# A Class Of Exact Solution Of Equations Governing Aligned Plane Rotating Magnetohydrodynamic Flows by Martin's Method

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**Abstract**

In this paper an approach has been made to determine the exact solution of steady, plane, aligned, MHD fluid flow having infinite electrical conductivity in a rotating frame by the application of Martin's method. The non-linear governing equations of the fluid flow are converted into a new form called Martin's form by employing differential geometry where in the curvilinear coordinates ($\phi$, $\psi$) in the plane of flow the co-ordinate lines $\psi$ =constant are the streamlines of flow and the co-ordinate lines $\phi$ = constant, are left arbitrary. Also von Mises co-ordinates (x, $\psi$) that require the use of $\phi$ =x in Martin's co-ordinates ($\phi$, $\psi$) are taken. Finally the value of magnetic field vector, velocity vector, pressure function, vorticity and the current density function are found out.

**Keywords:** MHD, Martin's method, exact solution, rotating frame, aligned flow, stream function.

## 1 Introduction

Martin [1] introduced a new method to study steady plane flows of viscous fluid. The study of fluids having infinite electrical conductivity is done for the mathematical idealization for completeness and to include super conducting liquid metal of the future. The theory of rotating fluid is very important because of its appearance in many natural phenomena and its application in various areas like oceanography, meteorology, atmospheric science and limnology.

Various transformation techniques are applied to convert the non-linear partial differential equations into linear differential equations. The Navier-Stokes equations and the basic equations governing the flow of magnetohydrodynamic (MHD) fluid are inherently non-linear partial differential equations and have no general solution. Only a small number of exact solutions have been found as the non-linear terms do not vanish automatically. Martin [1] used a new approach and employed differential geometry to convert equations into solvable form, where by using the curvilinear coordinates ($\phi$, $\psi$) in the plane of flow and keeping co-ordinate lines $\psi$ = constant are taken to be the streamlines of flow and the co-ordinate lines $\phi$ =constant, are left arbitrary. Martin's approach is followed in this paper. We have used von Mises co-ordinates (x, $\psi$) that require the use of $\phi$ = x in Martin's co-ordinates ($\phi$, $\psi$). C.S. Bagewadi and Siddabasappa [2] applied Martin's technique in the plane rotating viscous MHD flows to find the exact solution. O.P. Chandna, R.M. Barron and M.R. Garg [3] used Martin's method in plane compressible MHD flows study exact solution. O.P. Chandna, M.R. Garg [4] used Martin's method in the flow of viscous MHD fluid to find exact solutions. K.V. Govindaraju [5] applied Martin's technique in the flow of viscous fluid to determine exact solutions. V.I.Nath and O.P. Chandna [6] applied Martin's method on plane viscous magnetohydrodynamic flows to get exact solution. G. Ram and R.S. Mishra [7] determined exact solution of unsteady MHD flow of fluid through porous medium. O.P. Chandna, R.M. Barron and M.R. Garg [8] applied Martin's technique on plane compressible MHD flows to get exact solutions. O.P. Chandna and M.R. Garg [9] applied Martin's method on the flow of viscous MHD fluid to find exact solutions. R.K. Naeem [10] also used the Martin's method to find exact solutions for Navier-Stokes equations for viscous incompressible fluids. O.P. Chandna and F. Labropulu [11] found exact solutions of steady plane flows using von Mises co-ordinates. R.K. Naeem and S.A. Nadeem [12] studied steady plane flows of an incompressible fluid of variable viscosity using Martin's method. F. Labropulu and O.P. Chandna [14] determined exact solutions of steady plane MHD aligned flows using vonco-ordinates. S.A. Ali, A. Ara and N.A. Khan [15] applied Martin's method to steady plane flow of a second grade fluid and found out exact solution. C. Thakur, M. Kumar and M.K. Mahan [16] applied Martin's method to constantly inclined viscous MHD flows through porous media and got exact solutions. M. Kumar, C. Thakur, T.P. Singh and M.K. Mahan [17] found an exact solutions of steady plane aligned MHD flow using Martin's method. R.K. Naeem, A. Mansoor, W.A. Khan and Aurangzaib [18] determined exact solutions of steady plane flows of an incompressible fluid of variable viscosity using ($\epsilon$, $\psi$) or ($\eta$, $\psi$)-coordinates. R.K. Naeem, A. Mansoor and W.A. Khan [19] studied plane flows of an incompressible fluid of variable viscosity in the presence of unknown external force using ($\epsilon$, $\psi$)– or ($\eta$, $\psi$)-coordinates. C.S. Bagewadi and Siddabasappa [20] applied Martin's method to study exact solution of variably inclined rotating MHD flows in magnetograph plane. M Kumar and S. Sil [21] applied Martin's method to determine the exact solution of steady plane rotating aligned MHD flows in magnetograph plane. K.K. Singh and D.P. Singh [22] studied steady plane MHD flows through porous media with constant speed along each streamline. C. Thakur and M. Kumar [23] studied plane rotating viscous MHD flows through porous media and found exact solutions. S. Sil and M. Kumar [24] found a class of solution of orthogonal plane MHD flow through porous media in a rotating frame. Also S. Sil and M. Kumar [25] found exact solution of second grade fluid in a rotating frame through porous media using hodograph transformation method.

In this paper we find the exact solutions of steady plane aligned MHD flows of an incompressible rotating viscous fluid with infinite electrical conductivity using Martin's method.

## 2 BASIC EQUATIONS

The general governing equation of the plane, viscous, incompressible fluid of infinite electrical conductivity in the presence of magnetic field is given by

$$\nabla \cdot \mathbf{V} = \mathbf{0}, \tag{2.1}$$

$$\rho\left[(\mathbf{V}\cdot\nabla)\ \mathbf{V} + 2\mathbf{\Omega}\times\mathbf{V}\ + \mathbf{\Omega}\times(\mathbf{\Omega}\times\ \mathbf{r})\right] + \nabla \mathrm{p} = \eta\nabla^2\ \mathbf{V} + \mu(\nabla\times\mathbf{H})\times\mathbf{H}\ , \tag{2.2}$$

$$\nabla\times(\mathbf{V}\times\mathbf{H}) = 0, \tag{2.3}$$

$$\nabla\cdot\mathbf{H} = 0, \tag{2.4}$$

where $\mathbf{V}$ = velocity field vector, $\mathbf{H}$ = magnetic vector field , $p$ = dynamic pressure function, $\rho$ = the constant fluid field density, $\mathbf{\Omega}$ = angular velocity vector, $\mathbf{r}$ = radius vector $\eta$ = coefficient of dynamic viscosity, $\mu$ = constant magnetic permeability. The two dimensional flow is considered so that $\mathbf{V}$ and $\mathbf{H}$ lie in a plane having co-ordinates (x, y) and all the flow variables are functions of (x, y);

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \tag{2.5}$$

$$Q = \frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}, \tag{2.6}$$

$$B = \frac{1}{2}\rho V^2 + p^{'} + \frac{1}{2}\rho\left|\Omega\times r\right|^2, \tag{2.7}$$

where

$V^2 = u^2 + v^2$, $\omega$= vorticity, $Q$ = current density function and $p^{'}$ is the reduced pressure function given by $p^{'} = p - \frac{1}{2}\rho\left|\Omega\times r\right|^2$.

And the last term being the centrifugal contribution of pressure, u, v are the components of velocity vector $\mathbf{V}$, $H_1$, $H_2$ are components of magnetic field vector $\mathbf{H}$. Separating into components equations (2.1) to (2.4) are replaced by the following equations;

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{2.8}$$

$$\frac{\partial B}{\partial x} + \eta\frac{\partial \omega}{\partial y} - 2\rho\Omega v - \rho v\Omega + \mu H_2 Q = 0, \tag{2.9}$$

$$\frac{\partial B}{\partial y} - \eta\frac{\partial \omega}{\partial x} + 2\rho\Omega u - \rho u\Omega - \mu H_1 Q = 0, \tag{2.10}$$

$$uH_2 - vH_1 = C, \tag{2.11}$$

$$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0, \tag{2.12}$$

$$\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y} = 0, \tag{2.13}$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \tag{2.14}$$

where C is arbitrary constant of integration. We consider aligned flow in which magnetic field is everywhere parallel to velocity field, so that ;

$$\mathbf{H} = \beta \mathbf{V} \qquad i.e \;\; H_1 = \beta u, \qquad H_2 = \beta v, \tag{2.15}$$

where $\beta$ is some unknown scalar function such that

$$\mathbf{V} \cdot \boldsymbol{\nabla} \beta = 0, \tag{2.16}$$

is the condition satisfied by $\beta$ as obtained from equation (2.1),(2.4) and (2.15). Using (2.15) in the above system of equations (2.8) to (2.14) we get the following system of partial differential equations;

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{2.17}$$

$$\frac{\partial B}{\partial x} + \eta \frac{\partial \omega}{\partial y} - 2\rho\Omega v - \rho v \Omega + \mu \beta v Q = 0, \tag{2.18}$$

$$\frac{\partial B}{\partial y} - \eta \frac{\partial \omega}{\partial x} + 2\rho\Omega u - \rho u \Omega - \mu \beta u Q = 0, \tag{2.19}$$

$$u\frac{\partial \beta}{\partial x} + v\frac{\partial \beta}{\partial y} = 0, \tag{2.20}$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \tag{2.21}$$

$$\beta\omega + v\frac{\partial \beta}{\partial x} - u\frac{\partial \beta}{\partial y} = Q, \tag{2.22}$$

above are six partial differential equations in six unknown functions u(x, y), v(x, y), $\beta$(x, y), $\omega$(x, y), Q(x, y) and $\beta$(x, y). Once a solution of these equations are determined pressure function p(x, y) and the velocity vector $\mathbf{V}$ can be determined.

## 3 Some results of differential geometry

The equation of continuity of equation (2.17) implies the existence of a stream function $\psi = \psi(\text{x, y})$ such that;

$$\frac{\partial \psi}{\partial x} = -v \quad \text{and} \quad \frac{\partial \psi}{\partial y} = u, \tag{3.1}$$

We take $\phi$(x, y)= constant to be some arbitrary family of curves which generates with the streamline $\psi$(x, y) = constant, a curvilinear coordinate so that in the physical plane the independent variables x, y can be replaced by $\phi$, $\psi$, Let

$$x = x(\phi, \psi), y = y(\phi, \psi), \tag{3.2}$$

define a curvilinear coordinate in the (x, y) plane with the squared element of arc length along any curve given by

$$ds^2 = E(\phi, \psi)\, d\phi^2 + 2F(\phi, \psi)\, d\phi d\psi + G(\phi, \psi)\, d\psi^2, \tag{3.3}$$

where

$$\begin{aligned} E &= \left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial y}{\partial \phi}\right)^2, \\ F &= \frac{\partial x}{\partial \phi}\frac{\partial x}{\partial \psi} + \frac{\partial y}{\partial \phi}\frac{\partial y}{\partial \psi}, \\ G &= \left(\frac{\partial x}{\partial \phi}\right)^2 + \left(\frac{\partial y}{\partial \psi}\right)^2, \end{aligned} \tag{3.4}$$

Equation (3.2) can be solved to obtain $\phi$=$\phi$(x,y), $\psi$=$\psi$(x, y) such that

$$\frac{\partial x}{\partial \phi} = J\frac{\partial \psi}{\partial y}, \qquad \frac{\partial x}{\partial \psi} = -J\frac{\partial \phi}{\partial y}, \qquad \frac{\partial y}{\partial \phi} = -J\frac{\partial \psi}{\partial x}, \qquad \frac{\partial y}{\partial \psi} = J\frac{\partial \phi}{\partial x}, \tag{3.5}$$

Provided $0 < |J| < \infty$, where $J$ is the Jacobian transformation

$$J = \frac{\partial x}{\partial \phi}\frac{\partial y}{\partial \psi} - \frac{\partial x}{\partial \psi}\frac{\partial y}{\partial \phi} = \pm\sqrt{EG - F^2} = \pm W, \tag{3.6}$$

If $\alpha$ be the local angle of inclination of the tangent to the coordinate line $\psi$= constant, directed in the sense of increasing $\phi$, we have from differential geometry

$$\begin{aligned} &\frac{\partial x}{\partial \phi} = \sqrt{E}\cos\alpha, \qquad \frac{\partial y}{\partial \phi} = \sqrt{E}\sin\alpha, \qquad \frac{\partial x}{\partial \psi} = \frac{F}{\sqrt{E}}\cos\alpha - \frac{J}{\sqrt{E}}\sin\alpha, \\ &\frac{\partial y}{\partial \psi} = \frac{F}{\sqrt{E}}\sin\alpha + \frac{J}{\sqrt{E}}\cos\alpha, \qquad \frac{\partial \alpha}{\partial \phi} = \frac{J}{E}\Gamma_{11}, \qquad \frac{\partial \alpha}{\partial \psi} = \frac{J}{E}\Gamma^2{}_{12} \end{aligned} \tag{3.7}$$

and

$$K = \frac{1}{w}\left[\frac{\partial}{\partial \psi}\left(\frac{W}{E}{\Gamma^2}_{11}\right) - \frac{\partial}{\partial \phi}\left(\frac{W}{E}{\Gamma^2}_{12}\right)\right] = 0$$

Where

$$\begin{aligned}
{\Gamma^2}_{11} &= \frac{1}{2w^2}\left[-F\frac{\partial F}{\partial \phi} + 2E\frac{\partial F}{\partial \phi} - E\frac{\partial E}{\partial \Psi}\right], \\
{\Gamma^2}_{12} &= \frac{1}{2w^2}\left[E\frac{\partial G}{\partial \phi} - F\frac{\partial E}{\partial \Psi}\right],
\end{aligned}$$

here k is the Gaussian curvature.

## 4 Martins form of flow equations

Equation (3.1), (3.5) and (3.7) give

$$\begin{aligned}
\sqrt{E}\cos\alpha &= \frac{\partial x}{\partial \phi} = J\frac{\partial \psi}{\partial y} = Ju = JV\cos\theta, \\
\sqrt{E}\sin\alpha &= \frac{\partial y}{\partial \phi} = -J\frac{\partial \psi}{\partial x} = Jv = JV\sin\theta,
\end{aligned}$$

where $V = \sqrt{u^2 + v^2}$ and $\theta$ is the direction of flow in the physical plane. This pair of equations shows that the fluid flows along the streamlines towards higher or lower parameter values of $\phi$ according as to whether J>0 or J<0. We consider here fluid flows towards the higher parameter values of $\phi$ so that J= $W > 0$.

New forms of equations using (3.1) in equations (2.18) and (2.19) and considering (3.5), the equations in ($\phi$, $\psi$) co-ordinate are given by

$$\frac{\partial B}{\partial \phi}\frac{\partial x}{\partial \psi} - \frac{\partial B}{\partial \psi}\frac{\partial y}{\partial \phi} + \eta\left(-\frac{\partial \omega}{\partial \phi}\frac{\partial x}{\partial \psi} + \frac{\partial \omega}{\partial \psi}\frac{\partial x}{\partial \phi}\right) - \left[\rho\left(2\Omega + \omega\right) - \mu\beta Q\right]\frac{\partial y}{\partial \phi} = 0, \tag{4.1}$$

$$\frac{\partial B}{\partial \phi}\frac{\partial x}{\partial \psi} - \frac{\partial B}{\partial \psi}\frac{\partial x}{\partial \phi} + \eta\left(\frac{\partial \omega}{\partial \phi}\frac{\partial y}{\partial \psi} - \frac{\partial \omega}{\partial \psi}\frac{\partial y}{\partial \phi}\right) - \left[\rho\left(2\Omega + \omega\right) - \mu\beta Q\right]\frac{\partial x}{\partial \phi} = 0. \tag{4.2}$$

Multiplying (4.1) by $\frac{\partial x}{\partial \phi}$ and (4.2) by $\frac{\partial y}{\partial \phi}$ and subtracting we get

$$J\frac{\partial B}{\partial \phi} = \eta\left(F\frac{\partial \omega}{\partial \phi} - E\frac{\partial \omega}{\partial \psi}\right), \tag{4.3}$$

Again multiplying equation (4.1) by $\frac{\partial x}{\partial \psi}$ and (4.2) by $\frac{\partial y}{\partial \psi}$and subtracting

$$J\frac{\partial B}{\partial \psi} = \eta\left(-F\frac{\partial \omega}{\partial \psi} + G\frac{\partial \omega}{\partial \phi}\right) - J\left[\rho\left(2\Omega + \omega\right) - \mu\beta Q\right]. \tag{4.4}$$

### 4.1 Solenoidal equation

Using (3.1) in the equation (2.20) and transforming the resulting equations to ($\phi$, $\psi$) co-ordinate we get

$$\frac{\partial\psi}{\partial y}\left[\frac{\partial B}{\partial\phi}\frac{\partial\phi}{\partial x}+\frac{\partial\beta}{\partial\psi}\frac{\partial\psi}{\partial x}\right]-\frac{\partial\psi}{\partial x}\left[\frac{\partial\beta}{\partial\phi}\frac{\partial\phi}{\partial y}+\frac{\partial\phi}{\partial\psi}\frac{\partial\psi}{\partial y}\right]=0, \tag{4.5}$$

Which on simplification gives

$$\frac{\partial\beta}{\partial\phi}=0. \tag{4.6}$$

Current density equations. Employing (3.1) in equation (2.22) we have

$$\beta\omega-\frac{\partial\psi}{\partial x}\frac{\partial\beta}{\partial x}-\frac{\partial\psi}{\partial y}\frac{\partial\beta}{\partial y}=Q, \tag{4.7}$$

using (3.5) and employing (4.6) and (3.4) we get

$$\beta\omega-\frac{E}{J^2}\frac{\partial\psi}{\partial y}\frac{\partial\beta}{\partial y}=Q. \tag{4.8}$$

### 4.2 Equations of continuity and vorticity

Martins (2.1) obtained the necessary and sufficient condition for the flow of a fluid along the coordinate lines $\psi$ = constant of curvilinear co-ordinate system with $ds^2$ given by (3.3) to satisfy the principle of conservation of mass to be

$$WV=\sqrt{E}\,,\qquad u+iv=\frac{\sqrt{E}}{W}e^{i\alpha}, \tag{4.9}$$

He has also proven that the vorticity equation taken the form

$$\omega=\frac{1}{W}\left[\frac{\partial}{\partial\phi}\left(\frac{F}{W}\right)-\frac{\partial}{\partial\psi}\left(\frac{E}{W}\right)\right]. \tag{4.10}$$

## 5 Exact solution

We consider the given flow problem in the form (y-f(x))/g(x) = constant, where f(x) and g(x) $\neq$ 0 are continuously differentiable functions. For a given flow problem with (y-f(x))/g(x) = constant as the family of streamlines, we have

$$y=f(x)+g\left(x\right)\gamma\left(\psi\right), \tag{5.1}$$

where $\gamma(\psi)$is an unknown functions satisfying $\gamma^{'}(\psi)$ =0. Employing von Mises co-ordinates $\phi = (x, y)$ and equation (5.1) in equation (3.4) and (3.6) we get

$$\begin{aligned} E &= 1 + \left[f^{'}(x) + g^{'}(x)\gamma(\psi)\right]^2, \\ F &= \left[g(x) f^{'}(x) + g(x)\gamma(\psi)\right]\gamma^{'}(\psi), \\ G &= g^2(x)\gamma^{'2}(x), \\ J &= W = g(x)\gamma^{'}(\psi). \end{aligned} \tag{5.2}$$

Parabolic flows along $y - m_1x^2 - m_2x$ =constant.
Since the family of parabolic curves are the streamlines, it follows their exists some functions $\gamma(\psi)$such that

$$y = m_1x^2 + m_2x + \gamma(\psi), \quad \gamma^{'}(\psi) \neq 0 \tag{5.3}$$

Comparing (5.3) with (5.1), we have
$f(x) = m_1x^2 + m_2x$, $g(x) = 1$. Using these expressions for f(x), g(x) in (5.2) we get

$$\begin{aligned} E &= 1 + (2m_1x + m_2)^2, \ F = (2m_1x + m_2)\gamma^{'}(\psi) \qquad , \\ G &= \gamma^{'2}(\psi), \ J = W = \gamma^{'}(\psi). \end{aligned} \tag{5.4}$$

Martins (4.3), (4.4), (4.8) and (4.10) in von Mises co-ordinates $(\gamma, \psi)$ we have,

$$\frac{\partial B}{\partial \psi} = \eta\left[\gamma^1(\psi)\frac{\partial \omega}{\partial \phi} - (2m_1x + m_2)\frac{\partial \omega}{\partial \phi}\right] + \left[\mu\beta Q - \rho(2\Omega + \omega)\right], \tag{5.5}$$

$$Q = \beta(\psi)\omega - \left[\frac{1 + (2m_1x + m_2)^2}{\gamma^{'2}(\psi)}\right]\beta^{'}(\psi), \tag{5.6}$$

and

$$\omega = \frac{1}{\gamma^{'3}(\psi)}\left[2m_1\gamma^{'2}(\psi) + \gamma^{''}(\psi) + (2m_1x + m_2)^2\gamma^{''}(\psi)\right]. \tag{5.7}$$

Now eliminating $\omega$ and Q from the integrability condition $\frac{\partial^2 B}{\partial x \partial \psi} = \frac{\partial^2 B}{\partial \psi \partial x}$ using (5.5) to (5.7) we have

$$\sum_{n=0}^{4} a_n(\psi)\left[2m_1x + m_2\right]^n = 0, \tag{5.8}$$

where

$$a_0(\psi) = a_4(\psi) - 4m_1\left[\frac{\gamma^{''}(\psi)}{\gamma^{'3}(\psi)}\right]^{'} + 12{m^2}_1\frac{\gamma^{''}(\psi)}{\gamma^{'2}(\psi)},$$

$$a_2(\psi) = 2a_4(\psi) - 12m_1\left[\frac{\gamma''(\psi)}{\gamma'^3(\psi)}\right], a_3(\psi) = 0,$$

and

$$a_4(\psi) = \left[\frac{1}{\gamma'(\psi)}\left(\frac{\gamma''(\psi)}{\gamma'^3(\psi)}\right)'\right].'$$

Since x and $\psi$ are independent variables the identity (5.8) can only hold if $a_0(\psi)$, $a_1(\psi)$, $a_2(\psi)$, $a_3(\psi)$, and $a_4(\psi)$, vanish identically. Using the consequences $a_4(\psi)=0$, $a_2(\psi)=0$ in $a_0(\psi)=0$, $a_1(\psi)=0$, we find

$$\gamma(\psi) = c_1\psi + c_2 \text{ and } \beta(\psi) = \beta_0 \tag{5.9}$$

where $c_1 \neq 0$ and $\beta_0 \neq 0$ are arbitrary constants. From (5.9) and (5.3), we get

$$c_1\psi(x, y) + c_2 = y - m_1x^2 - m_2x, \tag{5.10}$$

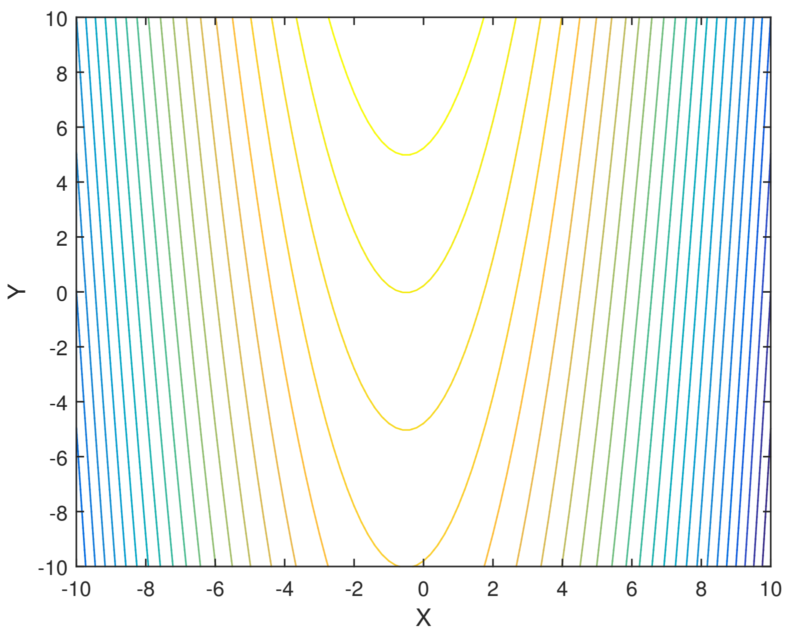


Fig. 1: Parabolic streamlines

For the chosen parabolic flow pattern, using (5.9),and (5.10) in (3.1), (2.15), (2.5), (2.6), (2.7) and (2.17) to (2.22) we find

$$u = \frac{1}{c_1}, \tag{5.11}$$

$$v = \left(\frac{2m_1x + m_2}{c_1}\right), \tag{5.12}$$

$$H_1 = \frac{\beta_0}{c^2{}_1}, \tag{5.13}$$

$$H_2 = \beta_0\left(\frac{2m_1x + m_2}{c_1}\right), \tag{5.14}$$

$$\begin{aligned} p &= \frac{2m_1}{c^2{}_1}\left[\mu\beta^2 - \rho\left(1 + \frac{c_1\Omega}{m_1}\right)\right] y - \frac{\rho}{2c^2{}_1}\left(1 + m^2{}_2\right) \\ &- 2\left[\left(\frac{m_1\mu\beta^2{}_0}{c^2{}_1}\right) - \left(\frac{\rho\Omega}{c_1}\right)\right]\left(m_1x^2 + m_2x\right) + p_0, \end{aligned} \tag{5.15}$$

$$\omega = \frac{2m_1}{c_1}, \tag{5.16}$$

$$Q = \frac{2m_1}{c_1}\beta_0, \tag{5.17}$$

where $p_o$ is arbitrary constant.

## 6 CONCLUSION

In this work we have considered steady plane fluid flow of infinite electrical conductivity. An approach for the determination of exact solution has been carried out. The expressions for velocity, magnetic field, vorticity, current density, and pressure distribution are found out. The streamline pattern of the flow is plotted as well. This work is more general and in the absence of rotating frame,we recover the results of O. P. Chandna and F. Labropulu [11], with expression for velocity, magnetic field, vorticity, current density remaining same and the pressure function obtained by $p = \frac{2m_1}{c^2{}_1}\left[\mu\beta^2 - \rho\right] y - 2\left(\frac{m_1\mu\beta^2{}_0}{c^2{}_1}\right)\left(m_1x^2 + m_2x\right) - \frac{\rho}{2c^2{}_1}\left(1 + m^2{}_2\right) + p_0$,
can be recovered by setting $\Omega = 0$ in equation (5.15). Likewise, if the magnetograph plane is considered for the flow and taking the co-ordinate system ($\phi$, $y$) in place of (x, $\psi$) then the results of M. Kumar and S. Sil [21] can be obtained after suitable transformations.

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