as $k = k_o r$.

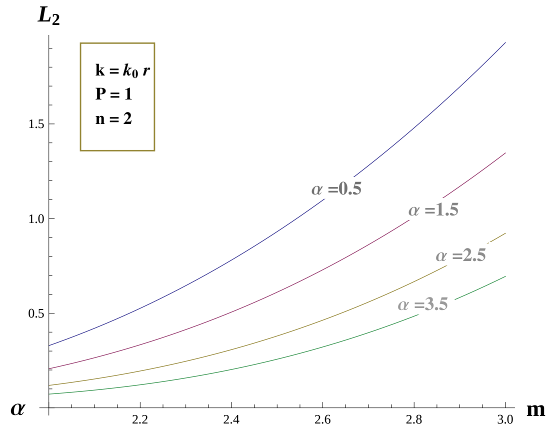

Fig. 7: Variation for Hydrodynamic permeability for different values of α when permeability of the porous region is vary as $k = k_o r$.

$$u_2(r) = C'_3 \cosh(2\log r) + D'_3 \sinh(2\log r) - \frac{1}{4}Pr^2 \log r, \quad \text{for} \quad \alpha = 2 \tag{4.18}$$

Solution of Stokes equation (2.8) for cell region-I is given by

$$u_1(r) = A_3 + B_3 \log r - \frac{Pr^2}{4}. \tag{4.19}$$

To find the constants A_3, B_3, C_3 and D_3 for $\alpha \neq 2$ we use boundary conditions (3.1), (3.2), (3.3) and (3.4). We get

$$\begin{aligned}
A_3 &= \frac{1}{4}P[\frac{\alpha n^2\{\alpha - 2\tanh(\alpha\log n)\} - 4\text{sech}(\alpha\log n)}{\alpha^2 - 4} + \frac{2m^2\tanh(\alpha\log n) - \alpha\log(n)}{\alpha}] \\
B_3 &= \frac{m^2 P}{2}, \\
C_3 &= -\frac{P}{\alpha^2 - 4}, \\
D_3 &= \frac{m^2 P\text{sech}(\alpha\log n)}{2\alpha} - \frac{P\{\alpha n^2\text{sech}(\alpha\log n) - 2\tanh(\alpha\log n)\}}{2\left(\alpha^2 - 4\right)}
\end{aligned}$$

To find the constants A_3', B_3', C_3' and D_3' for $\alpha = 2$ we use boundary conditions (3.1), (3.2), (3.3) and (3.4). We get

$$\begin{aligned} A_3' &= \frac{1}{8}P\{-2\left(2m^2+n^2\right)\log n + \left(2m^2-n^2+2n^2\log n\right)\tanh(2\log n)+2n^2\}, \\ B_3' &= \frac{m^2 P}{2}, \\ {C'}_3 &= 0, \\ {D'}_3 &= \frac{1}{8}P\left(2m^2-n^2+2n^2\log n\right)\operatorname{sech}(2\log n) \end{aligned} \tag{4.20}$$

The corresponding expressions for dimensionless velocity at any point in cell region and porous region, when permeability of medium is $k = k_0 r^2$ is given by Eq.s (4.19) and (4.17), on insertion of above values of constants.

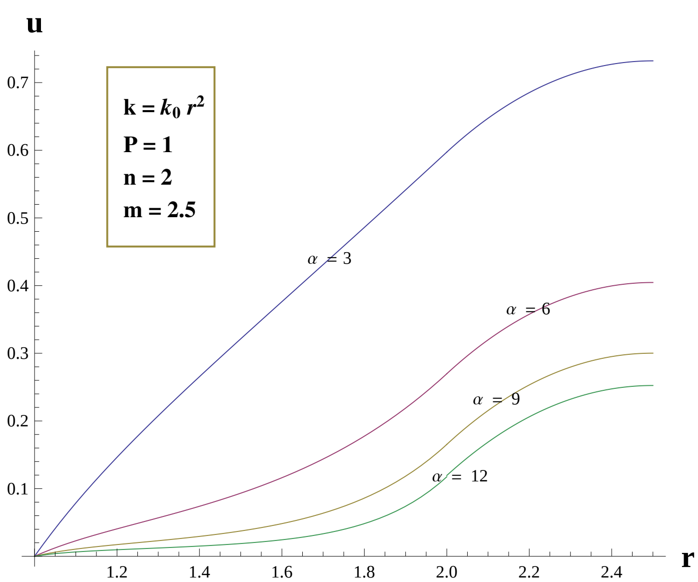

Fig. 8: Velocity profile for different values of α.

4.3.1 Rate of Volume Flow :

The dimensionless rate of volume flow through the cross - section of cylindrical particle enveloped by hypothetical cell when permeability of the porous region vary $k = k_o r^2$ is given by

$$Q = 2\pi\left[\int_n^m r u_1(r)dr + \int_1^n r u_2(r)dr\right] \tag{4.21}$$

Q for $\alpha \neq 2$ and $\alpha = 2$ is given by following equations.

$$Q = 2\pi \frac{4n^2((\alpha C_3 - 2D_3)\sinh(\alpha \log n) + (\alpha D_3 - 2C_3)\cosh(\alpha \log n)) + 8C_3 - 4\alpha D_3 + \left(n^4 - 1\right) P}{4\left(\alpha^2 - 4\right)}$$
$$+\frac{\pi}{8}[\left(m^2 - n^2\right)\{8A_3 - 4B - P\left(m^2 + n^2\right)\} + 8B_3\left(m^2 \log m - n^2 \log n\right)], \alpha \neq 2$$

$$Q = \frac{\pi}{8}\{8A_3\left(m^2 - n^2\right) + 4B_3\left(-m^2 + 2m^2 \log m + n^2 - 2n^2 \log n\right) + P\left(n^4 - m^4\right)\}$$
$$+\frac{\pi}{32}\{\left(n^4 - 1\right)\left(8C_3' + 8D' + 1\right) + 4\log n\left(8C_3' - 8D_3' - n^4\right)\}, \alpha = 2. \tag{4.22}$$

The variation for rate of volume flow through the cross section with α is shown in Fig.(9).

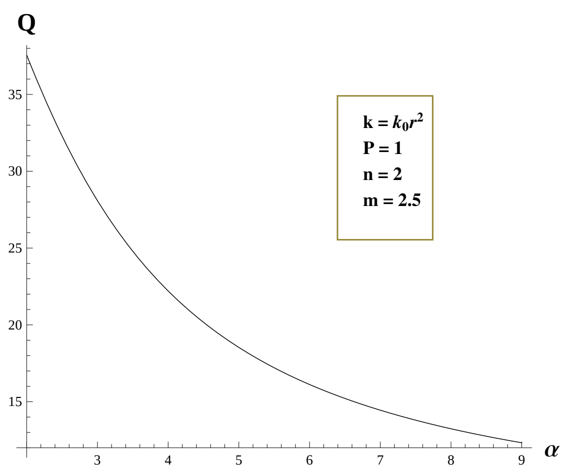

Fig. 9: Variation of rate volume flow Q with α when permeability of the porous region is vary as $k = k_o r^2$.

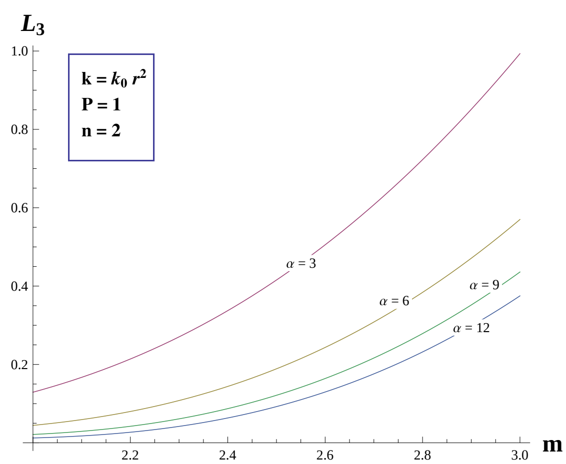

Fig. 10: Variation for Hydrodynamic permeability for different values of α when permeability of the porous region is vary as $k = k_o r^2$.

4.3.2 Hydrodynamic Permeability:

Hydrodynamic permeability of the membrane when permeability of the porous region is constant $k = k_o r^2$ is defined as

$$L_3 = \frac{Q}{\pi m^2 P}, \tag{4.23}$$

Where Q is given by equation no. 4.21 and 4.22. Variation of hydrodynamic permeability of the membrane with of m for different of α is shown in Fig.(10).

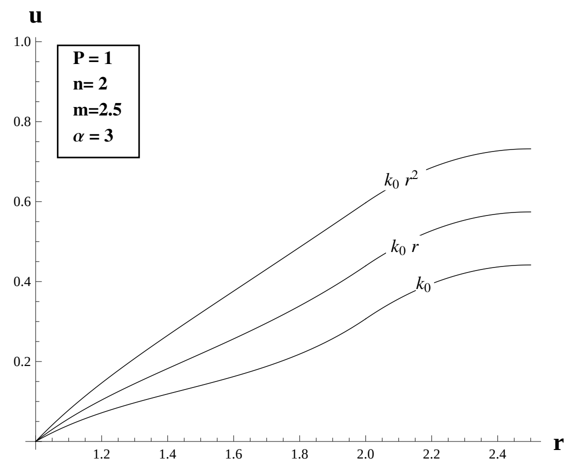

Fig. 11: Velocity Profile for different Cases.

5 Discussion and conclusion

Fig. (2), (5) and (8) shows the velocity profile of fluid velocity within the cell region and porous region when permeability of the porous particle vary according to the law $k = k_o$, $k = k_o r$ and $k = k_o r^2$, respectively. These figures are plotted for fixed value of $n = 2$, $m = 2.5$, $P = 1$ when $\alpha = 3, 6, 9$ and 12. Figures reveal that as permeability parameter α increases, velocity u of the fluid flow decreases. This is because increase in α caused decrease in the permeability of the porous region. Fig. (11) shows the effect of permeability variation on the fluid velocity for fixed value of $\alpha = 3$. We observe that velocity is maximum for permeability variation $k = k_0 r^2$ and minimum for the case $k = k_o$.

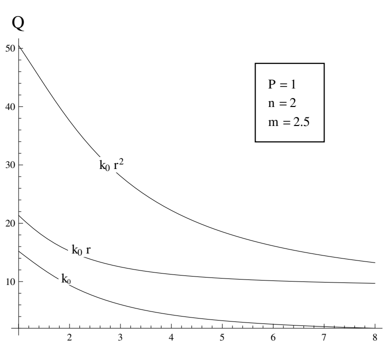

Fig. 12: Variation of volume flow rate for different cases of permeability variation .

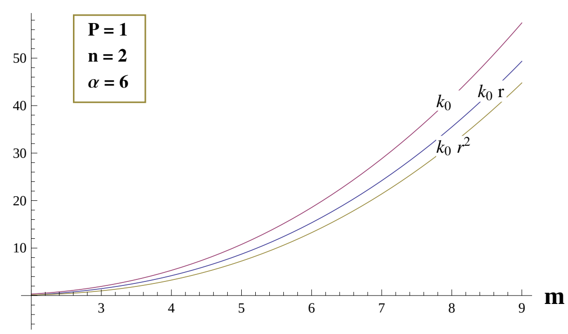

Fig. 13: Variation of hydrodynamic permeability for different cases of permeability variation.

Fig. (3), (6) and (9) shows that rate of volume flow Q decreases with increases in α i.e. with decrease in permeability for all the cases of permeability variation. In Fig.(12) we found that rate of volume flow Q is maximum when permeability of the porous medium varies as $k = k_0 r^2$ and it is minimum when permeability is constant.

Fig. (4), (7) and (10) shows the variation of hydrodynamic permeability of the membrane with particle volume fraction m when permeability of the porous particle vary according to the law $k = k_o$, $k = k_o r$ and $k = k_o r^2$, respectively. Figures reveal that hydrodynamic permeability of the membrane increases with increase in m. Fig.(13) shows that hydrodynamic permeability of the membrane is minimum in the case when permeability of the porous particle vary according to the law $k = k_o r^2$ and is maximum when it is constant.

It is observed that permeability parameter α has strong influence on the flow. Increase in α causes decrease in the fluid velocity and the volume flow rate through the membrane. It is also observed that permeability variation in the porous particle affects the flow through membrane considerably. Hydrodynamic permeability of the membrane increases with increase in m.

References

[1] Brinkman, H. C., A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles, Appl. Sci. Res. A, vol. 1, pp. 27 - 34, 1947.

[2] Chikh, S., Boumedien, A., Bouhadef, K., and Lauriat, G., Analytical solution of non-Darcian forced convection in an annular duct partially filled with a porous medium, Int. J. Heat Mass Transfer, vol. 38, pp. 15431551, 1995.

[3] Cunningham, E., On the Velocity of Study Fall of Spherical Particles through Fluid Medium, Roy. Soc. (London). A, 83 pp. 357 - 365, 1910.

[4] Darcy, H., Les Fontaines Publiques de la Ville de Dijon, Dalmont,Paris, 1856.

[5] Deo, S., Stokes flow past of porous cylinder with Happel and Kuwabara boundary conditions,Sadhana Vol.29 (4) pp. 381 - 387, 2004.

[6] Happel, J., Viscous flow relative to arrays of cylinders, A.I.Ch.E. 5(2), pp. 174 - 177, 1959.

[7] Kuwabara, S., The forces experienced by randomly distributed parallel circular cylinders or spheres in a viscous fluid at small Reynolds number. Phys. Soc. Japan,vol. 14, pp. 527 - 532, 1959.

[8] Kvashnin, A.G., Cell Model of suspension of spherical particles Fluid Dynamics, vol.14 pp.598 - 602, 1979.

[9] Liu, S. and Masliyah, J. H., Dispersion in porous media, in Handbook of Porous Media, ed. K. Vafai, pp. 81140, Taylor and Francis, Boca, Raton, FL, 2005.

[10] Mehta, G. and Morse, T., Flow through charged membranes, J. Chem. Phys., vol. 63, pp. 1877, 1975.

[11] Srivastava, B. and Deo, S., Effect of magnetic field on the viscous fluid flow in a channel filled with porous medium of variable permeability, Applied Mathematics and Computation, vol. 219, pp.89598964, 2013.

[12] Tiwari, A., Deo, S., Filippov, A.N., Effect of the magnetic field on the hydrodynamic permeability of a membrane, Colloid J., vol 74, pp. 515 - 522, 2012.

[13] Uchida, S., Viscous flow in multiparticle system: slow viscous flow through a mass of particles,Int. Sci. Technol. Univ.Tokyo (Translated by T.Motai in Eng.) 46, pp. 1194 - 1195, 1954.

[14] Vasin, S.I., Filippov, A.N. and Starov, V.M., Hydrodynamic permeability of membranes built up by particles covered by porous shells: Cell models, Advances in Colloid and Interface Science, vol. 139 , pp.83-96, 2008.

[15] Vasin, S.I. and Filippov, A.N., Cell Models for Flows in Concentrated Media Composed of Rigid Impenetrable Cylinders Covered with a Porous Layer, Colloid Journal, Vol. 71, No. 2, pp. 141-155, 2009,.

[16] Verma, V.K. and Datta, S., Flow In Annular Channel Filled With A Porous Medium Of Variable Permeability, Journal of Porous Media, vol 15, pp. 891–899, 2012.

[17] Yadav, P. K., Tewari, A., Deo, S., Filippov, A. and Vasin, S., Hydrodynamic permeability of membranes built up by spherical particles covered by porous shells: Effect of stress jump condition, Acta Mechanica, vol. 215, pp. 193–209, 2010.

[18] Yadav, P.K., Tiwari,A., Deo, S., Yadav, M.K., Filippov, A., Vasin, S. and Sherysheva, E., Hydrodynamic Permeability of Biporous Membrane, Colloid Journal, 2013, Vol. 75, No. 4, pp. 473-482, 2013.