Properties of k-Riemann-Liouville Fractional Integral Operator with Mittag-Leffler functions

Indu Bala Bapna [1] **Radhe Shyam Prajapat** [2]

[1,2] *Dept. of Mathematics*
M. L. V. Govt. PG College Bhilwara, Rajasthan, India
email: [1] *bapnain@yahoo.com* ; [2]*rprajapat71@yahoo.in*

Abstract

In this paper, we will introduce some properties based on the Laplace transform and Millen transform for k-Riemann Liouville fractional integral operator involving the Mittag-Leffler functions. The Laplace transform and Millen transform of k-Riemann Liouville operator will be obtained and also new interesting results will be introduced by the help of properties of the Mittag Leffler function.

Keywords: Mittag-Leffler function, Integral transform, Riemann Liouville fractional integral.

2010 Mathematics subject classification: 33E12, 35A22

1 Introduction

Resently,the special functions have wide applications in many areas of fractional calculas theory, statistics, plysical science and engineering (Haubold et. al. [3], Mathai et. al.[6],Bapna et. al. [1], [2],). In 1903, the classical Mittag-Laffler function is introduced by Swedish Methematician Gost Mittag-Leffler [4] and defined as

$$E_{\alpha}(z)=\sum_{n=0}^{\infty}\frac{z^{n}}{\Gamma(n\alpha+1)} \qquad Re(\alpha)>0, z\in C, \alpha\in C \tag{1.1}$$

Many researchers have expanded the research work of Mittag-Leffler function. They have investigated various forms and applications of generalization of Mittag-Leffler function $E_{\alpha}(z)$. The solution of kinetic equation and Voltra type integral equations have been introduced in term of Mittag-Leffler functions (Haubold et. al. [3], Mathai et. al.[6]). Some useful definitions, which are based on main results being given below as

1.1 Definitions

1. **Definitions1** In 1905, A. Wiman [11] has investigated generalization of $E_{\alpha}(u)$ with two parameters and defined by

(1.2) $$E_{\alpha,\beta}\left(u\right)=\sum_{n=0}^{\infty}\frac{u^{n}}{\Gamma\left(n\alpha+\beta\right)}\qquad ;\alpha,\beta\in C,\qquad \min\left(\mathrm{Re}\left(\alpha\right),\mathrm{Re}\left(\beta\right)\right)>0$$

2. **Definitionsn2** In 1971, Prabhakar [7] has introduced generalized Mittag Leffler function with three parameters as

(1.3) $$E_{\alpha,\beta}^{\gamma}\left(u\right)=\sum_{n=0}^{\infty}\frac{\left(\gamma\right)_{n}}{\Gamma\left(n\alpha+\beta\right)}\frac{u^{n}}{n!};\quad \alpha,\beta,\gamma\in C,\quad \min\left(\mathrm{Re}\left(\alpha\right),\mathrm{Re}\left(\beta\right),\mathrm{Re}\left(\gamma\right)\right)>0$$

3. **Definitions3** In 2007, Shukla and Prajapati [9] have introduced generalized Mittag Leffler function of with four parameters as

$$E_{\alpha,\beta}^{\gamma,k}\left(u\right)=\sum_{n=0}^{\infty}\frac{\left(\gamma\right)_{nk}}{\Gamma\left(n\alpha+\beta\right)}\frac{u^{n}}{n!};\alpha,\beta,\gamma,k\in C,$$

(1.4) $$\min\left(\mathrm{Re}\left(\alpha\right),\mathrm{Re}\left(\beta\right),\mathrm{Re}\left(\gamma\right)\right)>0$$

4. **Definitionsn4** In 2009, Salim [8] has introduced new generalized Mittag Leffler function with four parameters as

$$E_{\alpha,\beta}^{\gamma,\delta}\left(u\right)=\sum_{n=0}^{\infty}\frac{\left(\gamma\right)_{n}}{\Gamma\left(n\alpha+\beta\right)}\frac{u^{n}}{\left(\delta\right)_{n}};\alpha,\beta,\gamma,\delta\in C,$$

(1.5) $$\min\left(\mathrm{Re}\left(\alpha\right),\mathrm{Re}\left(\beta\right),\mathrm{Re}\left(\gamma\right)\right)>0$$

1.2 The k-Fractional Integral of Riemann Liouville Type

In 2012, Mubeen et. al. [5] have. λ be a real number such that $0<\lambda\leq 1, k>0$ then k-Riemann-Liouville fractional integral, k-Riemann-Liouville fractional singular kernel and k-Riemann-Liouville fractional derivative are given respectively as (Romero et. al. [10]) introduced k-fractional integral of Riemann Liouville type

(1.6) $$I_{k}^{\lambda}f\left(u\right)=\frac{1}{k\Gamma_{k}\left(\lambda\right)}\int_{0}^{u}\left(u-\xi\right)^{\frac{\lambda}{k}-1}f\left(\xi\right)d\xi,\qquad ;\lambda>0,t>0,k>0$$

Where $\Gamma_{k}\left(z\right)=\int\limits_{0}^{\infty}x^{z-1}e^{-\frac{x^{k}}{k}}dx=$ (Mubeen et. al. [5])

(1.7) $$j_{\lambda,k}f\left(u\right)=\frac{u^{\frac{\lambda}{k}-1}}{k\Gamma_{k}\left(\lambda\right)};t>0$$

(1.8) $$D_k^{\lambda} f(u) = \frac{d}{dt} I_k^{1-\lambda} f(u)$$

Equation (1.6) is written in term of (1.7) as

(1.9) $$I_k^{\lambda} f(u) = j_{\lambda,k}(u) * f(u)$$

Laplace transform of equation (1.7)

$$L\{j_{\lambda,k} f(u)\} = L\left\{\frac{u^{\frac{\lambda}{k}-1}}{k\Gamma_k(\lambda)}\right\}$$

$$\frac{1}{k\Gamma_k(\lambda)} \int_0^{\infty} e^{-su} u^{\frac{\lambda}{k}-1} du$$

$$= \frac{1}{k\Gamma_k(\lambda)} \frac{\Gamma\left(\frac{\lambda}{k}\right)}{s^{\frac{\lambda}{k}}}$$

(1.10) $$= \frac{1}{k k^{\frac{\lambda}{k}-1} \Gamma\left(\frac{\lambda}{k}\right)} \frac{\Gamma\left(\frac{\lambda}{k}\right)}{s^{\frac{\lambda}{k}}} = \frac{1}{(sk)^{\frac{\lambda}{k}}}$$

Using equation (1.9), Laplace transform of equation (1.6) is given as

$$L\left\{I_k^{\lambda} f(u); s\right\} = L\left\{j_{\lambda,k}(t) * f(u); s\right\}$$

$$= L\left\{j_{\lambda,k}(u)\right\} L\left\{f(u)\right\}$$

Using equation (1.10)

(1.11) $$= \frac{L(f(u))}{(sk)^{\frac{\lambda}{k}}}$$

2 Main Results

Theorem 1. *For $\beta > 0, k > 0$ and $0 < t < \infty$ then following results hold true*

(2.1) $$(i) L\left\{I_k^{\beta} E_{\alpha}(-\mu t^{\alpha}); p\right\} = \frac{1}{(pk)^{\frac{\beta}{k}}} \frac{p^{\alpha-1}}{p^{\alpha} + \mu}$$

$$(2.2) \qquad (ii)\int_0^\infty e^{-t} I_k^\beta E_\alpha\left(-\mu t^\alpha\right) dt = \frac{(k)^{-\frac{\beta}{k}}}{1+\mu}$$

Where L denotes the Laplace transform operator ***Proof:*** *Using equation (1.11)*

$$L\left\{I_k^\beta E_\alpha\left(-\mu t^\alpha\right);p\right\} = \frac{1}{(pk)^{\frac{\beta}{k}}} L\left[E_\alpha\left(-\mu t^\alpha\right)\right]$$

$$(2.3) \qquad \because L\left[E_\alpha\left(-\mu t^\alpha\right)\right] = \frac{p^{\alpha-1}}{p^\alpha+\mu} \qquad (Mathai\ et.\ al.[6])$$

Using equation (2.3)

$$L\left\{I_k^\beta E_\alpha\left(-\mu t^\alpha\right)\right\} = \frac{1}{(pk)^{\frac{\beta}{k}}}\frac{p^{\alpha-1}}{p^\alpha+\mu}$$

put $p = 1$ in equation (2.1)

$$\int_0^\infty e^{-t} I_k^\beta E_\alpha\left(-\mu t^\alpha\right) dt = \frac{(k)^{-\frac{\beta}{k}}}{1+\mu}$$

Theorem 2. *For $\beta > 0, k > 0$ and $0 < t < \infty$ then following results hold true*

$$(2.4) \qquad (i)L\left\{I_k^\beta t^{\delta-1} E_{\alpha,\delta,}\left(-\mu t^\alpha\right);p\right\} = \frac{1}{(pk)^{\frac{\beta}{k}}}\frac{p^{\alpha-\delta}}{p^\alpha+\mu}$$

$$(2.5) \qquad (ii)\int_0^\infty e^{-t} t^{\delta-1} I_k^\beta E_{\alpha,\delta}\left(-\mu t^\alpha\right) dt = \int_0^\infty e^{-t} I_k^\beta E_\alpha\left(-\mu t^\alpha\right) dt$$

Where L denotes the Laplace transform operator
Proof: *Using equation (1.11)*

$$L\left\{I_k^\beta t^{\delta-1} E_{\alpha,\delta}\left(-\mu t^\alpha\right);p\right\} = \frac{1}{(pk)^{\frac{\beta}{k}}} L\left[t^{\delta-1} E_{\alpha,\delta}\left(-\mu t^\alpha\right)\right]$$

$$(2.6) \qquad \because L\left[t^{\delta-1} E_{\alpha,\delta}\left(-\mu t^\alpha\right)\right] = \frac{p^{\alpha-\delta}}{p^\alpha+\mu} \qquad (Mathai\ et.\ al.[6])$$

By equation (2.6)

$$L\left\{I_k^\beta t^{\delta-1} E_{\alpha,\delta}\left(-\mu t^\alpha\right)\right\} = \frac{1}{(pk)^{\frac{\beta}{k}}}\frac{p^{\alpha-\delta}}{p^\alpha+\mu}$$

put $p=1$ in equation (2.4)

$$\int_{0}^{\infty} e^{-t} I_k^{\beta} t^{\delta-1} E_{\alpha}\left(-\mu t^{\alpha}\right) dt = \frac{(k)^{-\frac{\beta}{k}}}{1+\mu} \tag{2.7}$$

By equations (2.2) and (2.5)

$$\int_{0}^{\infty} e^{-t} t^{\delta-1} I_k^{\beta} E_{\alpha,\delta}\left(-\mu t^{\alpha}\right) dt = \int_{0}^{\infty} e^{-t} I_k^{\beta} E_{\alpha}\left(-\mu t^{\alpha}\right) dt$$

Theorem 3. *For $\beta > 0, k > 0$ and $0 < t < \infty$ then following results hold true*

$$(i) L\left\{ I_k^{\beta} t^{\delta-1} E_{\alpha,\delta}^{\gamma}\left(-\mu t^{\alpha}\right); p \right\} = \frac{1}{(pk)^{\frac{\beta}{k}}} \frac{p^{\alpha\gamma-\delta}}{\left(p^{\alpha}+\mu\right)^{\gamma}} \tag{2.8}$$

$$(ii) \int_{0}^{\infty} e^{-t} I_k^{\beta} t^{\delta-1} E_{\alpha,\delta}^{\gamma}\left(-t^{\alpha}\right) dt = 2^{-\gamma} k^{-\frac{\beta}{k}} \tag{2.9}$$

Where L denotes the Laplace transform operator

Proof: $L\left\{ I_k^{\beta} t^{\delta-1} E_{\alpha,\delta}^{\gamma}\left(-\mu t^{\alpha}\right); p \right\} = \frac{1}{(pk)^{\frac{\beta}{k}}} L\left[t^{\delta-1} E_{\alpha,\delta}^{\gamma}\left(-\mu t^{\alpha}\right) \right]$

$$\because L\left[t^{\delta-1} E_{\alpha,\delta}^{\gamma}\left(-\mu t^{\alpha}\right) \right] = \frac{p^{\alpha\gamma-\delta}}{\left(p^{\alpha}+\mu\right)^{\gamma}} \qquad (Mathai\ et.\ al. [6]) \tag{2.10}$$

Using equation (2.10)

$$L\left\{ I_k^{\beta} t^{\delta-1} E_{\alpha,\delta}^{\gamma}\left(-\mu t^{\alpha}\right) \right\} = \frac{1}{(pk)^{\frac{\beta}{k}}} \frac{p^{\alpha\gamma-\delta}}{\left(p^{\alpha}+\mu\right)^{\gamma}}$$

put $p=1$ and $\mu=1$ in equation (2.8)

$$\int_{0}^{\infty} e^{-t} I_k^{\beta} t^{\delta-1} E_{\alpha,\delta}^{\gamma}\left(-t^{\alpha}\right) dt = k^{-\frac{\beta}{k}} (1+\mu)^{-\gamma}$$

Theorem 4. *For $\beta > 0, k > 0$ and $0 < t < \infty$ then following results hold true*

$$(i) L\left\{ I_k^{\beta} t^{\alpha r+\delta-1} \frac{d^r}{dt^r} E_{\alpha,\delta}\left(\mu t^{\alpha}\right); p \right\} = \frac{1}{(pk)^{\frac{\beta}{k}}} \frac{r! p^{\alpha-\delta}}{\left(p^{\alpha}-\mu\right)^{r+1}} \tag{2.11}$$

$$\int_0^\infty e^{-at} I_k^\beta \frac{d^r}{dt^r} E_{\alpha,\delta}(t^\alpha)\,dt = \frac{1}{(ak)^{\frac{\beta}{k}}} \frac{r!a^{\alpha-\delta}}{(a^\alpha-1)^{r+1}} \tag{2.12}$$

Where L denotes the Laplace transform operator

Proof: $L\left\{I_k^\beta t^{\alpha\gamma+\delta-1} \frac{d^r}{dt^r} E_{\alpha,\delta}(-\mu t^\alpha);p\right\}$

$$= \frac{1}{(pk)^{\frac{\beta}{k}}} L\left[t^{\alpha\gamma+\delta-1} \frac{d^r}{dt^r} E_{\alpha,\delta}^\gamma(-\mu t^\alpha)\right]$$

$$L\left[t^{\alpha\gamma+\delta-1} \frac{d^r}{dt^r} E_{\alpha,\delta}(-\mu t^\alpha)\right] = \frac{r!p^{\alpha-\delta}}{(p^\alpha-\mu)^{r+1}} \qquad (Mathai\ et.\ al.\,[6]) \tag{2.13}$$

Using equation (2.13)

$$L\left\{I_k^\beta t^{\alpha r+\delta-1} \frac{d^r}{dt^r} E_{\alpha,\delta}(\mu t^\alpha)\right\} = \frac{1}{(pk)^{\frac{\beta}{k}}} \frac{r!p^{\alpha-\delta}}{(p^\alpha-\mu)^{r+1}}$$

put $p=a$ *and* $\mu=1$ *in equation (2.11)*

$$\int_0^\infty e^{-at} I_k^\beta \frac{d^r}{dt^r} E_{\alpha,\delta}(t^\alpha)\,dt = \frac{1}{(ak)^{\frac{\beta}{k}}} \frac{r!a^{\alpha-\delta}}{(a^\alpha-1)^{r+1}}$$

Theorem 5. *For* $\beta>0, k>0$ *and* $0<t<\infty$ *then following results hold true*

$$(i)\ M\left[I_k^\beta E_{\alpha,\lambda}^\gamma(-\rho t)\right] = \frac{B_k(\beta,1-\beta-pk)}{\Gamma_k(\beta)} \frac{\Gamma\left(p+\frac{\beta}{k}\right)\Gamma\left(\gamma-p-\frac{\beta}{k}\right)}{\Gamma(\gamma)\Gamma\left(\lambda-\left(p\alpha+\frac{\beta\alpha}{k}\right)\right)}\rho^{-p} \tag{2.14}$$

$$(ii)\ \int_0^\infty I_k^\beta E_{\alpha,\lambda}^\gamma(-\rho t)\,dt = \frac{B_k(\beta,1-\beta-k)}{\Gamma_k(\beta)} \frac{\Gamma\left(\gamma-\frac{\beta}{k}\right)}{\Gamma(\gamma)\Gamma\left(\lambda-\alpha-\frac{\alpha\beta}{k}\right)}\rho^{-1} \tag{2.15}$$

Where M denotes the Mellin transform operator and

$$B_k(l_1,l_2) = \frac{\Gamma_k(l_1)\Gamma_k(l_2)}{\Gamma_k(l_1+l_2)} = \frac{1}{k}\int_0^1 \xi^{\frac{l_1}{k}-1}(1-\xi)^{\frac{l_2}{k}-1}d\xi \tag{2.16}$$

Proof: *Millen transform of* $I_k^\beta f(t)$ *as*

$$M\left\{I_k^\beta f(t);p\right\} = \frac{1}{k\Gamma_k(\beta)} M\left\{\int_0^t (t-z)^{\frac{\beta}{k}-1} f(z)\,dz\right\}$$

$$= \frac{1}{k\Gamma_k(\beta)} \int_0^\infty \int_0^t t^{p-1}(t-z)^{\frac{\beta}{k}-1} f(z)\, dz dt$$

Change the order of double integration

$$M\left\{I_k^\beta f(t);p\right\} = \frac{1}{k\Gamma_k(\beta)} \int_0^\infty f(z)\, dz \int_t^\infty t^{p-1}(t-z)^{\frac{\beta}{k}-1} dt \text{ put } t = \frac{z}{u}$$

$$= \frac{1}{k\Gamma_k(\beta)} \int_0^\infty f(z)\, dz \int_0^1 \left(\frac{z}{u}\right)^{p-1}\left(\frac{z}{u}-z\right)^{\frac{\beta}{k}-1} \frac{du}{u^2}$$

$$= \frac{1}{k\Gamma_k(\beta)} \int_0^\infty f(z)\, dz \int_0^1 z^{p+\frac{\beta}{k}-1} u^{-\frac{\beta}{k}-p+2}(1-u)^{\frac{\beta}{k}-1} \frac{du}{u^2}$$

$$= \frac{1}{k\Gamma_k(\beta)} \int_0^\infty z^{\frac{\beta}{k}+p-1} f(z)\, dz \int_0^1 u^{-\frac{\beta}{k}-p}(1-u)^{\frac{\beta}{k}-1} du$$

$$= \frac{1}{\Gamma_k(\beta)} \int_0^\infty z^{\frac{\beta}{k}+p-1} f(z)\, dz B_k(\beta, 1-\beta-pk)$$

$$= \frac{B_k(\beta.1-\beta-pk)}{\Gamma_k(\beta)} \int_0^\infty z^{\frac{\beta}{k}+p-1} f(z)\, dz$$

$$(2.17) \qquad \Rightarrow M\left\{I_k^\beta f(t);p\right\} = \frac{B_k(\beta.1-\beta-pk)}{\Gamma_k(\beta)} M\left\{f(t);p+\frac{\beta}{k}\right\}$$

Using equation (2.17), L.H.S of equation (2.14) will be become

$$M\left[I_k^\beta E_{\alpha,\lambda}^\gamma(-\rho t)\right] = \frac{B_k(\beta, 1-\beta-pk)}{\Gamma_k(\beta)} M\left[E_{\alpha,\lambda}^\gamma(-\rho t);p+\frac{\beta}{k}\right]$$

$$(2.18) \qquad \because M\left[E_{\alpha,\lambda}^\gamma(-\rho t)\right] = \frac{\Gamma(p)\Gamma(\gamma-p)}{\Gamma(\gamma)\Gamma(\lambda-p\alpha)} \rho^{-p} \left(Mathai\ et.\ al.[6]\right)$$

$$\Rightarrow M\left[I_k^{\beta}E_{\alpha,\lambda}^{\gamma}\left(-\rho t\right)\right]=\frac{B_k\left(\beta,1-\beta-pk\right)}{\Gamma_k\left(\beta\right)}\frac{\Gamma\left(p+\frac{\beta}{k}\right)\Gamma\left(\gamma-p-\frac{\beta}{k}\right)}{\Gamma\left(\gamma\right)\Gamma\left(\lambda-\left(p\alpha+\frac{\beta\alpha}{k}\right)\right)}\rho^{-p}$$

$$\Rightarrow\int_0^{\infty}t^{p-1}I_k^{\beta}E_{\alpha,\lambda}^{\gamma}\left(-\rho t\right)dt=\frac{B_k\left(\beta,1-\beta-pk\right)}{\Gamma_k\left(\beta\right)}\frac{\Gamma\left(p+\frac{\beta}{k}\right)\Gamma\left(\gamma-p-\frac{\beta}{k}\right)}{\Gamma\left(\gamma\right)\Gamma\left(\lambda-p\alpha-\frac{\alpha\beta}{k}\right)}\rho^{-p}$$

put $p=1$ in equation (2.14)

$$\int_0^{\infty}I_k^{\beta}E_{\alpha,\lambda}^{\gamma}\left(-\rho t\right)dt=\frac{B_k\left(\beta,1-\beta-k\right)}{\Gamma_k\left(\beta\right)}\frac{\Gamma\left(\gamma-\frac{\beta}{k}\right)}{\Gamma\left(\gamma\right)\Gamma\left(\lambda-\alpha-\frac{\alpha\beta}{k}\right)}\rho^{-1}$$

Corollary 1. *For $\beta>0,k>0$ and $0<t<\infty$ then following formulae hold true*

$(i)M\left[I_k^{\beta}E_{\alpha,\lambda}^{\gamma,\delta}\left(-\rho t\right);p+\frac{\beta}{k}\right]$

$$(2.19)\quad\frac{B_k\left(\beta,1-\beta-pk\right)}{\Gamma_k\left(\beta\right)}\frac{\Gamma\left(\delta\right)\Gamma\left(p+\frac{\beta}{k}\right)\Gamma\left(1-p-\frac{\beta}{k}\right)\Gamma\left(\gamma-\frac{\beta}{k}\right)}{\Gamma\left(\gamma\right)\Gamma\left(\lambda-p\alpha-\frac{\alpha\beta}{k}\right)\Gamma\left(\delta-p-\frac{\beta}{k}\right)}\rho^{-\left(p+\frac{\beta}{k}\right)}$$

$$(2.20)\quad(ii)\int_0^{\infty}t^{-1}I_k^{\beta}E_{\alpha,\lambda}^{\gamma,\delta}\left(-\rho t\right)dt=\frac{\pi\cos ec\left(\frac{\beta}{k}\right)\rho^{-\frac{\beta}{k}}k^{-\frac{\beta}{k}}\Gamma\left(\frac{1-\beta}{k}\right)\Gamma\left(\delta\right)}{\Gamma\left(\frac{1}{k}\right)\Gamma\left(\gamma\right)\Gamma\left(\lambda-\frac{\alpha\beta}{k}\right)\Gamma\left(\delta-\frac{\beta}{k}\right)}$$

Where M denotes the Mellin transform operator

3 References

[1] Bapna I.B. and Jain N. (2008), *Application of modified fractional calculus applied in statistical distribution*, Math. Sci. res. J., vol. 12(11), pp. 258-269

[2] Bapna, I.B. and Mathur, N., (2012),*application of fractional calculus in statistics*, International Journal Contemporary Mathematical science, vol. 7, no. 18, pp.849- 856

[3] Haubold, H., J., Mathai M. A. and Saxena, R.K. (2011),*Mittag Leffler Functions and their Applications*, Hindawi Publishing Corporation Journal of applied mathematics vol. 2011 Article ID298628, 51 pages

[4] Mittag-Leffler,S.G.(1903) *Sur la function*,C.R Acad. Sci. Paris,No.137 pp 554-558

[5] Mubeen S., and Habibullah G.,M.(2012), *k- fractional integral and applications*, International Journal Contemporary Mathematical Science, vol. 7, pp.89-94

[6] Mathai , A., and Haubold, H.(2008), *Special function for applied scientists*, Springer

[7] Prabhakar, T.R.(1971), *A singular integral equation with a generalized Mittag-Leffler function in the kernel*, Yokohama Math. J., No. 19, pp.7–15

[8] Salim T.O., (2009), *Some properties relating to the generalized Mittag-Leffler function.* Adv Appl Math Anal 4: 21–30

[9] Shukla, A.K. and Prajapati, J.C.(2007), *On a generalization of Mittag-Leffler function and its properties*, J. Math. Anal. Appl. No 336 (2007), 797-811.

[10] Romero,L.G, Luciano L. Luque, L.,Dorrego G.A., and Cerutti ,R.A,(2013), *On the k-Riemann-Liouville Fractional Derivative*, Int. J. Contemp. Math. Sciences,Vol. 8, no. 1, 41 - 51,

[11] Wiman, A. *Uber den fundamental satz in der theory functionen*, Acta Math.,No.29, pp 191- 201