Growth analysis of $(p,q)^{th}$ relative Gol'dberg order of composite entire functions

Dharmendra Kumar Gautam and Anupma Rastogi

Department of Mathematics and Astronomy
University of Lucknow, Lucknow, INDIA-226007

Abstract

In this paper we discuss about the growth properties and some results depending on $(p,q)^{th}$ relative Gol'dberg order of composite entire functions.

Keywords: Composite entire functions,$(p,q)^{th}$ relative Gol'dberg order.

1 Introduction

Let $\mathbb{C}^n$ and $\mathbb{R}^n$ respectively denote the complex and real n-space. Also let us indicate the point $(z_1, z_2, ..., z_n), (m_1, m_2, ..., m_n)$ of $\mathbb{C}^n$ or $\mathbb{I}^n$ by their corresponding unsuffixed symbols z, m respectively, where $\mathbb{I}$ denotes the set of non-negative integers. The modulus of z, denoted by $|z|$, is defined as $|z| = \left(|z_1|^2 + ... + |z_n|^2\right)^{\frac{1}{2}}$. If the coordinates of the vector m are non-negative integers, then z^m will denote $z_1^{m_1}...z_n^{m_n}$ and $||m|| = m_1 + ... + m_n$.

If $D \subset \mathbb{C}^n$ ($\mathbb{C}^n$ denote the n-dimensional complex space) be an arbitrary bounded complex n-circular domain with center at the origin of coordinates then for any entire function $f(z)$ of n-complex variables and $R > 0$, $M_{f,D}(R)$ may be defined as $M_{f,D} = \sup_{z \epsilon D_R} |f(z)|$,where a point $z \epsilon D_R$ if and only if $\frac{z}{R} \epsilon D$. If $f(z)$ is non-constant, then $M_{f,D}(R)$ is strictly increasing and its inverse $M_{f,D}^{-1} : (|f(0)|, \infty) \to (0, \infty)$ exists such that $\lim_{R\to\infty} M_{f,D}^{-1}(R) = \infty$.

Definition 1.1. *([1],[2]) Let $f(z)$ be an entire function of n-variables with respect to any bounded complete n-circular domain D with center at the origin in $\mathbb{C}^n$. Then Gol'dberg order of an entire function $f(z)$ for any bounded complete n-circular domain D is defined as*

$$\rho_{f,D} = \limsup_{R\to\infty} \frac{log^{[2]} M_{f,D}(R)}{logR}.$$

The Gol'dberg lower order of an entire function $f(z)$ for any bounded complete n-circular domain D is defined as

$$\lambda_{f,D} = \liminf_{R\to\infty} \frac{log^{[2]} M_{f,D}(R)}{logR},$$

where $log^{[k]}R = log\left(log^{[k-1]}R\right)$ for $k = 1, 2, 3, ...; log^{[0]}R = R$, and
$exp^{[k]}R = exp\left(exp^{[k-1]}R\right)$ for $k = 1, 2, 3, ...; exp^{[0]}R = R$.

For any bounded complete n-circular domain D, an entire function of n-complex variables for which Goldberg order and Goldberg lower order are the same is said to be of regular growth. Functions which are not of regular growth are said to be of irregular growth. To compare the relative growth of entire functions of n-complex variables having same non zero finite Goldberg order.

Definition 1.2. *([3]) Let $f(z)$ and $g(z)$ be any entire function of n-variables and D be a bounded complete n-circular domain with center at the origin in $\mathbb{C}^n$. Then relative Gol'dberg order of an entire function $f(z)$ for any bounded complete n-circular domain D is defined as*

$$\rho_{g,D}(f) = \inf\{\mu > 0 : M_{f,D}(R) < M_{g,D}(R^{\mu}), \forall\ R > R_0(\mu) > 0\}\ .$$

$$\rho_{g,D}(f) = \limsup_{R\to\infty} \frac{logM_{g,D}^{-1}M_{f,D}(R)}{logR}\ .$$

The Gol'dberg lower order of an entire function $f(z)$ for any bounded complete n-circular domain D is defined as

$$\lambda_{g,D}(f) = \liminf_{R\to\infty} \frac{logM_{g,D}^{-1}M_{f,D}(R)}{logR}\ .$$

In the case of relative Goldberg order, it therefore seems reasonable to define suitably the $(p,q)^{th}$ relative Goldberg order of entire function of n-complex variables and for any bounded complete n-circular domain D with center at the origin in $\mathbb{C}^n$. With this in view one may introduce the definition of $(p,q)^{th}$ relative Goldberg order of an entire function $f(z)$ with respect to another entire function $g(z)$ where both $f(z)$ and $g(z)$ are of n-complex variables and D be any bounded complete n-circular domain with center at the origin in $\mathbb{C}^n$, in the light of index-pair.

Definition 1.3. *([7]) Let $f(z)$ be an entire function of n-variables and D be a bounded complete n-circular domain with center at the origin in $\mathbb{C}^n$. Then $(p,q)^{th}$ Gol'dberg order of an entire function $f(z)$ for any bounded complete n-circular domain D is defined as*

$$\rho_{f,D}(p,q) = \limsup_{R\to\infty} \frac{log^{[p]}M_{f,D}(R)}{log^{[q]}R}, \qquad where\ p \geq q \geq 1\ .$$

The $(p,q)^{th}$ Gol'dberg lower order of an entire function $f(z)$ for any bounded complete n-circular domain D is defined as

$$\lambda_{f,D}(p,q) = \liminf_{R\to\infty} \frac{log^{[p]}M_{f,D}(R)}{log^{[q]}R}, \qquad where\ p \geq q \geq 1\ .$$

Definition 1.4. *([3],[8]) Let $f(z)$ and $g(z)$ be entire function of n-complex variables with index pair (m,q) and (m,p) respectively, where p,q and m are positive integers such that $m \geq q \geq 1$ and $m \geq p \geq 1$ and D be any bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Then the $(p,q)^{th}$ relative Gol'dberg order of $f(z)$ with respect to $g(z)$ is defined as*

$$\rho_{g,D}^{(p,q)}(f) = \inf\{\mu > 0 : M_{f,D}(r) < M_{g,D}(exp^{[p]}(\mu \log^{[q]} R)), \forall R > R_0(\mu) > 0\}\ .$$

$$= \limsup_{R\to\infty} \frac{log^{[p]}M_{g,D}^{-1}M_{f,D}(R)}{log^{[q]}R}\ .$$

Similarly, the $(p,q)^{th}$ relative Gol'dberg lower order of an entire function $f(z)$with respect to another entire function $g(z)$ is defined as

$$\lambda_{g,D}^{(p,q)}(f) = \liminf_{R\to\infty} \frac{log^{[p]}M_{g,D}^{-1}M_{f,D}(R)}{log^{[q]}R}\ .$$

Definition 1.5. *([8]) Let $f(z)$ and $g(z)$ be any entire function of n-complex variables with index-pairs $(m,q)and(m,p)$ respectively, where $m \geq q \geq 1, m \geq p \geq 1$ and D be any*

bounded complete n-circular domain. Then the entire function $f(z)$ is said to have relative index-pair (p,q) with respect to another entire function $g(z)$ if $b < \rho_{g,D}^{(p,q)}(f) < \infty$, and $\rho_{g,D}^{(p-1,q-1)}(f)$ is not a non-zero finite number, where $b = 1$ if $p = q = m$ and $b = 0$ for otherwise.
Moreover, if $0 < \rho_{g,D}^{(p,q)}(f) < \infty$,then

$$\rho_{g,D}^{(p-n,q)}(f) = \infty \qquad \text{for } n < p,$$

and

$$\rho_{g,D}^{(p,q-n)}(f) = 0 \qquad \text{for } n < q,$$

$$\rho_{g,D}^{(p+n,q+n)}(f) = 1 \qquad \text{for } n = 1, 2, 3, \ldots .$$

Similarly, for $0 < \lambda_{g,D}^{(p,q)}(f) < \infty$, we can easily verify that

$$\lambda_{g,D}^{(p-n,q)}(f) = \infty \qquad \text{for } n < p,$$

and

$$\rho_{g,D}^{(p,q-n)}(f) = 0 \qquad \text{for } n < q,$$

$$\lambda_{g,D}^{(p+n,q+n)}(f) = 1 \qquad \text{for } n = 1, 2, 3, \ldots$$

Further an entire function $f(z)$ for which $(p,q)^{th}$ relative Goldberg order and $(p,q)^{th}$ relative Goldberg lower order with respect to another entire function $g(z)$ are the same is called a function of regular relative $(p,q)^{th}$- Goldberg growth with respect to $g(z)$. Otherwise, $f(z)$ is said to be irregular relative $(p,q)^{th}$-Goldberg growth.with respect to $g(z)$.

During the past decades, several authors (see [1],[4],[6],[7],[8]) made closed investigations on the properties of relative order of entire functions of several complex variable using different growth indicator such as Goldberg order, $(p,q)^{th}$ Goldberg order. In this paper we wish to study some relative growth properties of entire functions of n-complex variables using Goldberg order, $(p,q)^{th}$ relative Goldberg order.

2 Main Results

Theorem 2.1. *Let $f(z)$ and $g(z)$ be an entire function of n-complex variables and D be bounded complete n-circular domain with center at origin in $\mathbb{C}^n$. Also let $0 < \lambda_D^{(p,q)}(fog) < \rho_D^{(p,q)}(fog) < \infty$ and $0 < \lambda_D^{(m,q)}(g) < \rho_D^{(m,q)}(g) < \infty$. Then,*

$$\frac{\lambda_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)} \le \liminf_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[q]} M_{g,D}(R)} \le \min\left\{ \frac{\lambda_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)}, \frac{\rho_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)} \right\} \le \max\left\{ \frac{\lambda_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)}, \frac{\rho_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)} \right\}$$
$$\le \limsup_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[q]} M_{g,D}(R)} \le \frac{\rho_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)}$$

Proof. From the definition of $(p,q)^{th}$ Gol'dberg order and $(p,q)^{th}$ lower Gol'dberg order of entire composite functions fog for $\epsilon > 0$ and value of R tending to infinity.

$$log^{[p]} M_{fog,D}(R) \le \left(\rho_D^{(p,q)}(fog) + \epsilon\right) log^{[q]} R \tag{2.1}$$

and

$$log^{[p]} M_{fog,D}(R) \ge \left(\lambda_D^{(p,q)}(fog) - \epsilon\right) log^{[q]} R\ . \tag{2.2}$$

Again, for all sufficiently large value of R

$$log^{[p]} M_{fog,D}(R) \leq \left(\lambda_D^{(p,q)}(fog) + \epsilon\right) log^{[q]} R \tag{2.3}$$

and

$$log^{[p]} M_{fog,D}(R) \geq \left(\rho_D^{(p,q)}(fog) - \epsilon\right) log^{[q]} R\ . \tag{2.4}$$

Now, from the definition of $(p,q)^{th}$ Gol'dberg order and $(p,q)^{th}$ lower Gol'dberg order of entire function $g(z)$ for $\epsilon > 0$ and for all sufficiently large value of R

$$log^{[m]} M_{g,D}(R) \leq \left(\rho_D^{(m,q)}(g) + \epsilon\right) log^{[q]} R \tag{2.5}$$

and

$$log^{[m]} M_{g,D}(R) \geq \left(\lambda_D^{(m,q)}(g) - \epsilon\right) log^{[q]} R\ . \tag{2.6}$$

Also for a sequence of value of R, tending to infinity

$$log^{[m]} M_{g,D}(R) \leq \left(\lambda_D^{(m,q)}(g) + \epsilon\right) log^{[q]} R \tag{2.7}$$

and

$$log^{[m]} M_{g,D}(R) \geq \left(\rho_D^{(m,q)}(g) - \epsilon\right) log^{[q]} R\ . \tag{2.8}$$

Now from (2) and (5) it follows for $R \to \infty$ that

$$\frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \geq \frac{\left(\lambda_D^{(p,q)}(fog) - \epsilon\right)}{\left(\rho_D^{(m,q)}(g) + \epsilon\right)}\ .$$

As $\epsilon(> 0)$ is arbitrary we obtained that

$$\liminf_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \geq \frac{\lambda_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)}\ . \tag{2.9}$$

Again, from (3) and (6) it follows for $R \to \infty$ that

$$\frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \leq \frac{\left(\lambda_D^{(p,q)}(fog) + \epsilon\right)}{\left(\lambda_D^{(m,q)}(g) - \epsilon\right)}\ .$$

As $\epsilon(>0)$ arbitrary, we obtained that

$$\liminf_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \leq \frac{\lambda_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)} \ . \tag{2.10}$$

Similarly, from (1) and (8) it follows for a sequence of values of R tending to infinity that

$$\frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \leq \frac{\left(\rho_D^{(p,q)}(fog)+\epsilon\right)}{\left(\rho_D^{(m,q)}(g)-\epsilon\right)} \ .$$

As $\epsilon(>0)$, we obtained that

$$\liminf_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \leq \frac{\rho_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)} \ . \tag{2.11}$$

Now combining (9),(10) and (11) we get that

$$\frac{\lambda_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)} \leq \liminf_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \leq \min\left\{ \frac{\lambda_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)}, \frac{\rho_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)} \right\} \ . \tag{2.12}$$

From (2) and (7) it follows for $R\to\infty$ that

$$\frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \geq \frac{\left(\lambda_D^{(p,q)}(fog)-\epsilon\right)}{\left(\lambda_D^{(m,q)}(g)+\epsilon\right)} \ .$$

As $\epsilon(>0)$, we obtained that

$$\limsup_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \geq \frac{\lambda_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)} \ . \tag{2.13}$$

From (1) and (6) it follows for $R\to\infty$ that

$$\frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \leq \frac{\left(\rho_D^{(p,q)}(fog)+\epsilon\right)}{\left(\lambda_D^{(m,q)}(g)-\epsilon\right)} \ .$$

As $\epsilon(>0)$, we obtained that

$$\limsup_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \leq \frac{\rho_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)} \ . \tag{2.14}$$

Similarly, from (4) and (5) it follows for $R \to \infty$ that

$$\frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \geq \frac{\left(\rho_D^{(p,q)}(fog) - \epsilon\right)}{\left(\rho_D^{(m,q)}(g) + \epsilon\right)} .$$

As $\epsilon(>0)$, we obtained that

$$\limsup_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \geq \frac{\rho_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)} . \tag{2.15}$$

Now combining (13),(14) and (15) we obtained that

$$\max\left\{\frac{\lambda_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)}, \frac{\rho_D^{(p,q)}(fog)}{\rho_D^{(m,q)}(g)}\right\} \leq \limsup_{R\to\infty} \frac{log^{[p]} M_{fog,D}(R)}{log^{[m]} M_{g,D}(R)} \leq \frac{\rho_D^{(p,q)}(fog)}{\lambda_D^{(m,q)}(g)} . \tag{2.16}$$

Hence theorem follows from (12) and (16).

□

Theorem 2.2. *Let $f(z), g(z)$ and $h(z)$ be three entire functions and D be bounded complete n-circular domain with center at origin in $\mathbb{C}^n$.*
Also let $0 < \rho_{h,D}^{(p,q)}(fog) < \infty$ and $0 < \rho_{g,D}^{(m,q)}(f) < \infty$. Then,
$\liminf_{R\to\infty} \frac{log^{[p]} M_{h,D}^{-1} M_{fog,D}(R)}{log^{[q]} M_{g,D}^{-1} M_{f,D}(R)} \leq \frac{\rho_{h,D}^{(p,q)}(fog)}{\rho_{g,D}^{(m,q)}(f)} \leq \limsup_{R\to\infty} \frac{log^{[p]} M_{h,D}^{-1} M_{fog,D}(R)}{log^{[q]} M_{g,D}^{-1} M_{f,D}(R)}$,
where p, q and m are positive integer such that $p > q$ and $m > q$ that is $q < min\{p, m\}$.

Proof. From the definition of $(p,q)^{th}$ relative Gol'dberg order with respect to another entire function, we get for a sequence of value of R tending to infinity

$$\log^{[p]} M_{h,D}^{-1} M_{fog,D}(R) \leq \left(\rho_{h,D}^{(p,q)}(fog) + \epsilon\right) \log^{[q]} R \tag{2.17}$$

and

$$\log^{[p]} M_{h,D}^{-1} M_{fog,D}(R) \geq \left(\rho_{h,D}^{(p,q)}(fog) - \epsilon\right) \log^{[q]} R . \tag{2.18}$$

Also

$$\log^{[m]} M_{g,D}^{-1} M_{f,D}(R) \leq \left(\rho_{g,D}^{(m,q)}(f) + \epsilon\right) \log^{[q]} R \tag{2.19}$$

and

$$\log^{[m]} M_{g,D}^{-1} M_{f,D}(R) \geq \left(\rho_{g,D}^{(m,q)}(f) - \epsilon\right) \log^{[q]} R . \tag{2.20}$$

From (18) and (19) it follows for a sequence of values of R tending to infinity that

$$\frac{log^{[p]}M_{h,D}^{-1}M_{fog,D}(R)}{log^{[m]}M_{g,D}^{-1}M_{f,D}(R)} \geq \frac{\left(\rho_{h,D}^{(p,q)}(fog)-\epsilon\right)}{\left(\rho_{g,D}^{(p,q)}(f)+\epsilon\right)} .$$

As $\epsilon(>0)$ is arbitrary we obtained that

$$\limsup_{R\to\infty} \frac{log^{[p]}M_{h,D}^{-1}M_{fog,D}(R)}{log^{[m]}M_{g,D}^{-1}M_{f,D}(R)} \geq \frac{\rho_{h,D}^{(p,q)}(fog)}{\rho_{g,D}^{(p,q)}(f)} . \tag{2.21}$$

Again, from (17) and (20) we get a sequence of values of R tending to infinity that

$$\frac{log^{[p]}M_{h,D}^{-1}M_{fog,D}(R)}{log^{[m]}M_{g,D}^{-1}M_{f,D}(R)} \leq \frac{\left(\rho_{h,D}^{(p,q)}(fog)+\epsilon\right)}{\left(\rho_{g,D}^{(p,q)}(f)-\epsilon\right)} .$$

As $\epsilon(>0)$ is arbitrary we obtained that

$$\liminf_{R\to\infty} \frac{log^{[p]}M_{h,D}^{-1}M_{fog,D}(R)}{log^{[m]}M_{g,D}^{-1}M_{f,D}(R)} \leq \frac{\rho_{h,D}^{(p,q)}(fog)}{\rho_{g,D}^{(p,q)}(f)} . \tag{2.22}$$

Hence the theorem follows from (21) and (22).

□

Theorem 2.3. *Let $f(z), g(z)$ and $h(z)$be three entire functions and D be bounded complete n-circular domain with center at origin in $\mathbb{C}^n$.*
Also let $0<\lambda_{h,D}^{(p,q)}(fog)<\infty$ and $0<\lambda_{g,D}^{(m,q)}(f)<\infty$.Then,

$$\liminf_{R\to\infty} \frac{log^{[p]}M_{h,D}^{-1}M_{fog,D}(R)}{log^{[m]}M_{g,D}^{-1}M_{f,D}(R)} \leq \frac{\lambda_{h,D}^{(p,q)}(fog)}{\lambda_{g,D}^{(p,q)}(f)} \leq \limsup_{R\to\infty} \frac{log^{[p]}M_{h,D}^{-1}M_{fog,D}(R)}{log^{[m]}M_{g,D}^{-1}M_{f,D}(R)}$$

where p,q and m are positive integer such that $p>q$ and $m>q$ that is $q<min\{p,m\}$.

Proof. From the definition of $(p,q)^{th}$ relative lower Gol'dberg order with respect to another entire function, we get for a sequence for large value of R

$$\log^{[p]} M_{h,D}^{-1}M_{fog,D}(R) \geq \left(\lambda_{h,D}^{(p,q)}(fog)-\epsilon\right)\log^{[q]} R \tag{2.23}$$

and

$$\log^{[p]} M_{h,D}^{-1}M_{fog,D}(R) \leq \left(\lambda_{h,D}^{(p,q)}(fog)+\epsilon\right)\log^{[q]} R . \tag{2.24}$$

Also for a sequence of values of $R \to \infty$, that

$$\log^{[p]} M_{g,D}^{-1} M_{f,D}(R) \geq \left(\lambda_{g,D}^{(m,q)}(f) - \epsilon\right) \log^{[q]} R \tag{2.25}$$

and

$$\log^{[p]} M_{g,D}^{-1} M_{f,D}(R) \leq \left(\lambda_{g,D}^{(m,q)}(f) + \epsilon\right) \log^{[q]} R \ . \tag{2.26}$$

Now from (23) and (26) we get a sequence of values of $R \to \infty$, that

$$\frac{log^{[p]} M_{h,D}^{-1} M_{fog,D}(R)}{log^{[m]} M_{g,D}^{-1} M_{f,D}(R)} \geq \frac{\left(\lambda_{h,D}^{(p,q)}(fog) - \epsilon\right)}{\left(\lambda_{g,D}^{(m,q)}(f) + \epsilon\right)} \ .$$

As $\epsilon(>0)$ is arbitrary, we obtained that

$$\limsup_{R\to\infty} \frac{log^{[p]} M_{h,D}^{-1} M_{fog,D}(R)}{log^{[m]} M_{g,D}^{-1} M_{f,D}(R)} \geq \frac{\lambda_{h,D}^{(p,q)}(fog)}{\lambda_{g,D}^{(m,q)}(f)} \ . \tag{2.27}$$

Now from (24) and (25) we get a sequence of values of $R \to \infty$, that

$$\frac{log^{[p]} M_{h,D}^{-1} M_{fog,D}(R)}{log^{[m]} M_{g,D}^{-1} M_{f,D}(R)} \leq \frac{\left(\lambda_{h,D}^{(p,q)}(fog) + \epsilon\right)}{\left(\lambda_{g,D}^{(m,q)}(f) - \epsilon\right)} \ .$$

As $\epsilon(>0)$ is arbitrary, we obtained that

$$\liminf_{R\to\infty} \frac{log^{[p]} M_{h,D}^{-1} M_{fog,D}(R)}{log^{[m]} M_{g,D}^{-1} M_{f,D}(R)} \leq \frac{\lambda_{h,D}^{(p,q)}(fog)}{\lambda_{g,D}^{(m,q)}(f)} \ . \tag{2.28}$$

Hence the theorem follows from (27) and (28).

□

ACKNOWLEDGMENTS.
The author is thankful to the referee for his/her valuable suggestions towards the improvement of the paper.

References

[1] A.A.Gol'dberg, Elementary remarks on the formulas defining order and type of function of several complex variables,*S.S.R. Dokl.*,Vol. 29 (1959) pp. 145-151.

[2] B.A. Fuks:Introduction to the theory of analytic function of several complex variables,*American mathematical Society*, R.I.,(1963).

[3] B.C. Mondal and C. Roy, Relative Gol'dberg order of an entire function of several variables,*Bull. cal. Math. Society*,Vol-102, No-4(2010),pp. 371-380.

[4] G.Valiron, lecture on the general theory of intgral functions,*Chelsea publishing company*,(1949).

[5] O. P. Juneja, G. P. Kapoor and S. K. Bajpai, On the (p,q)-order and lower (p,q)-order of an entire function,*J. Reine Angew. Math.*, 282 (1976), 53-67.

[6] S. Halvarsson, Growth properties of entire functions depending on a parameter,*Annales Polonici Mathe-matici*, 14(1) (1996), pp. 71-96.

[7] S.K. Datta and A.R. Maji, Study of growth properties on the basis of generalized Gol'dberg order of composite entire functions of several complex variables,*International Journal of Math.Sci. and Engg.Appls,* Vol 5 No.V(2011),pp. 297-311.

[8] Tanmay Biswas, Some Results relating to $(p,q)^{th}$ relative Gol'dberg order and $(p,q)^{th}$ -relative Gol'dberg type of entire functions of several variables, (2018).