Motion of porous spherical shell of variable permeability in a spherical container: Effect of stress jump condition

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Abstract

In this paper slow motion of a porous spherical shell with variable permeability in a spherical container at the instant, it passes through the center of the spherical container is discussed. The container is filled by a viscous incompressible fluid. Flow outside the porous spherical shell is governed by the Stokes equation and inside the spherical shell by the Brinkman equation. We consider the permeability of the porous sphere varies quadratically with radial distance. Analytical solution of the problem is obtained by using the continuity of the velocity and normal stress and jump in tangential shear stress at the interface of fluid and porous sphere as the boundary condition. The exact expression of the relevant hydrodynamical quantities such as stream lines, velocity, pressure, wall correction factor and drag on the surface of the shell is obtained. The influence of various parameters such as stress jump coefficient, permeability parameter and separation parameter on wall correction factor and drag force has been discussed and exhibited graphically.

Keywords: Porous media, Spherical shell, Variable permeability flow, Brinkman model, Stokes flow, Viscous flow.

1 Introduction

The problems of the motion of a particle in a bounded region serves as a model of interaction in multi-particle systems. These type of problems are very important because it provides some information on wall effects. A survey of literature regarding the flow past and within porous bodies indicates that while abundant information is available for flows in an infinite expanse of fluid, very little information is available for flows in enclosures. The problems of flow through porous particles have been modeled by using the Stokes equation for the flow in clear region and Brinkman equation for the porous region. Initially Cunningham (1910) and Williams (1915) independently considered the motion of a solid sphere in a spherical container. Haberman et.al.(1958) investigated wall effects for rigid and fluid spheres in slow motion with moving fluid. Ramkissoon and Rahman (2003) studied the motion of a solid spherical particle in a spheroidal container at the instant it passes the center of a spheroidal container using the no-slip condition at the surfaces. They obtained drag on the particle and examined the wall effects. Raja Sekhar et.al.(2003) have used stress jump boundary condition while discussing two dimensional viscous flow in a granular material with a void of arbitrary shape. Bhattacharyya and Raja Sekhar (2004, 2005) have used stress jump boundary condition while discussing the Stokes flow of a viscous fluid inside a sphere with internal singularities, enclosed by a porous spherical shell and discussed arbitrary viscous flow past a porous sphere with an impermeable core. They concluded that the fluid velocity at a porous-liquid interface varies with the stress jump coefficient and it plays an important role in describing the flow field associated with porous medium. Srinivasacharya (2005) investigated the motion of a porous sphere in a spherical container. He evaluated the drag force and wall correction factor and found the drag force decreases as the permeability parameter increases and wall correction increases as the permeability parameter increases. Deo and Gupta (2009) investigated the symmetrical creeping flow of an incompressible viscous fluid past a swarm of porous approximately spheroidal particles with the Kuwabara boundary condition. They evaluated the drag force experienced by each porous oblate spheroid in a cell and discussed the dependence of the drag coefficient on permeability for a porous oblate spheroid in an unbounded medium and a solid oblate spheroid in a cell on the solid volume fraction. Saad (2010) investigated the flow of an incompressible axis-symmetrical quasi-steady translation and a steady rotation of a porous spheroid in a concentric spheroidal container. Srinivasacharya and Prasad (2012) studied the creeping motion of a porous approximate sphere with an impermeable core in a spherical container. They use the stress jump boundary condition for tangential stresses, continuity of the normal stresses and velocity components and disused the drag decreases with the increasing permeability along with increasing the stress jump coefficient and the correction factor depends on the stress jump coefficient and separation parameter. Srinivasacharya (2013) studied the motion of a porous approximate sphere at the instant it passes the center of an approximate spherical container with Ochoa-Tapia and Whitaker stress jump boundary condition. They obtained drag force and wall correction factor and discussed drag force decreases as the stress jump coefficient increasing and the wall corrector factor increases as stress jump coefficient increases. Zaytoon et.al.(2016) studied the flow through a variable permeability Brinkman porous layer with quadratic permeability function, underlain by a Darcy porous layer of variable linear permeability function. Verma and Gupta (2017) studied the effects of stress jump coefficient on flow past a porous spherical shell of varying permeability with a solid core. They used the Brinkman model. They observed that the velocity increases as the stress jump coefficient increases. Miari et.al.(2018) studied the unsteady rotational motion of a composite sphere, consisting of a solid core surrounded by a porous shell, in an incompressible viscous fluid. They used the stress jump boundary condition for tangential stresses.

In the present problem, we have considered the motion of a porous spherical shell of variable permeability in a spherical container filled with a viscous incompressible fluid. The porous sphere is moving along the center line of a spherical container. The resulting flow inside the porous sphere is described by the Brinkman equation and flow within the spherical container is governed by the Stokes equation. The boundary conditions used at the clear fluid/porous interface are the continuity of velocity and normal stress and jump in tangential shear stress.

2 Mathematical formulation

We consider the motion of a porous spherical shell in the spherical container as shown in Fig. (1). Shell consist of two regions, the inner region is solid and the outer region is porous. Radius of rigid sphere is $r^* = a$, radius of porous sphere is $r^* = b$. Shell is placed in a spherical container of radius c which is filled with a viscous incompressible fluid. We assume that fluids have uniform velocity U far away from the spherical shell and flow is axis-symmetric. The flow field is divided into two regions. Region I is the clear fluid region outside the porous spherical shell, i.e., $c \geqslant r^* \geqslant b$ and region II is the porous region within the spherical shell, i.e., $a \leqslant r^* \leqslant b$, $(b > a)$. The flow in the region I is governed by the Stokes equation and equation of continuity that are given by

$$\nabla p_1^* = \mu \nabla^2 V_1^*; \qquad b \leqslant r^* \leqslant c, \tag{2.1}$$

$$\nabla . V_1^* = 0; \qquad b \leqslant r^* \leqslant c, \tag{2.2}$$

where V_1^*, p_1^* and μ are the velocity, pressure and viscosity of fluid in region I, respectively. The flow in region II is governed by the Brinkman equation (1947) and equation of continuity

$$\nabla p_2^* = -\frac{\mu}{k(r^*)} V_2^* + \mu_e \nabla^2 V_2^*; \qquad a \leqslant r^* \leqslant b, \tag{2.3}$$

$$\nabla . V_2^* = 0; \qquad a \leqslant r^* \leqslant b, \tag{2.4}$$

where V_2^*, p_2^* and μ are the velocity, pressure and viscosity of the fluid in region II, respectively. μ_e is effective viscosity in a porous medium and k is permeability of the porous spherical shell. We assume the permeability of porous spherical shell of thickness $(b-a)$ increases with radial distance according to the law $k = k_o(r^*/a)^2$. Here k_o is the permeability on the inner surface of the porous spherical shell. It is a debatable point whether the effective viscosity μ_e is the same as the viscosity of the fluid μ or not. According to Liu and Masliyah (2005), depending upon the type of porous media, μ_e may be either greater or smaller than μ but it is common practice to take $\mu_e = \mu$ for high porosity cases. Many investigators including Brinkman preferred to consider use $\mu_e = \mu$ for weak flow in porous media. Chikh et al. (1995) also assume that $\mu_e = \mu$. For the present problem, we assume that $\mu_e = \mu$, with this consideration Brinkman equation (2.3) becomes

$$\nabla p_2^* = -\frac{\mu}{k(r^*)} V_2^* + \mu \nabla^2 V_2^*. \tag{2.5}$$

We choose spherical polar coordinate system $(r^*, \ \theta, \ \phi)$ with center of spherical shell as origin and the line $\theta = 0$ as axis of symmetry along the direction of uniform flow U as shown in Fig.1 Due to axis symmetry of the problem we have $\partial/\partial\phi = 0$. It is convenient to use the following dimensionless variables

$$r = \frac{r^*}{a}, \qquad u_j = \frac{u_j^*}{U}, \qquad v_j = \frac{v_j^*}{U}, \qquad p_j = \frac{a p_j^*}{\mu U}, \tag{2.6}$$

where u_j^* and v_j^* are the radial and azimuthal component of velocity V_j^* in the increasing direction of r^* and θ, respectively. Here $j = 1$ corresponds to region I and $j = 2$ corresponds to region II. Using dimensionless variables, equation of continuity (2.2) and (2.4) in spherical polar coordinates can be written as

$$\frac{\partial}{\partial r}\left(r^2 u_j\right) + \frac{r}{\sin\theta}\frac{\partial}{\partial \theta}\left(v_j \sin\theta\right) = 0. \tag{2.7}$$

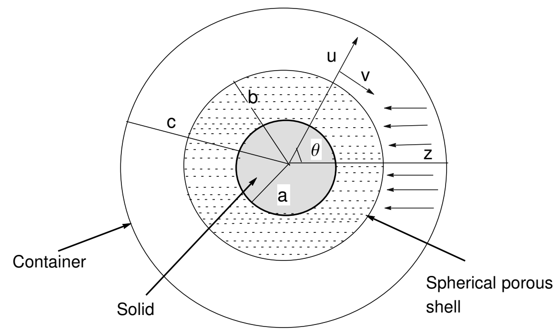

Fig. 1: Sketch of the problem

Equation (2.1) in dimensionless variables in spherical polar coordinates provides two component equations as follows

$$(2.8)\quad \frac{\partial p_1}{\partial r}=\frac{\partial^2 u_1}{\partial r^2}+\frac{2}{r}\frac{\partial u_1}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_1}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial u_1}{\partial \theta}-\frac{2u_1}{r^2}-\frac{2}{r^2}\frac{\partial v_1}{\partial \theta}-\frac{2v_1\cot\theta}{r^2};\qquad q\leqslant r\leqslant \frac{1}{\lambda},$$

$$(2.9)\quad \frac{1}{r}\frac{\partial p_1}{\partial \theta}=\frac{\partial^2 v_1}{\partial r^2}+\frac{2}{r}\frac{\partial v_1}{\partial r}+\frac{1}{r^2}\frac{\partial^2 v_1}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial v_1}{\partial \theta}+\frac{2}{r^2}\frac{\partial u_1}{\partial \theta}-\frac{v_1\mathrm{cosec}^2\theta}{r^2};\qquad q\leqslant r\leqslant \frac{1}{\lambda}.$$

Here $q=b/a$ and $\lambda=a/c$. Brinkman equation (2.5) for porous region ($1\leqslant r\leqslant q$) in dimensionless variables in spherical polar coordinates provides two component equations as follows

$$(2.10)\quad -\frac{\partial p_2}{\partial r}=\frac{a^2u_2}{k(r)}-\frac{\partial^2 u_2}{\partial r^2}-\frac{2}{r}\frac{\partial u_2}{\partial r}-\frac{1}{r^2}\frac{\partial^2 u_2}{\partial \theta^2}-\frac{\cot\theta}{r^2}\frac{\partial u_2}{\partial \theta}+\frac{2u_2}{r^2}+\frac{2}{r^2}\frac{\partial v_2}{\partial \theta}+\frac{2v_2\cot\theta}{r^2},$$

$$(2.11)\quad -\frac{1}{r}\frac{\partial p_2}{\partial \theta}=\frac{a^2v_2}{k(r)}-\frac{\partial^2 v_2}{\partial r^2}-\frac{2}{r}\frac{\partial v_2}{\partial r}-\frac{1}{r^2}\frac{\partial^2 v_2}{\partial \theta^2}-\frac{\cot\theta}{r^2}\frac{\partial v_2}{\partial \theta}-\frac{2}{r^2}\frac{\partial u_2}{\partial \theta}+\frac{v_2\mathrm{cosec}^2\theta}{r^2}.$$

Here $q=b/a$ is a thickness parameter of porous spherical shell. The matching conditions at the surface of spherical shell are continuity of tangential and normal velocity and stress.

$$\begin{aligned}u_1&=u_2 &&\text{at}\quad r=q,\\ v_1&=v_2 &&\text{at}\quad r=q,\\ \tau_{r\theta(1)}&=\tau_{r\theta(2)}-\frac{a\beta}{\sqrt{k}}v_2 &&\text{at}\quad r=q,\\ \tau_{rr(1)}&=\tau_{rr(2)} &&\text{at}\quad r=q,\end{aligned}\qquad (2.12)$$

where $\tau_{r\theta(j)}$ and $\tau_{rr(j)}$ are dimensionless shear and normal stress, respectively and are given by

$$(2.13)\quad \tau_{r\theta(j)}=\frac{1}{r}\frac{\partial u_j}{\partial \theta}+\frac{\partial v_j}{\partial r}-\frac{v_j}{r},$$

$$(2.14)\quad \tau_{rr(j)}=-p_j+2\frac{\partial u_j}{\partial r}.$$

No-slip condition on the surface of impermeable sphere (at $r = 1$) provides us

$$u_2 = 0 \quad \text{at} \;\; r = 1, \tag{2.15}$$
$$v_2 = 0 \quad \text{at} \;\; r = 1.$$

On the surface of spherical container the condition of impenetrability is as fallows

$$u_1 \to \cos\theta \;\; \text{and} \;\; v_1 \to -\sin\theta \;\; \text{as} \;\; r \to \frac{1}{\lambda}. \tag{2.16}$$

3 Solution of the problem

We now introduce the stream function ψ such that

$$u_j = \frac{1}{r^2 \sin\theta}\frac{\partial \psi_j}{\partial \theta}, \qquad v_j = -\frac{1}{r\sin\theta}\frac{\partial \psi_j}{\partial r}; \quad i \;=\; 1, 2. \tag{3.1}$$

Where ψ_1 and ψ_2 are stream function corresponding to regions I and II, respectively. Eliminating pressure p_1 from Eq.(2.8) and Eq.(2.9), p_2 from Eq.(2.10) and Eq.(2.11) and then using Eq.(3.1), we get

$$E^4\psi_1 = 0; \qquad\qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{3.2}$$

$$E^4\psi_2 - \frac{a^2}{k(r)}\left(E^2 - \frac{1}{k}\frac{\partial k}{\partial r}\frac{\partial}{\partial r}\right)\psi_2 = 0; \qquad 1 \leqslant r \leqslant q. \tag{3.3}$$

Where E^2 is Stokes stream function operator, defined as

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right), \tag{3.4}$$

$$\begin{aligned} E^4 \;\; = \;\; & \frac{\partial^4}{\partial r^4} + \frac{6\sin\theta}{r^4}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta} \times \left[\frac{\partial^2}{\partial r^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^3}{\partial\theta\partial r^2}\right. \\ & \left. + \frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta} \times \left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)\right]. \end{aligned} \tag{3.5}$$

We consider permeability of the porous region II as a function of radial distance and vary according to the law $k(r) = k_o r^2$, $(1 \leqslant r \leqslant q)$. For this $k(r)$ Eq.(3.3) becomes

$$E^4\psi_2 - \frac{\sigma^2}{r^2}\left(E^2 - \frac{2}{r}\frac{\partial}{\partial r}\right)\psi_2 = 0; \qquad 1 \leqslant r \leqslant q, \tag{3.6}$$

where $\sigma^2 = a^2/k_o$ is permeability variation parameter. Outside porous spherical shell boundary condition (2.16) in terms of Stokes stream function can be expressed as

$$\psi_1 \to \frac{r^2}{2}\sin^2\theta \qquad \text{as} \qquad r \to \frac{1}{\lambda}. \tag{3.7}$$

Boundary condition (3.7) leads to consideration of solution of Eqs.(3.2) and (3.6) in the form

$$\psi_1(r,\theta) = f_1(r)\sin^2\theta; \qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{3.8}$$

$$\psi_2(r,\theta) = f_2(r)\sin^2\theta; \qquad 1 \leqslant r \leqslant q. \tag{3.9}$$

Substituting ψ_1 and ψ_2 from the above equations in Eqs.(3.2) and (3.6), respectively, we get the following ordinary differential equations:

$$r^4 f_1^{''''} - 4r^2 f_1^{''} + 8r f_1^{'} - 8 f_1 = 0; \qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{3.10}$$

$$r^4 f_2^{''''} - 4r^2 f_2^{''} + 8r f_2^{'} - 8 f_2 - \sigma^2 (r^2 f_2^{''} - 2r f_2^{'} - 2 f_2) = 0; \qquad 1 \leqslant r \leqslant q. \tag{3.11}$$

Equation (3.10) is a homogeneous ordinary differential equation. We can simply find its solution as

$$f_1(r) = \frac{A_1}{r} + A_2 r + A_3 r^2 + A_4 r^4. \tag{3.12}$$

Equation (3.11) is also a homogeneous ordinary differential equation. Its solution is

$$\begin{aligned} f_2(r) \; = \; & C_1 r^{1/2(3-\sqrt{13+2\sigma^2-2\sqrt{36-4\sigma^2+\sigma^4}})} + C_2 r^{1/2(3+\sqrt{13+2\sigma^2-2\sqrt{36-4\sigma^2+\sigma^4}})} \\ & + C_3 r^{1/2(3-\sqrt{13+2\sigma^2+2\sqrt{36-4\sigma^2+\sigma^4}})} + C_4 r^{1/2(3+\sqrt{13+2\sigma^2+2\sqrt{36-4\sigma^2+\sigma^4}})}. \end{aligned} \tag{3.13}$$

Let
$\alpha_1 = \frac{1}{2}\left(3 - \sqrt{2\sigma^2 - 2\sqrt{\sigma^4 - 4\sigma^2 + 36} + 13}\right)$, $\alpha_2 = \frac{1}{2}\left(3 - \sqrt{2\sigma^2 + 2\sqrt{\sigma^4 - 4\sigma^2 + 36} + 13}\right)$,
$\beta_1 = \frac{1}{2}\left(3 + \sqrt{2\sigma^2 - 2\sqrt{\sigma^4 - 4\sigma^2 + 36} + 13}\right)$, $\beta_2 = \frac{1}{2}\left(3 + \sqrt{2\sigma^2 + 2\sqrt{\sigma^4 - 4\sigma^2 + 36} + 13}\right)$.
Then Eqn.(3.13) can be written as

$$f_2(r) = C_1 r^{\alpha_1} + C_2 r^{\beta_1} + C_3 r^{\alpha_2} + C_4 r^{\beta_2}$$

where $A_1,\ A_2,\ A_3,\ A_4,\ C_1,\ C_2,\ C_3$ and C_4 are constants of integration which can be determined by using boundary conditions. With the above expressions for $f_1(r)$ and $f_2(r)$, Eqs.(3.8) and (3.9) represent stream function in the region I and II, respectively. Boundary conditions (2.12) in terms of $f_1(r)$ and $f_2(r)$ can be written as

$$\begin{aligned} f_1(q) &= f_2(q), \\ f_1^{'}(q) &= f_2^{'}(q), \\ f_1^{''}(q) &= f_2^{''}(q) - \frac{\beta\sigma}{q} f_2^{'}(q) \\ \text{and} \qquad f_1^{'''}(q) &= f_2^{'''}(q) - \sigma^2 f_2^{'}(q). \end{aligned} \tag{3.14}$$

No-slip condition (2.15) on impermeable sphere at $r = 1$ in terms of functions $f_1(r)$ and $f_2(r)$ can be expressed as

$$\begin{aligned} f_2(1) &= 0, \\ f_2'(1) &= 0. \end{aligned} \tag{3.15}$$

Condition of impenetrability (2.16) on the surface of container at $r = 1/\lambda$ in terms of functions $f_1(r)$ and $f_2(r)$ can be expressed as

$$\begin{aligned} f_1\left(\frac{1}{\lambda}\right) &= \frac{1}{2\lambda^2}, \\ f_1'\left(\frac{1}{\lambda}\right) &= \frac{1}{\lambda}. \end{aligned} \tag{3.16}$$

Using the above boundary conditions (3.14), (3.15) and (3.16), we get the values of arbitrary constants $A_1,\ A_2,\ A_3,\ A_4,\ C_1,\ C_2,\ C_3$ and C_4. As the expressions for these constants are lengthy, we do not present them here. Thus the stream functions in region I (outside the spherical shell) and in region II (within the porous spherical shell) are given by

$$\psi_1(r,\theta) = \left(\frac{A_1}{r} + A_2 r + A_3 r^2 + A_4 r^4\right)\sin^2\theta; \qquad q \leqslant r \leqslant \frac{1}{\lambda}, \tag{3.17}$$

$$\psi_2(r,\theta) = \left(C_1 r^{\alpha_1} + C_2 r^{\beta_1} + C_3 r^{\alpha_2} + C_4 r^{\beta_2}\right)\sin^2\theta; \qquad 1 \leqslant r \leqslant q. \tag{3.18}$$

Using Eq.(3.1) the velocity of fluid in region I ($q \leqslant r \leqslant 1/\lambda$) is given by

$$u_1 = \frac{2\cos\theta}{r^2}\left(\frac{A_1}{r} + A_2 r + A_3 r^2 + A_4 r^4\right), \tag{3.19}$$

$$v_1 = -\frac{\sin\theta}{r}\left(-\frac{A_1}{r^2} + A_2 + 2A_3 r + 4A_4 r^3\right) \tag{3.20}$$

and velocity of fluid in the region II ($1 \leqslant r \leqslant q$) by

$$u_2 = \frac{2\cos\theta}{r^2}\left(C_1 r^{\alpha_1} + C_2 r^{\beta_1} + C_3 r^{\alpha_2} + C_4 r^{\beta_2}\right), \tag{3.21}$$

$$v_2 = -\frac{\sin\theta}{r}\left(C_1\alpha_1 r^{\alpha_1 - 1} + C_2\beta_1 r^{\beta_1 - 1} + C_3\alpha_2 r^{\alpha_2 - 1} + C_4\beta_2 r^{\beta_2 - 1}\right) \tag{3.22}$$

Substituting velocity from Eqs.(3.19) and (3.20) in Eq.(2.9) and integrating the resulting equation, we get the pressure p_1 outside the spherical shell as

$$p_1 = 2\cos\theta\left(\frac{A_2}{r^2} + 10A_4 r\right); \qquad q \leqslant r \leqslant \frac{1}{\lambda}. \tag{3.23}$$

Similarly, the pressure p_2 inside the spherical shell $(1 \leqslant r \leqslant q)$ is obtained by using Eqs.(3.21) and (3.22) in Eq.(2.11) as given below:

$$p_2 = \cos\theta \Big(-C_1 \left(\alpha_1\sigma^2 + \alpha_1^3 - 3\alpha_1^2 + 4\right) r^{\alpha_1 - 3} - C_2 \left(\beta_1\sigma^2 + \beta_1^3 - 3\beta_1^2 + 4\right) r^{\beta_1 - 3}$$

$$-C_3 \left(\alpha_2\sigma^2 + \alpha_2^3 - 3\alpha_2^2 + 4\right) r^{\alpha_2 - 3} - C_4 \left(\beta_2\sigma^2 + \beta_2^3 - 3\beta_2^2 + 4\right) r^{\beta_2 - 3}\Big). \tag{3.24}$$

3.1 Drag Force on the Shell:

The non-dimensional drag force acting on the surface of porous spherical shell is given by

$$D = 2\pi \int_0^{\pi} \left(\tau_{rr(1)} \cos\theta - \tau_{r\theta(1)} \sin\theta\right)_{r=q} \sin\theta \, d\theta \tag{3.25}$$

Substituting $\tau_{rr(2)}$ and $\tau_{r\theta(2)}$ from Eqs.(2.13) and (2.14) in the above equation, we get

$$D = 24\pi \int_0^{\pi/2} \left[-\left(\frac{2A_1}{q^4} + \frac{A_2}{q^2} + 2A_4 q\right) \cos^2\theta \sin\theta + \left(\frac{A_1}{q^4} + A_4 q\right) \sin^3\theta \right] d\theta. \tag{3.26}$$

After integration, we get

$$D = -\frac{8\pi A_2}{q^2}, \tag{3.27}$$

In the limiting case when $q \to 1$, we get

$$\lim_{q\to 1} D = -\frac{24\pi\left(\lambda^4 + \lambda^3 + \lambda^2 + \lambda + 1\right)}{(\lambda - 1)^3(4\lambda^2 + 7\lambda + 4)}. \tag{3.28}$$

This is verify the result given by Ramkissoon(2003), which is the dimensionless drag on the solid sphere moving in the spherical container. Now

$$D_\infty = \lim_{\lambda\to 0} D = 6\pi\nabla, \tag{3.29}$$

where ∇ is given in the appendix. Which is drag on the porous spherical shell moving in infinite fluid, when permeability of the porous shell varying according to rule $k = k_0 r^2$, again

$$\lim_{q\to 1} D_\infty = 6\pi, \tag{3.30}$$

which is drag on the solid sphere moving with uniform velocity in an infinite fluid.

3.2 Wall Correction Factor:

The wall correction factor W_c is defined as the ratio of the actual drag force experienced by the particle in the enclosure and the drag force on a particle in an infinite expanse of fluid. Then

$$W_c = D/D_\infty. \tag{3.31}$$

Clearly for $\lambda = 0$, $W_c = 1$ and when $0 < \lambda \leq 0.5$ then $W_c > 1$.

4 Discussion

Fig.(2) represents the variation of drag force D with σ on the surface of porous spherical shell within the container at the instant it passing through the centre of container. This drag force D is sketch for stress jump coefficient $\beta = 0.5$, thickness parameter $q = 2$ when separation parameter $\lambda = 0.2$ and $\lambda = 0.3$.

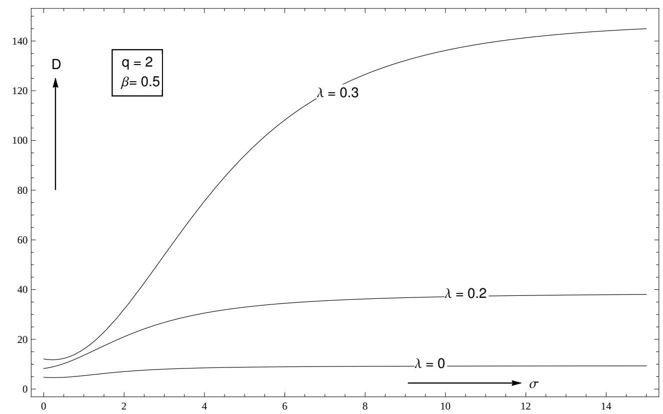

Fig. 2: Variation of Drag Force D with permeability parameter σ for different values of separation parameter λ when stress jump coefficient β is 0.5 and $q = 2$.

Figure reveal that drag force D increases with permeability parameter σ. We also observe that drag on the shell is large when separation parameter λ is large. Drag on the shell is minimum for $\lambda = 0$, i.e when shell is in unbounded region.

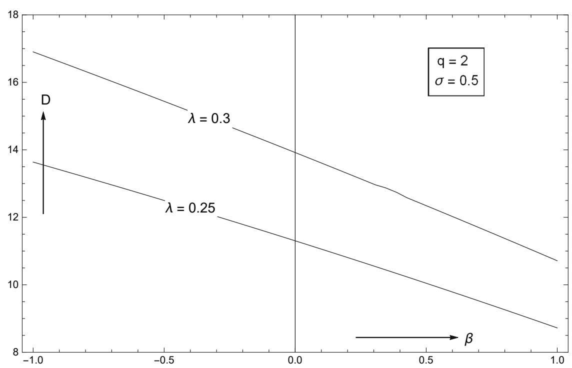

Fig. 3: Variation of Drag Force D with stress jump coefficient β for different values of separation parameter λ when permeability parameter σ is 0.5 and $q = 2$.

Fig.(3) shows the effect of stress jump coefficient β on the drag force D on the shell. In fig.(3) variation of drag D with β has been sketched for $q = 2$ and $\sigma = 0.5$ when $\lambda = 0.25$ and $\lambda = 0.3$. We observe that drag on the shell decreases as β increases$(-1 \leq \beta \leq 1)$.

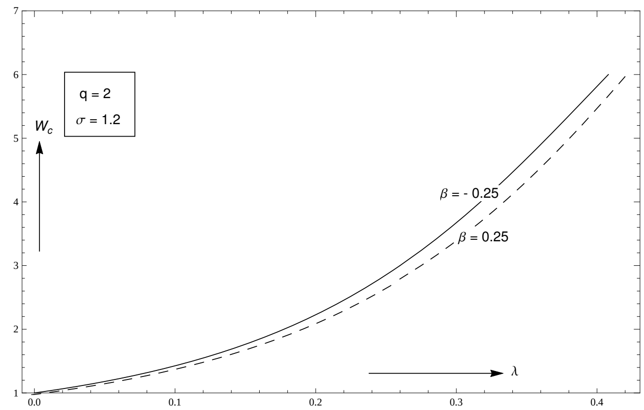

Fig. 4: Variation of Wall correction factor W_c with λ for different values of stress jump coefficient β when permeability parameter $\sigma = 1.2$ and $q = 2$.

When $\lambda = 0.25$ for $\beta = -1$, D is 13 and for $\beta = +1$, D is 8. This shows that stress jump coefficient have very strong effect on the drag force. Stress jump coefficient dependent on the structure of the porous medium. Thus structure of porous medium have large effect on the drag force of the porous shell. Therefore error in the value of β may cause large error in the measurement of drag force.Variation of wall correction factor W_c with separation parameter λ for different values of stress jump coefficient β when $q = 2$ and $\sigma = 1.2$ is shown in Fig.(4).

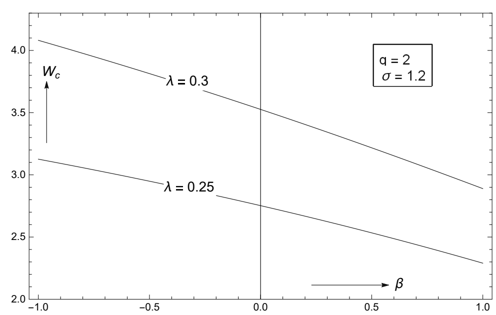

Fig. 5: Variation of Wall correction factor W_c with stress jump coefficient β for different values of separation parameter λ when permeability parameter $\sigma = 1.2$ and $q = 2$.

We observe that wall correction factor W_c increases with increases in separation parameter λ (i.e. decrease in gap between porous sphere and container) and wall correction factor increases with the decreases in the value of stress jump coefficient β. Variation of wall correction factor W_c with stress jump coefficient β for the different values of separation parameter λ when $q = 2$ and $\sigma = 1.2$ is shown in Fig.(5). We observe that the wall correction factor W_c decreases as the stress jump coefficient β increase from -1 to 1. This shows that β have strong effect on the wall correction factor. We also observe that the wall correction factor W_c increases with the increase in the value of separation parameter

λ(i.e. decrease in gap between porous sphere and container) and value of wall correction factor W_c is greater then one($W_c > 1$) when $0 < \lambda \leq 0.5$ and $W_c = 1$ for $\lambda = 0$.

5 Conclusion

In this work, an exact solution of the problem for the motion of a porous spherical shell with variable permeability in a spherical container is obtained. We use the Brinkman equation in the porous sphere and Stokes equations in the clear fluid region. At the fluid/porous interface continuity of velocity and normal stress and jump in the tangential shear stress have been used. An expression for the drag force on the shell and wall correction factor are obtained and exhibited graphically. It has been found that the stress jump coefficient β and separation parameter λ has a significant effect on drag force and wall correction factor. We have find that as stress jump coefficient β increases drag on the porous shell and wall correction factor decreases. Also as separation parameter λ increases D and W_c increases. Therefore β and λ are very strong effect on the drag and wall correction factor.

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Appendix

$$
\begin{aligned}
\triangledown \;=\;& \alpha_1^2(\alpha_2^3(\beta_1-\beta_2)q^{\alpha_2}+\beta_1^3\beta_2 q^{\beta_1}+2(q^{\alpha_2}-q^{\beta_1})\beta_2+\alpha_2(-\beta_1^3 q^{\beta_1}+\beta_2^3 q^{\beta_2}+2(q^{\beta_1}-q^{\beta_2})\\
-\;& (q^{\alpha_2}-q^{\beta_1})(q^2\sigma^2+3\mu\sigma+3)\beta_1+(q^{\alpha_2}-q^{\beta_2})(q^2\sigma^2+3\mu\sigma+3)\beta_2)+\beta_1(-2q^{\alpha_2}-\beta_2^3\\
\times\;& q^{\beta_2}+2q^{\beta_2}-(q^{\beta_1}-q^{\beta_2})(q^2\sigma^2+3\mu\sigma+3)\beta_2))q^{\alpha_1}+\alpha_1^3(-\beta_1^2\beta_2 q^{\beta_1}-q^{\alpha_2}\alpha_2^2(\beta_1-\beta_2)\\
-\;& 2(q^{\alpha_2}-q^{\beta_1})\beta_2+\alpha_2(\beta_1^2 q^{\beta_1}-2q^{\beta_1}+2q^{\beta_2}+(q^{\alpha_2}-q^{\beta_1})(\mu\sigma+1)\beta_1+(q^{\beta_2}(\mu\sigma-\beta_2\\
+\;& 1)-q^{\alpha_2}(\mu\sigma+1))\beta_2)+\beta_1(2(q^{\alpha_2}-q^{\beta_2})+\beta_2((\mu\sigma+1)q^{\beta_1}+(-\mu\sigma+\beta_2-1)q^{\beta_2})))q^{\alpha_1}\\
+\;& \alpha_2^3(\beta_1^2\beta_2 q^{\beta_1}+2(q^{\alpha_1}-q^{\beta_1})\beta_2+\beta_1(-2q^{\alpha_1}+2q^{\beta_2}+(q^{\beta_2}(\mu\sigma-\beta_2+1)-q^{\beta_1}(\mu\sigma+1))\\
\times\;& \beta_2))q^{\alpha_2}+\alpha_2^2(-\beta_1^3\beta_2 q^{\beta_1}-2(q^{\alpha_1}-q^{\beta_1})\beta_2+\beta_1(\beta_2^3 q^{\beta_2}+2(q^{\alpha_1}-q^{\beta_2})+(q^{\beta_1}-q^{\beta_2})\\
\times\;& (q^2\sigma^2+3\mu\sigma+3)\beta_2))q^{\alpha_2}+2(q^{\alpha_1}-q^{\alpha_2})\beta_1\beta_2(-(\beta_1-1)\beta_1 q^{\beta_1}+(\beta_2-1)\beta_2 q^{\beta_2}\\
+\;& (q^{\beta_1}-q^{\beta_2})(q^2\sigma^2+2\mu\sigma+2))+\alpha_1(\alpha_2^2(\beta_1^3 q^{\beta_1}-2q^{\beta_1}-\beta_2^3 q^{\beta_2}+2q^{\beta_2}+(q^{\alpha_1}-q^{\beta_1})\\
\times\;& (q^2\sigma^2+3\mu\sigma+3)\beta_1-(q^{\alpha_1}-q^{\beta_2})(q^2\sigma^2+3\mu\sigma+3)\beta_2)q^{\alpha_2}+\alpha_2^3(-\beta_1^2 q^{\beta_1}+2(q^{\beta_1}-q^{\beta_2})\\
-\;& (q^{\alpha_1}-q^{\beta_1})(\mu\sigma+1)\beta_1+\beta_2((\mu\sigma+1)q^{\alpha_1}+(-\mu\sigma+\beta_2-1)q^{\beta_2}))q^{\alpha_2}+\beta_1^2(\beta_2^3 q^{\beta_2}\\
+\;& 2(q^{\alpha_2}-q^{\beta_2})+(q^{\alpha_1}-q^{\beta_2})(q^2\sigma^2+3\mu\sigma+3)\beta_2)q^{\beta_1}-\beta_1^3(2(q^{\alpha_2}-q^{\beta_2})+\beta_2((\mu\sigma+1)q^{\alpha_1}\\
+\;& (-\mu\sigma+\beta_2-1)q^{\beta_2}))q^{\beta_1}+2(q^{\alpha_2}-q^{\beta_1})\beta_2((\beta_2-1)\beta_2 q^{\beta_2}+(q^{\alpha_1}-q^{\beta_2})(q^2\sigma^2+2\mu\sigma
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 2)) + (q^{\alpha_1} - q^{\beta_1})\beta_1(q^{\beta_2}\beta_2^2(-\sigma(\sigma q^2 + 3\mu) + (\mu\sigma + 1)\beta_2 - 3) - 2(q^{\alpha_2} - q^{\beta_2})(q^2\sigma^2 \\
&+ \quad 2\mu\sigma + 2)) + (q^{\alpha_1} - q^{\alpha_2})\alpha_2(\beta_1^2(-\sigma(\sigma q^2 + 3\mu) + (\mu\sigma + 1)\beta_1 - 3)q^{\beta_1} - \beta_2^2(-\sigma(\sigma q^2 \\
&+ \quad 3\mu) + (\mu\sigma + 1)\beta_2 - 3)q^{\beta_2} + 2(q^{\beta_1} - q^{\beta_2})(q^2\sigma^2 + 2\mu\sigma + 2))) + \alpha_2(\beta_1^2(-2q^{\alpha_1} - \beta_2^3 q^{\beta_2} \\
&+ \quad 2q^{\beta_2} - (q^{\alpha_2} - q^{\beta_2})(q^2\sigma^2 + 3\mu\sigma + 3)\beta_2)q^{\beta_1} + \beta_1^3(2(q^{\alpha_1} - q^{\beta_2}) + \beta_2((\mu\sigma + 1)q^{\alpha_2} \\
&+ \quad (-\mu\sigma + \beta_2 - 1)q^{\beta_2}))q^{\beta_1} + 2(q^{\alpha_1} - q^{\beta_1})\beta_2((q^{\beta_2} - q^{\alpha_2})(q^2\sigma^2 + 2\mu\sigma + 2) - q^{\beta_2}(\beta_2 - 1) \\
&\times \quad \beta_2) + (q^{\alpha_2} - q^{\beta_1})\beta_1(2(q^{\alpha_1} - q^{\beta_2})(q^2\sigma^2 + 2\mu\sigma + 2) - q^{\beta_2}\beta_2^2(-\sigma(\sigma q^2 + 3\mu) \\
&+ \quad (\mu\sigma + 1)\beta_2 - 3)))/(q(\alpha_1^2(\alpha_2^3(\beta_1 - \beta_2)q^{\alpha_2} + \beta_1\beta_2(\beta_1^2 q^{\beta_1} - \beta_2^2 q^{\beta_2} + (q^{\beta_2} - q^{\beta_1})(q^2\sigma^2 \\
&+ \quad 3\mu\sigma + 1)) + \alpha_2(-\beta_1^3 q^{\beta_1} + (q^{\beta_1} - q^{\alpha_2})(q^2\sigma^2 + 3\mu\sigma + 1)\beta_1 + \beta_2(\beta_2^2 q^{\beta_2} + (q^{\alpha_2} - q^{\beta_2})(q^2\sigma^2 \\
&+ \quad 3\mu\sigma + 1))))q^{\alpha_1} + \alpha_1^3(-\beta_1^2\beta_2 q^{\beta_1} - q^{\alpha_2}\alpha_2^2(\beta_1 - \beta_2) - (q^{\alpha_2} - q^{\beta_1})\beta_2 + \beta_1(q^{\alpha_2} \\
&+ \quad \mu\sigma\beta_2 q^{\beta_1} + (\beta_2^2 - \mu\sigma\beta_2 - 1)q^{\beta_2})\alpha_2(\beta_1^2 q^{\beta_1} - q^{\beta_1} + q^{\beta_2} + (q^{\alpha_2} - q^{\beta_1})\mu\sigma\beta_1 - \beta_2(\mu\sigma q^{\alpha_2} \\
&+ \quad (\beta_2 - \mu\sigma)q^{\beta_2})))q^{\alpha_1} + \alpha_2^2\beta_1\beta_2(-\beta_1^2 q^{\beta_1} + \beta_2^2 q^{\beta_2} + (q^{\beta_1} - q^{\beta_2})(q^2\sigma^2 + 3\mu\sigma + 1))q^{\alpha_2} \\
&+ \quad \alpha_2^3(\beta_1^2\beta_2 q^{\beta_1} + (q^{\alpha_1} - q^{\beta_1})\beta_2\beta_1(-q^{\alpha_1} - \mu\sigma\beta_2 q^{\beta_1} + (-\beta_2^2 + \mu\sigma\beta_2 + 1)q^{\beta_2}))q^{\alpha_2} + (q^{\alpha_1} \\
&- \quad q^{\alpha_2})\beta_1\beta_2(-\beta_1^2 q^{\beta_1} + \beta_2^2 q^{\beta_2}(q^{\beta_1} - q^{\beta_2})(q^2\sigma^2 + 3\mu\sigma + 1)) + \alpha_2(\beta_1^2\beta_2((q^{\beta_2} - q^{\alpha_2})(q^2\sigma^2 \\
&+ \quad 3\mu\sigma + 1) - q^{\beta_2}\beta_2^2)q^{\beta_1}\beta_1^3(q^{\alpha_1} - q^{\beta_2} + \beta_2(\mu\sigma q^{\alpha_2} + (\beta_2 - \mu\sigma)q^{\beta_2}))q^{\beta_1} + (q^{\alpha_1} - q^{\beta_1}) \\
&\times \quad \beta_2((q^{\beta_2} - q^{\alpha_2})(q^2\sigma^2 + 3\mu\sigma + 1)q^{\beta_2}\beta_2^2) + (q^{\alpha_2} - q^{\beta_1})\beta_1((q^{\alpha_1} - q^{\beta_2})(q^2\sigma^2 + 3\mu\sigma + 1) \\
&- \quad q^{\beta_2}\beta_2^2(-\sigma(\sigma q^2 + 3\mu) + \mu\sigma\beta_2 - 1)))\alpha_1(\alpha_2^2(\beta_1^3 q^{\beta_1} + (q^{\alpha_1} - q^{\beta_1})(q^2\sigma^2 + 3\mu\sigma + 1)\beta_1 \\
&- \quad \beta_2(\beta_2^2 q^{\beta_2} + (q^{\alpha_1} - q^{\beta_2})(q^2\sigma^2 + 3\mu\sigma + 1)))q^{\alpha_2} + \alpha_2^3(-\beta_1^2 q^{\beta_1} + q^{\beta_1} - q^{\beta_2} - (q^{\alpha_1} - q^{\beta_1}) \\
&\times \quad \mu\sigma\beta_1 + \beta_2(\mu\sigma q^{\alpha_1} + (\beta_2 - \mu\sigma)q^{\beta_2}))q^{\alpha_2} + \beta_1^2\beta_2(\beta_2^2 q^{\beta_2} + (q^{\alpha_1} - q^{\beta_2})(q^2\sigma^2 + 3\mu\sigma + 1)) \\
&\times \quad q^{\beta_1} - \beta_1^3(q^{\alpha_2} - q^{\beta_2} + \beta_2(\mu\sigma q^{\alpha_1} + (\beta_2 - \mu\sigma)q^{\beta_2}))q^{\beta_1} + (q^{\alpha_2} - q^{\beta_1})\beta_2(\beta_2^2 q^{\beta_2} + (q^{\alpha_1} \\
&- \quad q^{\beta_2})(q^2\sigma^2 + 3\mu\sigma + 1)) + (q^{\alpha_1} - q^{\beta_1})\beta_1(\beta_2^2(-\sigma(\sigma q^2 + 3\mu) + \mu\sigma\beta_2 - 1)q^{\beta_2} + (q^{\beta_2} \\
&- \quad q^{\alpha_2})(q^2\sigma^2 + 3\mu\sigma + 1)) + (q^{\alpha_1} - q^{\alpha_2})\alpha_2((q^2\sigma^2 + 3\mu\sigma + \beta_1^2(-\sigma(\sigma q^2 + 3\mu) + \mu\sigma\beta_1 \\
&- \quad 1) + 1)q^{\beta_1} + ((q^2\sigma^2 + 3\mu\sigma - \mu\beta_2\sigma + 1)\beta_2^2 - \sigma(\sigma q^2 + 3\mu) - 1)q^{\beta_2})))).
\end{aligned}
$$