Motion of a porous spherical shell in a container: Effect of stress jump condition

Vineet Kumar Verma[1] **and Hariom Verma**[2]

Department of Mathematics & Astronomy
University of Lucknow, Lucknow, India

vinlkouniv@gmail.com[1] *& hariom2407@gmail.com*[2]

Abstract

In this paper we have studied slow motion of a porous spherical shell in a spherical container at the instant it passes through the center of the spherical container. Container is filled by a viscous incompressible fluid. Flow outside the porous spherical shell is governed by the Stokes equation and inside the spherical shell by the Brinkman equation. Analytical solution of the problem is obtained by using continuity of the velocity and normal stress and jump in tangential shear stress at the interface of fluid and porous shell as boundary condition. Exact expression of the relevant hydrodynamical quantities such as stream lines, velocity, pressure, wall correction factor and drag on surface of the sphere are obtained. The influence of various parameters such as stress jump coefficient, permeability parameter and separation parameter on the wall correction factor and drag force has been discussed and exhibited graphically.

Keywords: Porous media, Spherical shell, Brinkman model, Stokes flow, Viscous flow.

1 Introduction

Motion of solid or porous particles in a fluid of bounded region have numerous applications in chemical engineering, civil engineering, environmental engineering, industrial field and biomechanics etc. The hydrodynamical quantities such as drag on the particle, velocity and pressure are very much effected by shape of the surface by which particle is bounded. Few examples of such flow are flow through porous beds, floc settling processes, sedimentation of particulate suspensions, flow in reservoirs, flow of fine suspensions in blood etc. Some author have been investigated the motion of sphere in bounded region. Cunningham (1910) and Williams (1915) independently considered the motion of a solid sphere in a spherical container. Haberman et.al.(1958) investigated wall effects for rigid and fluid spheres in slow motion with moving fluid. Ramkissoon and Rahman (2003) studied the motion of a solid spherical particle in a spheroidal container at the instant it passes the center of a spheroidal container using the no-slip condition at the surfaces. They obtained drag on the particle and examined the wall effects. Keh and Chou (2004) have been investigated analytically the quasisteady translation and steady rotation of a spherically symmetric composite particle composed of a solid core and a surrounding porous shell located at the center of a spherical cavity filled with an incompressible Newtonian fluid. Bhattacharyya and Raja Sekhar (2004, 2005) have used stress jump boundary condition while discussing the Stokes flow of a viscous fluid inside a sphere with internal singularities, enclosed by a porous spherical shell and in discussing arbitrary viscous flow past a porous sphere with an impermeable core. They concluded that the fluid velocity at a porous-liquid interface varies with the stress jump coefficient and it plays an important role in describing the flow field associated with porous medium. Srinivasacharya (2005) investigated the creeping motion of a porous sphere at the instant it passes the center of a spherical container . They used Brinkman's model for the flow inside the porous sphere and the Stokes equation for the flow in the spherical container. They evaluated that, as the inner porous particle size increases, the drag coefficient decreases. The drag coefficient is decreasing as the permeability parameter is increasing. Deo and Gupta (2009) investigated symmetrical creeping flow of an incompressible viscous fluid past a swarm of porous approximately spheroidal particles with Kuwabara boundary condition. They evaluated drag force experienced by each porous oblate spheroid in a cell and discussed the dependence of the drag coefficient on permeability for a porous oblate spheroid in an unbounded medium and for a solid oblate spheroid in a cell on the solid volume fraction. D. Srinivasacharya and M. Krishna Prasad (2012) studied the Creeping motion of a porous approximate sphere with an impermeable core in a spherical container. They use the stress jump boundary condition for tangential stresses, continuity of the normal stresses and velocity components and disused the drag decreases with the increasing permeability along with increasing the stress jump coefficient and the correction factor depends on the stress jump coefficient and separation parameter. Srinivasacharya (2013) studied the motion of a porous approximate sphere at the instant it passes the center of an approximate spherical container with Ochoa-Tapia and Whitaker stress jump boundary condition. Saad and Faltas (2014) investigated flow of an incompressible axisymmetrical quasisteady motion of a porous sphere translating in a viscous fluid along the axis of a circular cylindrical pore using a combined analytical numerical technique. They use the stress jump boundary condition for the tangential stress at fluid porous interface. D. Srinivasacharya and M. Krishna Prasad (2015) investigated rotation of a porous approximate sphere in an approximate spherical container they use Brinkmans model for the flow inside the porous approximate sphere and the Stokes equation for the flow in an approximate spherical container. They obtained torque experienced by the porous approximate spherical particle in the presence of cavity and wall correction factor. Mukesh, Datta and Pandya(2015) investigated the creeping motion of a spherically symmetric fluid-permeable composite sphere composed by a uniform porous core and a uniformly surrounded porous shell located at the center of a spherical cavity filled with an incompressible Newtonian fluid. Verma and Dixit (2016) investigated creeping flow of an incompressible, viscous fluid past a heterogeneous porous spherical shell with concentric impermeable sphere. They obtained an analytical solution of the governing equations for the flow inside and outside the spherical shell. Verma and Dixit (2017) investigated Steady flow of a viscous incompressible fluid past a porous sphere of radially increasing permeability embedded in another homogeneous porous medium. They obtained an analytical solution of the problem and relevant quantities such as stream lines, velocity, pressure, and drag on surface of sphere. Jai Prakash and Raja Sekhar (2017) investigated viscous flow past a porous spherical particle composed of a rigid core inside. They discussed hydrodynamic drag and torque exerted on the surface of the composite sphere analytically which help us to find the translational and rotational mobility of the particle. Deo and Ansari(2018) investigated effect of magnetic field on an incompressible viscous fluid flow past and through a porous sphere embedded in another porous medium at low Reynolds number. They obtained drag on a porous sphere embedded in another porous medium.

In the present problem we have consider motion of a porous spherical shell in a spherical container filled with a viscous incompressible fluid. porous shell is moving along the center line of a spherical container. The resulting flow inside the porous region of shell is described by Brinkman equation and flow outside the shell and within spherical container is governed by Stokes equation. The boundary conditions used at clear fluid/porous interface are the continuity of velocity and normal stress and jump in tangential shear stress.

2 Mathematical formulation

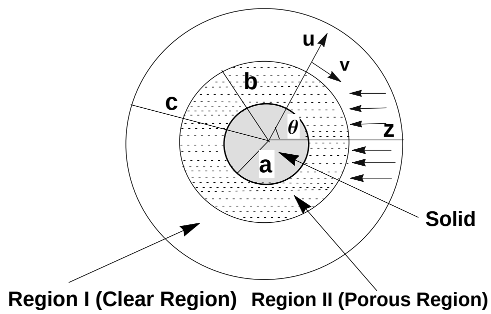

Fig. 1: Sketch of the problem

We consider the motion of porous spherical shell in spherical container as shown in Fig.(1). Shell consist of two regions, inner region is solid and outer region is porous. Radius of inner rigid sphere is $r^* = a$ and radius of shell is $r^* = b$. Shell is placed in a spherical container of radius c, which is filled with a viscous incompressible fluid. We assume that fluids have uniform velocity U far away from the spherical shell and flow is axis symmetric. Flow field is divided into two regions. Region I is the clear fluid region outside the porous spherical shell, i.e., $c \geqslant r^* \geqslant b$ and region II is the porous region within the spherical shell, i.e., $a \leqslant r^* \leqslant b$, $(b > a)$. The flow in region I is governed by the Stokes equation and equation of continuity that are given by

$$\nabla p_1^* = \mu \nabla^2 V_1^*; \qquad b \leqslant r^* \leqslant c, \tag{2.1}$$

$$\nabla . V_1^* = 0; \qquad b \leqslant r^* \leqslant c, \tag{2.2}$$

where V_1^*, p_1^* and μ are the velocity, pressure and viscosity of fluid in region I, respectively. The flow in region II is governed by the Brinkman equation (1947) and equation of continuity

$$\nabla p_2^* = -\frac{\mu}{k_o} V_2^* + \mu_e \nabla^2 V_2^*; \qquad a \leqslant r^* \leqslant b, \tag{2.3}$$

$$\nabla . V_2^* = 0; \qquad a \leqslant r^* \leqslant b, \tag{2.4}$$

where V_2^*, p_2^* and μ are the velocity, pressure and viscosity of fluid in region II, respectively. μ_e is effective viscosity in porous medium and k_o is permeability of the porous spherical shell. It is a debatable point whether the effective viscosity μ_e is the same as the viscosity of the fluid μ or not. According to Liu and Masliyah (2005), depending upon the type of porous media, μ_e may be either greater or smaller than μ but it is common practice to take $\mu_e = \mu$ for high porosity cases. Many investigators including Brinkman preferred to consider use $\mu_e = \mu$ for weak flow in porous media. Chikh et al. (1995) also assume that $\mu_e = \mu$. For present problem we assume that $\mu_e = \mu$, with this consideration Brinkman equation (2.3) becomes

$$\nabla p_2^* = -\frac{\mu}{k_o} V_2^* + \mu \nabla^2 V_2^*. \tag{2.5}$$

We choose spherical polar coordinate system $(r^*,\ \theta,\ \phi)$ with center of spherical shell as origin and the line $\theta = 0$ as axis of symmetry along the direction of uniform flow U as shown in Fig.(1). Due to axis symmetry of the problem we have $\partial/\partial\phi = 0$. It is convenient to use the following dimensionless variables

$$r = \frac{r^*}{a}, \qquad u_j = \frac{u_j^*}{U}, \qquad v_j = \frac{v_j^*}{U}, \qquad p_j = \frac{a p_j^*}{\mu U}, \tag{2.6}$$

where u_j^* and v_j^* are the radial and azimuthal component of velocity V_j^* in the increasing direction of r^* and θ, respectively. Here $j = 1$ corresponds to region I and $j = 2$ corresponds to region II. Using dimensionless variables, equation of continuity (2.2) and (2.4) in spherical polar coordinates can be written as

$$\frac{\partial}{\partial r}\left(r^2 u_j\right) + \frac{r}{\sin\theta}\frac{\partial}{\partial\theta}\left(v_j \sin\theta\right) = 0. \tag{2.7}$$

Equation (2.1) in dimensionless variables in spherical polar coordinates provides two component equations as follows

$$\frac{\partial p_1}{\partial r} = \frac{\partial^2 u_1}{\partial r^2} + \frac{2}{r}\frac{\partial u_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_1}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u_1}{\partial\theta} - \frac{2u_1}{r^2} - \frac{2}{r^2}\frac{\partial v_1}{\partial\theta} - \frac{2v_1\cot\theta}{r^2}; \qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{2.8}$$

$$\frac{1}{r}\frac{\partial p_1}{\partial\theta} = \frac{\partial^2 v_1}{\partial r^2} + \frac{2}{r}\frac{\partial v_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_1}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial v_1}{\partial\theta} + \frac{2}{r^2}\frac{\partial u_1}{\partial\theta} - \frac{v_1 \text{cosec}^2\theta}{r^2}; \qquad q \leqslant r \leqslant \frac{1}{\lambda}. \tag{2.9}$$

Here $q = b/a$ and $\lambda = a/c$. Brinkman equation (2.5) for porous region $(1 \leqslant r \leqslant q)$ in dimensionless variables in spherical polar coordinates provides two component equations as follows

$$-\frac{\partial p_2}{\partial r} = \frac{a^2 u_2}{k_o} - \frac{\partial^2 u_2}{\partial r^2} - \frac{2}{r}\frac{\partial u_2}{\partial r} - \frac{1}{r^2}\frac{\partial^2 u_2}{\partial\theta^2} - \frac{\cot\theta}{r^2}\frac{\partial u_2}{\partial\theta} + \frac{2u_2}{r^2} + \frac{2}{r^2}\frac{\partial v_2}{\partial\theta} + \frac{2v_2\cot\theta}{r^2}, \tag{2.10}$$

$$-\frac{1}{r}\frac{\partial p_2}{\partial\theta} = \frac{a^2 v_2}{k_o} - \frac{\partial^2 v_2}{\partial r^2} - \frac{2}{r}\frac{\partial v_2}{\partial r} - \frac{1}{r^2}\frac{\partial^2 v_2}{\partial\theta^2} - \frac{\cot\theta}{r^2}\frac{\partial v_2}{\partial\theta} - \frac{2}{r^2}\frac{\partial u_2}{\partial\theta} + \frac{v_2 \text{cosec}^2\theta}{r^2}. \tag{2.11}$$

Here $q = b/a$ is a thickness parameter of porous spherical shell. The matching conditions at the surface of spherical shell are continuity of tangential and normal velocity and stress.

$$\begin{aligned} u_1 &= u_2 && \text{at} \quad r = q, \\ v_1 &= v_2 && \text{at} \quad r = q, \\ \tau_{r\theta(1)} &= \tau_{r\theta(2)} - \frac{a\beta}{\sqrt{k_o}} v_2 && \text{at} \quad r = q, \\ \tau_{rr(1)} &= \tau_{rr(2)} && \text{at} \quad r = q, \end{aligned} \tag{2.12}$$

where $\tau_{r\theta(j)}$ and $\tau_{rr(j)}$ are dimensionless shear and normal stress, respectively and are given by

$$\tau_{r\theta(j)} = \frac{1}{r}\frac{\partial u_j}{\partial \theta} + \frac{\partial v_j}{\partial r} - \frac{v_j}{r}, \tag{2.13}$$

$$\tau_{rr(j)} = -p_j + 2\frac{\partial u_j}{\partial r}. \tag{2.14}$$

No-slip condition on the surface of impermeable sphere (at $r = 1$) provides us

$$u_2 = 0 \quad \text{and} \qquad v_2 = 0 \qquad \text{at} \ \ r = 1$$

On the surface of spherical container the condition of impenetrability is as fallows

$$u_1 \to \cos\theta \ \ \text{and} \ \ v_1 \to -\sin\theta \ \ \text{as} \ \ r \to \frac{1}{\lambda}. \tag{2.15}$$

3 Solution of the problem

We now introduce the stream function ψ such that

$$u_j = \frac{1}{r^2 \sin\theta}\frac{\partial \psi_j}{\partial \theta}, \qquad v_j = -\frac{1}{r\sin\theta}\frac{\partial \psi_j}{\partial r}; \quad i \ = \ 1, 2. \tag{3.1}$$

Where ψ_1 and ψ_2 are stream function corresponding to regions I and II, respectively. Eliminating pressure p_1 from Eq.(2.8) and Eq.(2.9), p_2 from Eq.(2.10) and Eq.(2.11) and then using Eq.(3.1), we get

$$E^4\psi_1 = 0; \qquad\qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{3.2}$$

$$E^4\psi_2 - \sigma^2 E^2 \psi_2 = 0; \qquad 1 \leqslant r \leqslant q. \tag{3.3}$$

Where $\sigma^2 = a^2/k_o$ is permeability variation parameter and E^2 is Stokes stream function operator, defined as

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\right), \tag{3.4}$$

$$E^4 = \frac{\partial^4}{\partial r^4} + \frac{6\sin\theta}{r^4}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta} \times \left[\frac{\partial^2}{\partial r^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^3}{\partial\theta\partial r^2} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta} \times \left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)\right]. \tag{3.5}$$

Outside porous spherical shell boundary condition (2.15) in terms of Stokes stream function can be expressed as

$$\psi_1 \to \frac{r^2}{2}\sin^2\theta \qquad \text{as} \qquad r \to \frac{1}{\lambda}. \tag{3.6}$$

Boundary condition (3.6) leads to consideration of solution of Eqs.(3.2) and (3.3) in the form

$$\psi_1(r,\theta) = f_1(r)\sin^2\theta; \qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{3.7}$$

$$\psi_2(r,\theta) = f_2(r)\sin^2\theta; \qquad 1 \leqslant r \leqslant q. \tag{3.8}$$

Substituting ψ_1 and ψ_2 from the above equations in Eqs.(3.2) and (3.3), respectively, we get the following ordinary differential equations:

$$r^4 f_1^{''''} - 4r^2 f_1^{''} + 8r f_1^{'} - 8f_1 = 0; \qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{3.9}$$

$$r^4 f_2^{''''} - 4r^2 f_2^{''} + 8r f_2^{'} - 8f_2 - \sigma^2 r^2 (r^2 f_2^{''} - 2r f_2^{'} - 2f_2) = 0; \qquad 1 \leqslant r \leqslant q. \tag{3.10}$$

Equation (3.9) is a homogeneous ordinary differential equation. We can simply find its solution as

$$f_1(r) = \frac{P_1}{r} + Q_1 r + R_1 r^2 + S_1 r^4. \tag{3.11}$$

Solution of the equation (3.10) is given as

$$f_2(r) = \frac{P_2}{r} + Q_2 r^2 + R_2\left(\cosh(r\sigma) - \frac{\sinh(r\sigma)}{r\sigma}\right) + S_2\left(\sinh(r\sigma) - \frac{\cosh(r\sigma)}{r\sigma}\right), \tag{3.12}$$

where $P_1,\ Q_1,\ R_1,\ S_1,\ P_2,\ Q_2,\ R_2$ and S_2 are constants of integration which can be determined by using boundary conditions. With the above expressions for $f_1(r)$ and $f_2(r)$, Eqs.(3.7) and (3.8) represent stream function in the region I and II, respectively. Boundary conditions (2.12) in terms of $f_1(r)$ and $f_2(r)$ can be written as

$$\begin{aligned} f_1(q) &= f_2(q), \\ f_1^{'}(q) &= f_2^{'}(q), \\ f_1^{''}(q) &= f_2^{''}(q) - \beta\sigma f_2^{'}(q) \\ \text{and} \quad f_1^{'''}(q) &= f_2^{'''}(q) - \sigma^2 f_2^{'}(q). \end{aligned} \tag{3.13}$$

No-slip condition (2.15) on impermeable sphere at $r = 1$ in terms of functions $f_1(r)$ and $f_2(r)$ can be expressed as

$$\begin{aligned} f_2(1) &= 0, \\ f_2'(1) &= 0. \end{aligned} \tag{3.14}$$

Condition of impenetrability (2.15) on the outer sphere at $r = 1/\lambda$ in terms of functions $f_1(r)$ and $f_2(r)$ can be expressed as

$$\begin{aligned} f_1\left(\frac{1}{\lambda}\right) &= \frac{1}{2\lambda^2}, \\ f_1'\left(\frac{1}{\lambda}\right) &= \frac{1}{\lambda}. \end{aligned} \tag{3.15}$$

Using the above boundary conditions (3.13), (3.14) and (3.15), we get the values of arbitrary constants $P_1,\ Q_1,\ R_1,\ S_1,\ P_2,\ Q_2,\ R_2$ and S_2. As the expressions for these constants are lengthy, we do not present them here. Thus the stream functions in region I (outside the spherical shell) and in region II (within the porous spherical shell) are given by

$$\psi_1(r,\theta) = \left(\frac{P_1}{r} + Q_1 r + R_1 r^2 + S_1 r^4\right)\sin^2\theta; \qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{3.16}$$

$$\begin{aligned} \psi_2(r,\theta) &= \left[\frac{P_2}{r} + Q_2 r^2 + R_2\left(\cosh(r\sigma) - \frac{\sinh(r\sigma)}{r\sigma}\right)\right. \\ &\left. + S_2\left(\sinh(r\sigma) - \frac{\cosh(r\sigma)}{r\sigma}\right)\right]\sin^2\theta; \qquad 1 \leqslant r \leqslant q. \end{aligned} \tag{3.17}$$

Using Eq.(3.1) the velocity of fluid in region I ($q \leqslant r \leqslant 1/\lambda$) is given by

$$u_1 = \frac{2\cos\theta}{r^2}\left(\frac{P_1}{r} + Q_1 r + R_1 r^2 + S_1 r^4\right), \tag{3.18}$$

$$v_1 = -\frac{\sin\theta}{r}\left(-\frac{P_1}{r^2} + Q_1 + 2R_1 r + 4S_1 r^3\right) \tag{3.19}$$

and velocity of fluid in the region II ($1 \leqslant r \leqslant q$) by

$$u_2 = \frac{2\cos\theta}{r^2}\left[\frac{P_2}{r} + Q_2 r^2 + R_2\left(\cosh(r\sigma) - \frac{\sinh(r\sigma)}{r\sigma}\right) + S_2\left(\sinh(r\sigma) - \frac{\cosh(r\sigma)}{r\sigma}\right)\right], \tag{3.20}$$

$$\begin{aligned} v_2 = -\frac{\sin\theta}{r}&\left[-\frac{P_2}{r^2} + 2Q_2 r + R_2\left(\frac{\sinh(r\sigma)}{r^2\sigma} + \sigma\sinh(r\sigma) - \frac{\cosh(r\sigma)}{r}\right)\right. \\ &\left. + S_2\left(\frac{\cosh(r\sigma)}{r^2\sigma} - \frac{\sinh(r\sigma)}{r} + \sigma\cosh(r\sigma)\right)\right], \end{aligned} \tag{3.21}$$

Substituting velocity from Eqs.(3.18) and (3.19) in Eq.(2.9) and integrating the resulting equation, we get the pressure p_1 outside the spherical shell as

$$p_1 = 2\cos\theta\left(\frac{Q_1}{r^2} + 10S_1 r\right); \qquad q \leqslant r \leqslant \frac{1}{\lambda}. \tag{3.22}$$

Similarly, the pressure p_2 inside the spherical shell ($1 \leqslant r \leqslant q$) is obtained by using Eqs.(3.20) and (3.21) in Eq.(2.11) as given below:

$$p_2 = \frac{\sigma^2\cos\theta\left(P_2 - 2Q_2 r^3\right)}{r^2}$$

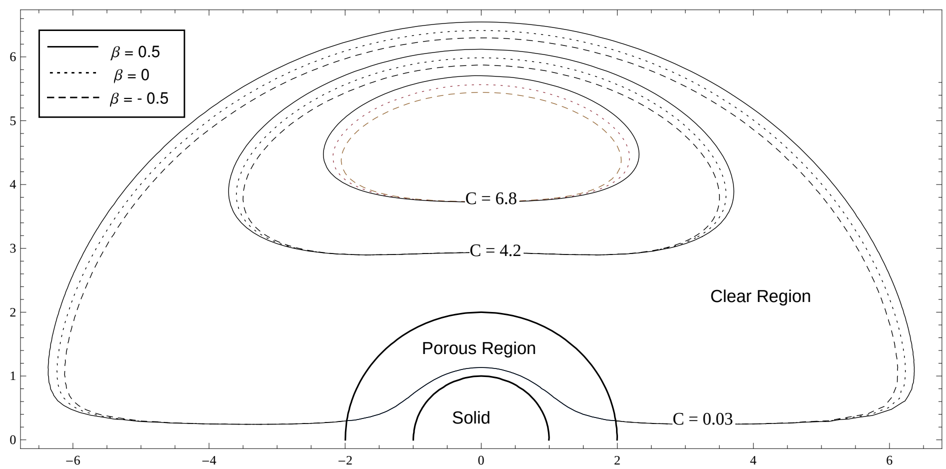

Fig. 2: Stream lines $\psi = C$ of the flow for $C = 0.03$, 4.2 and 6.8 when $q = 2$, permeability parameter σ is 0.5, separation parameter λ is 0.3 and for different values of stress jump coefficient β are -0.5, 0 and 0.5.

3.1 Drag Force on the Shell:

The non-dimensional drag force acting on the surface of porous spherical shell is given by

$$D = 2\pi\int_0^{\pi}\left(\tau_{rr(1)}\cos\theta - \tau_{r\theta(1)}\sin\theta\right)_{r=q}\sin\theta\, d\theta \tag{3.23}$$

Substituting $\tau_{rr(2)}$ and $\tau_{r\theta(2)}$ from Eqs.(2.13) and (2.14) in the above equation, we get

$$D = 24\pi\int_0^{\pi/2}\left[-\left(\frac{2P_1}{q^4} + \frac{Q_1}{q^2} + 2S_1 q\right)\cos^2\theta\sin\theta + \left(\frac{P_1}{q^4} + S_1 q\right)\sin^3\theta\right] d\theta. \tag{3.24}$$

After integration, we get

$$D = -\frac{8\pi Q_1}{q^2}, \tag{3.25}$$

In the limiting case when $q \to 1$, we get

$$\lim_{q\to 1} D = -\frac{24\pi\left(\lambda^4+\lambda^3+\lambda^2+\lambda+1\right)}{(\lambda-1)^3(4\lambda^2+7\lambda+4)}. \tag{3.26}$$

This is verify the result given by Ramkissoon(2003), which is the dimensionless drag on the solid sphere moving in the spherical container. Now

$$D_\infty = \lim_{\lambda\to 0} D,$$

$$\begin{aligned}
D_\infty \;=\; & -24\pi((3\beta\sigma + q(2q^3-3q+1)\sigma^4 + \beta q^2(2q^3+1)\sigma^5 + \beta(q(q(2q+3)-3)+1)\sigma^3 \\
+\; & 3q\sigma^2)\sinh(\sigma - q\sigma) - \sigma(-\beta q(2q^3-3q+1)\sigma^3 - q^2(2q^3+1)\sigma^4 - 3\beta(q-1)\sigma \\
+\; & 3(1-q)q\sigma^2)\cosh(\sigma - q\sigma))/q^2(-(12\beta\sigma + 4\beta(2q^4+q)\sigma^5 - 3\beta(q(-4q-4)+4)\sigma^3 \\
-\; & 12q\sigma^4 + 12q\sigma^2)\sinh(\sigma - q\sigma) - \sigma(4(2q^4+q)\sigma^4 + 4\beta(2q^3-3q+1)\sigma^3 - 3\beta(-4q \\
-\; & 4)\sigma + 3q(4q+4)\sigma^2)\cosh(\sigma - q\sigma) + 12\sigma(2\beta\sigma + 2q\sigma^2)),
\end{aligned} \tag{3.27}$$

Which is drag on the porous spherical shell moving in infinite fluid, when permeability of the porous shell is constant. Again

$$\lim_{q\to 1} D_\infty = 6\pi, \tag{3.28}$$

which is drag on the solid sphere moving with uniform velocity in an infinite fluid.

3.2 Wall Correction Factor:

The wall correction factor W_c is defined as the ratio of the actual drag force experienced by the particle in the enclosure and the drag force on a particle in an infinite expanse of fluid. Then

$$W_c = D/D_\infty. \tag{3.29}$$

Clearly for $\lambda = 0$, $W_c = 1$ and when $0 < \lambda \leq 0.5$ then $W_c > 1$.

4 Discussion

Stream lines are shown in Fig.(2) for stress jump coefficient $\beta = -0.5$, 0 and 0.5. These stream lines are sketched for permeability parameter $\sigma = 0.5$ and separation parameter $\lambda = 0.3$. We observe that the stress jump coefficient β has remarkable effect on the flow. There is deviation in the shape of the stream lines as they pass from clear region to porous region. The angle of deviation increases as stress jump coefficient β increases. Thus there is a decreases in fluid flow through porous region as stress jump coefficient β increases.

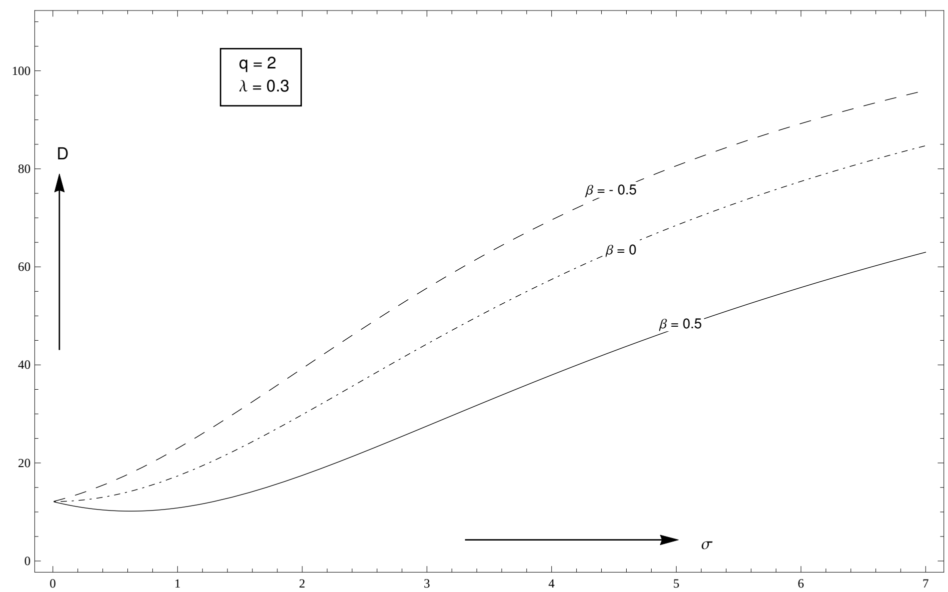

Fig. 3: Variation of Drag Force D with permeability parameter σ for different values stress jump coefficient β when separation parameter λ is 0.3 and $q = 2$.

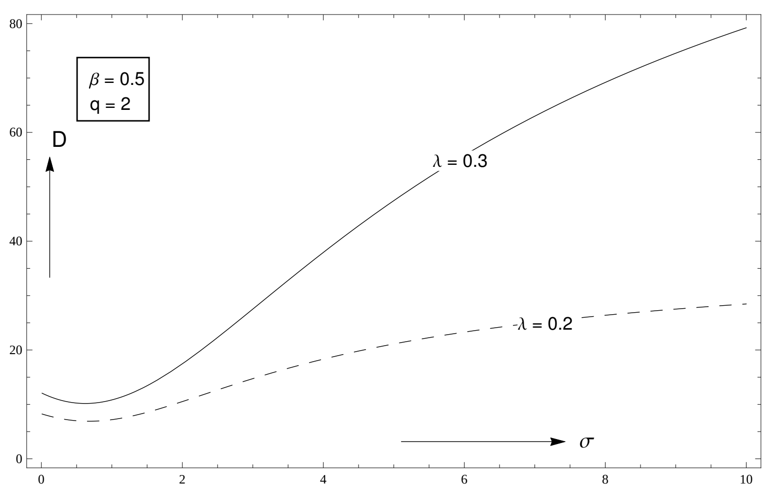

Fig. 4: Variation of Drag force D with σ for different values of Wall correction factor λ when stress jump coefficient $\beta = 0.5$ and $q = 2$.

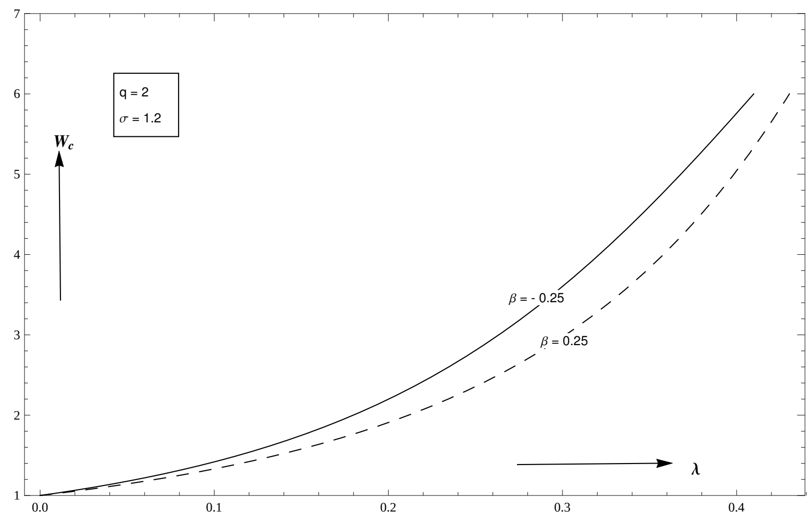

Fig. 5: Variation of Wall correction factor W_c with λ for different values of stress jump coefficient β when permeability parameter $\sigma = 1.2$ and $q = 2$.

The drag force D of the flow past porous spherical shell within the container is obtained by Eq.(3.25). Fig.(3) shows variation of drag force D on the porous sphere with permeability parameter σ for the different values of stress jump coefficient β when $q = 2$ and separation parameter λ is 0.3. We observe that the drag on the porous sphere increases with the increase in σ (i.e. decrease in permeability of sphere) and the drag force increases with the decreases in the value of stress jump coefficient β. Stress jump coefficient β has large effect on the Drag force D. Fig.(4) shows variation of drag force D on the porous sphere with permeability parameter σ for the different values of separation parameter λ(i.e. gap between porous sphere and container) when $q = 2$ and stress jump coefficient β is 0.5. We observe that the drag on the porous sphere increases with the increase in σ (i.e. decrease in permeability of sphere) and the drag force increases with the increasing in the value of separation parameter λ. Separation parameter λ has large effect on the drag force D.

The wall correction factor W_c of the flow past porous spherical shell within the container is obtained by $W_c = D/D_\infty$. Here we also find that for $\lambda = 0$, $W_c = 1$ and when $0 < \lambda \leq 0.5$ then $W_c > 1$. Fig.(5) shows Variation of wall correction factor W_c with separation parameter λ for different values of β. We observe that wall correction factor W_c increases with increases in separation parameter λ. The wall correction factor W_c increases with the decreases in the stress jump coefficient β. Figure shows that stress jump coefficient β has significant effect on wall correction factor.

5 Conclusion

The exact solution of the problem for the motion of a porous spherical shell in a spherical container is obtained. We use Brinkman equation in the porous sphere and Stokes equations in the clear fluid region outside the porous spherical shell and inside the container. At the fluid/porous interface continuity of velocity and normal stress and jump in the tangential shear stress have been used. Solution for the stream lines, fluid velocity, drag force on the sphere and wall correction factor are obtained. Obtained results are exhibited graphically. It has been found that stress jump coefficient β has significant effect on flow quantities such as stream line, drag force and wall correction factor. We observe that as the value of stress jump coefficient β increases the drag force D decreases and the wall correction factor W_c increases. Therefore, while studying flow through porous particles one has to take the variation of stress jump coefficient into consideration, which has a significant impact on the flow characteristics.

References

[1] Ansari, I. A. and Deo, S.; Magnetohydrodynamic viscous fluid flow past a porous sphere embedded in another porous medium, Special Topics and Reviews in Porous Media: An International Journal, vol.9(2), pp. 191-200, 2018.

[2] Awasthi, M., Pandya, N. and Datta, S.; Effect of Permeabilities on the Translational Motion of a Spherical Particle with Porous Core in a Concentric Spherical Cavity, European Journal of Advances in Engineering and Technology, vol. 2(9), pp. 52-58, 2015.

[3] Brinkman, H.C., A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles, Appl. Sci. Res. A, vol.1, pp. 27-34, 1947.

[4] Bhattacharyya, A. and Raja Sekhar, G.P.; Viscous flow past a porous sphere with an impermeable core: effect of stress jump condition, Chem. Eng. Sci. 59, pp. 4481 - 4492, 2004.

[5] Bhattacharyya, A. and Raja Sekhar, G.P.; Stokes flow inside a porous spherical shell: stress jump boundary condition, Z. Angew. Math. Phys. 56, pp. 475 - 496, 2005.

[6] Chikh, S., Boumedien, A., Bouhadef, K. and Lauriat, G., Analytical solution of non-Darcian forced convection in an annular duct partially filled with a porous medium, International Journal of Heat and Mass Transfer, vol. 38, pp. 1543 - 1551, 1995.

[7] Cunningham, E.; On the velocity of steady fall of spherical particles through fluid medium, Proc. R. Soc. Lond. Ser. vol., 83, pp. 357 - 365, 1910.

[8] Deo, S. and Gupta, B. R., Stokes flow past a swarm of porous approximately spheroidal particles with Kuwabara boundary condition, Acta Mech., vol. 203, pp. 241 - 254, 2009.

[9] Haberman, W.L. and Sayre,R.M., Wall effects for rigid and fluid spheres in slow motion with a moving liquid. David Taylor model, Basin Report No. 1143, Washington. D.C, 1958.

[10] Keh, H.J. and Chou, J., Creeping motion of a composite sphere in spherical cavity, Chem. Eng. Sci., vol. 59, pp. 407 - 415, 2004.

[11] Liu, S. and Masliyah, J. H.; Dispersion in porous media, Handbook of Porous Media, Taylor and Francis, pp. 81 - 140, 2005.

[12] Prakash, J. and Raja Sekhar, G. P., Slow motion of a porous spherical particle with a rigid core in a spherical fluid cavity, Meccanica, vol. 52(1), pp. 91-105, 2017.

[13] Prasad, M. K. and Srinivasacharya, D., Rotation of a porous approximate sphere in an approximate spherical container, Latin American Applied Research, vol. 45(2), pp. 107-112, 2015.

[14] Ramkissoon, H., Rahman, K., Wall effects on a spherical particle, Int.J. Eng. Sci., vol. 41, pp. 283 - 290, 2003.

[15] Saad, E.I. and Faltas, M.S., Slow Motion of a Porous Sphere Translating Along the Axis of a Circular Cylindrical Pore Subject to a Stress Jump Condition, Transp. Porous. Med., Vol. 102(1) , pp. 91 - 109, 2014.

[16] Srinivasacharya, D., Motion of a porous sphere in a spherical container, C.R. Mecanique, vol. 333, pp. 612 - 616, 2005.

[17] Srinivasacharya, D. and Krishna Prasad, M.; Creeping motion of a porous approximate sphere with an impermeable core in a spherical container,European Journal of Mechanics B/Fluids, Vol. 36, pp. 104 - 114, 2012.

[18] Srinivasacharya, D., Axi-symmetric motion of a porous approximate spherical container Arch. Mech., vol.65 (6), pp. 485 - 509, 2013.

[19] Verma, V. K. and Dixit, P. K., Flow past porous spherical shell with radial variation of permeability; Special Topics and Reviews in Porous Media: An International Journal, vol. 7(3), pp. 299-307, 2016.

[20] Verma, V. K. and Dixit, P. K., Flow past a porous sphere of radially varying permeability embedded in another porous medium, Special Topics and Reviews in Porous Media: An International Journal, vol. 8(3), pp. 231-243, 2017.

[21] Williams, W.E., On the motion of a sphere in a viscous fluid, Philos. Mag. 29, 526-550, 1915.

★ ★ ★