Mathematical Modelling on the Effect of Chemical Reaction on the Air Pollutant under Mesoscale Wind

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Abstract

A steady state two dimensional model is proposed to study the effect of chemical reaction on the air pollutant under the large-scale and mesoscale wind so generated by the urban heat island effect. The chemical reaction so considered is not constant but vary with height and follow power law profile. Also, the large-scale wind and diffusivity is considered to be the function of height and follows power law profile. In this paper,we have considered an inter-conversion process of primary to secondary pollutant in the presence of variable chemical reaction under the presence of large-scale and mesoscale wind.The result revealed that the urban heat island effect aggravate the concentration of pollutant. During the analysis it is found that the variable chemical reaction makes the concentration to decrease along downwind distance as well as along vertical height. Further in the process of inter-conversion, the variable chemical reaction and variable removal parameter makes the concentration of primary and secondary pollutant to decrease but their existence remain upto a significant level in the domain of urban heat island (UHI).

Subject class: 93A30, 97Mxx.
Keywords: Urban heat island effect, Mesoscale wind, Chemical reaction, Primary pollutant, Secondary pollutant.

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1 Introduction:

Air pollution is one of the major environmental problem causing different diseases especially respiratory diseases in human beings. The matter of air pollution becomes a global concern involving ambient air quality by adding different air pollutant in the towns and cities across the world. To study such problems, mathematical modelling plays an important role through which one can understand the dispersion of air pollutant as well as the influence of individual parameter on pollutant concentration.

In this regard Gaussian model is widely used (Seinfeld, 1986) to study the dispersion of pollutant. Hana et. al. (1982) analytically solved the governing equation in the Gaussian model while considering eddy diffusivity and wind velocity to be constant. Several studies made by Hinrichsen (1986) and Lin & Hildemann (1996, 1997) suggest that under certain conditions, it is difficult to define constant wind profile and diffusivity.

In previous years, the effect of urban heat islands on the dispersion of pollutant has been investigated by the use of mathematical models. Griffith (1970) pointed out that the knowledge of large-scale wind is not only sufficient for air pollution forecast in urban area but mesoscale wind play an important role in shaping urban air pollution pattern.

A considerable attention has been received during past years to understand the effect of mesoscale wind like Chander (1968), Findlay & Hirt (1969), Dilley and Yen (1971), Agarwal and Tandon (2010). Chander (1968) found that due to urban heat island effect, the wind so produced sharpens the pollution gradient between urban and rural area. Findley and Hirt (1969) measured the surface inflow of an urban induced meso-circulation. Dilley and Yen (1971) studied the effect of mesoscale on the pollutant distribution from an infinite line source without a removal parameter. Agarwal and Tandon (2010) presented a two dimensional mathematical model under mesoscale wind where pollutants are emitted from an area source, the solution was obtained by considering the numerical approach under neutral, stable and unstable conditions. These papers so considered above either do not consider removal parameter or take removal parameter as constant.

In parallel to above such studies, an analytical investigation on the effect of first order chemical reaction on the air pollutant has been carried out by Heines & Peters (1973). Shukla & Chauhan (1991) studied the removal of air pollutant by rain. The other aspect of the chemical reaction in the form of inter-conversion of primary to secondary pollutant has been studied by Shukla and Chauhan (1988) where dispersion of pollutant from a time dependent point source forming a secondary pollutant has been investigated by taking into account the dry and wet deposition. Agarwal & Joseph (2016) studied the inter-conversion of primary to secondary pollutant in an urban dome where the diffusion equation is considered in cylindrical polar co-ordinates.Recently, Agarwal & Joseph (2019)proposed a model on the formation of secondary pollutant due to the chemical reaction of two primary pollutants emitted from two different point sources under variable wind and diffusivity profile. Naresh & Nath (1991) studied the effect of chemical reaction on the dispersion of air pollutant over area source where they considered the variation of chemical reaction with height.

In view of the above studies, it was found that the variation of chemical reaction with height is not considered under mesoscale wind. Thus, in this paper, we formulate a model where we shall considered that pollutant is released in the atmosphere at steady rate from a ground based point source. In the desired region it is assumed that along with large-scale wind, mesoscale wind also exist due to urban heat island effect. The large-scale wind is considered along the downwind direction and taken as function of height which follows power law profile. Further we assumed that in the specified region of consideration the variation of chemical reaction with height exist. For simplicity of the problem we neglect the crosswind advection and diffusion term. The diffusivity is considered along the vertical height and considered as function of height which also follows power law profile. Beside this a deposition of pollutant on the ground is considered. To understand the impact of the presence of mesoscale wind we investigate the effect of chemical reaction on the pollutant in the presence and absence of the mesoscale wind.

Another form of chemical reaction as inter-conversion of primary pollutant to secondary pollutant is considered under the presence of large-scale and mesoscale wind where we analyse the impact of variable reaction on the pollutants in the domain of urban heat island.

2 Mathematical formulation:

Consider a steady state dispersion of a point source situated at the origin. The diffusion equation for the concentration of pollutant can be written as

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{\partial}{\partial x}\left(K_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right)+R+S \tag{2.1}$$

where C is a concentration of pollutant. R represents the removal term and S is a source term. u, v, w represent the wind components. K_x, K_y, K_z are the diffusivity coefficient along x, y, z direction respectively. R represents the rate of chemical reaction/removal parameter and S represent the source term.

The pollutant is considered to be transported by the large-scale wind along the downwind direction only as well as by the mesoscale wind. The large scale wind is considered to be the function of z and follow power law profile. The mesoscale wind is considered to be produced by the urban heat island effect.

The urban heat island effect causes a rising of air above the centre of the island. In such situation a thermally induced convective current is set up. To understand mesoscale wind so formed under heat island effect we follow a reverse stagnation point flow for which

$$u_e \propto -x \qquad \text{and} \qquad w_e \propto z \ \ \text{(Dilley and Yen, 1971)} \tag{2.2}$$

The above assumption provides a surface influx. Further u$_e$ and w$_e$ are horizontal and vertical mesoscale wind components and can be written as

$$u_e=-ax\left(\frac{z}{H}\right)^m \qquad \text{and} \qquad w_e=\frac{az}{(m+1)}\left(\frac{z}{H}\right)^m \ \ \text{(Dilley and Yen, 1971)} \tag{2.3}$$

where a is the proportionality constant and $m \geq 0$. H represent the vertical height.

The large-scale wind and vertical diffusivity are assumed to vary with height and taken as power law profile.

$$u(z)=u_1\left(\frac{z}{H}\right)^n \qquad \text{and} \qquad K_z=K_H\left(\frac{z}{H}\right)^n$$

where u_1 and K_H are the wind speed and diffusivity respectively at a reference height H and $n \geq 0$.

Also, it is assumed that the downwind diffusion is neglected in comparison to advection. For the simplicity of the problem we eliminate the advection term and diffusion term along y direction. With these assumptions the equation (2.1) becomes

$$(u_1-ax)\left(\frac{z}{H}\right)^m\frac{\partial C}{\partial x}+\frac{az}{(m+1)}\left(\frac{z}{H}\right)^m\frac{\partial C}{\partial z}=\frac{\partial}{\partial z}\left[K_H\left(\frac{z}{H}\right)^n\frac{\partial C}{\partial z}\right]-\alpha C \tag{2.4}$$

The rate of chemical reaction α is considered to be function of vertical height and follow the power law profile. Hence the removal parameter α is represented as

$$\alpha=\alpha_1\left(\frac{z}{H}\right)^q \ \ \text{where} \ \ q \geq 0 \ \ \text{and} \ \ \alpha_1 \ \ \text{is constant.}$$

Therefore equation (2.4) becomes

$$(u_1 - ax)\left(\frac{z}{H}\right)^m \frac{\partial C}{\partial x} + \frac{az}{(m+1)}\left(\frac{z}{H}\right)^m \frac{\partial C}{\partial z} = \frac{\partial}{\partial z}\left[K_H\left(\frac{z}{H}\right)^n \frac{\partial C}{\partial z}\right] - \alpha_1\left(\frac{z}{H}\right)^q C \tag{2.5}$$

For convenience we take $m = n = q = 1/2$. Hence equation (2.5) becomes

$$(u_1 - ax)\left(\frac{z}{H}\right)^{1/2} \frac{\partial C}{\partial x} + \frac{2}{3}az\left(\frac{z}{H}\right)^{1/2} \frac{\partial C}{\partial z} = \frac{\partial}{\partial z}\left[K_H\left(\frac{z}{H}\right)^{1/2} \frac{\partial C}{\partial z}\right] - \alpha_1\left(\frac{z}{H}\right)^{1/2} C \tag{2.6}$$

The pollutantis assumed to be emitted at steady rate from a point source at the origin and hence mathematically expressed as

$$\frac{Q\delta(z)}{u} = C \text{ at } x = 0 \tag{2.7}$$

where Q is the emission rate of the pollutant and $\delta(z)$ represent the Dirac Delta Function.

The pollutant which are emitted through steady rate into the atmosphere by a point source is considered to be removed from the atmosphere by the ground absorption and mathematically expressed as

$$K_z\frac{\partial C}{\partial z} = V_d C \text{ at } z = 0 \tag{2.8}$$

where V_d represent the deposition velocity of the pollutant. The pollutant is confined within the inversion layer H and is not able to penetrate through it so we have

$$K_z\frac{\partial C}{\partial z} = 0 \text{ at } z = H \tag{2.9}$$

Special Case I: In absence of mesoscale wind the two dimensional partial differential equation governing the dispersion of pollutant is represented as

$$u_1\left(\frac{z}{H}\right)^{\frac{1}{2}} \frac{\partial C}{\partial x} = \frac{\partial}{\partial z}\left[K_H\left(\frac{z}{H}\right)^{\frac{1}{2}} \frac{\partial C}{\partial z}\right] - \alpha_1\left(\frac{z}{H}\right)^{\frac{1}{2}} C \tag{2.10}$$

The boundary conditions associated with above equations are same as (2.7) to (2.9)

Special Case II: In case of inter-conversion of primary to secondary pollutant due to the chemical reaction in presence of large-scale as well as mesoscale wind, it is assumed that the rate of conversion as well as removal parameter is height dependent. Thus in such conditions, the dispersion of primary pollutant is represented as

$$(u_1 - ax)\left(\frac{z}{H}\right)^{\frac{1}{2}} \frac{\partial C_1}{\partial x} + \frac{2}{3}az\left(\frac{z}{H}\right)^{\frac{1}{2}} \frac{\partial C_1}{\partial z} = \frac{\partial}{\partial z}\left[K_H\left(\frac{z}{H}\right)^{\frac{1}{2}} \frac{\partial C_1}{\partial z}\right] - (\alpha_1 + \alpha_2)\left(\frac{z}{H}\right)^{\frac{1}{2}} C_1 \tag{2.11}$$

where $\alpha = \alpha_1 \left(\frac{z}{H}\right)^{\frac{1}{2}}$ represent the rate of conversion and $\alpha = \alpha_2 \left(\frac{z}{H}\right)^{\frac{1}{2}}$ represent the removal rate of primary pollutant. C_1 represent the concentration of primary pollutant.

The boundary conditions are

$$\frac{Q\delta(z)}{u} = C_1 \qquad \text{at } x = 0 \tag{2.12}$$

$$K_z \frac{\partial C_1}{\partial z} = V_{d_1} C_1 \qquad \text{at } z = 0 \tag{2.13}$$

$$K_z \frac{\partial C_1}{\partial z} = 0 \qquad \text{at } z = H \tag{2.14}$$

where V_{d_1} represent the deposition velocity of the primary pollutant.

The dispersion of secondary pollutant is represented as

$$\begin{aligned}(u_1 - ax)\left(\frac{z}{H}\right)^{\frac{1}{2}} \frac{\partial C_2}{\partial x} + \frac{2}{3} az \left(\frac{z}{H}\right)^{\frac{1}{2}} \frac{\partial C_2}{\partial z} &= \frac{\partial}{\partial z}\left[K_H \left(\frac{z}{H}\right)^{\frac{1}{2}} \frac{\partial C_2}{\partial z}\right] \\ &- \alpha_3 \left(\frac{z}{H}\right)^{\frac{1}{2}} C_2 + \alpha_1 \left(\frac{z}{H}\right)^{1/2} C_1 \end{aligned} \tag{2.15}$$

where $\alpha = \alpha_3 \left(\frac{z}{H}\right)^{\frac{1}{2}}$ represent the removal rate of secondary pollutant.C_2 represent the concentration of secondary pollutant.

The boundary conditions are

$$K_z \frac{\partial C_2}{\partial z} = V_{d_2} C_2 \qquad \text{at } z = 0 \tag{2.16}$$

$$K_z \frac{\partial C_2}{\partial z} = 0 \qquad \text{at } z = H \tag{2.17}$$

where V_{d_2} represent the deposition velocity of the secondary pollutant.

3 Solution:

Using the following dimensionless quantities to solve the equation (2.6), (2.10), (2.11) and (2.15)

$$\bar{x} = \frac{xK_H}{u_1 H^2},\ \bar{z} = \frac{z}{H},\ \bar{C} = \frac{u_1 H}{Q} C,\ \bar{\alpha} = \frac{H^2 \alpha}{K_H},\ \bar{C}_1 = \frac{u_1 H}{Q} C_1,\ \bar{C}_2 = \frac{u_1 H}{Q} C_2$$

Dropping bar for convenience the partial differential equation (2.6) becomes

$$(1-\beta x)\, z^{1/2}\frac{\partial C}{\partial x}+\frac{2}{3}z^{3/2}\beta\frac{\partial C}{\partial z}=\frac{1}{2}z^{-1/2}\frac{\partial C}{\partial z}+z^{1/2}\frac{\partial^2 C}{\partial z^2}-\alpha_1 z^{1/2}C \tag{3.1}$$

where $\beta=\frac{aH^2}{K_H}$ is a dimensionless quantity.

The solution of equation (3.1) along with the boundary conditions (2.7) to (2.9) is represented as

$$C(x,z)=\sum_{i=1}^{\infty}\frac{M_i(0)}{\int_0^1 z^{1/2}M_i^2 dz}\,(1-\beta x)^{\frac{p_i^2}{\beta}}\,M_i(z) \tag{3.2}$$

where

$$\begin{aligned} M_i\left(z\right) \;&=\; \left(1+\frac{\left(\alpha_1-p_i^2\right)}{3}z^2+\frac{2}{21}\left(\frac{\beta}{3}+\frac{\left(\alpha_1-p_i^2\right)}{4}\right)\left(\alpha-p_i^2\right)z^4+.....\right)\\ &+\; N_1\left(2z^{\frac{1}{2}}+\frac{2}{5}\left(\frac{\beta}{3}+\left(\alpha_1-p_i^2\right)\right)z^{\frac{5}{2}}+\frac{16}{45}\left(\frac{5\beta}{12}+\left(\alpha_1-p_i^2\right)\right)\right.\\ &\left.\left(\frac{\beta}{12}+\left(\alpha_1-p_i^2\right)\right)z^{\frac{9}{2}}+...\right) \end{aligned} \tag{3.3}$$

$$\text{and}\qquad N_1=\frac{V_d H}{K_H} \tag{3.4}$$

Special Case I: In absence of mesoscale wind the dimensionless concentration of pollutant is expressed as

$$C(x,z)=\sum_{i=1}^{\infty}\frac{M_i(0)}{\int_0^1 z^{1/2}M_i^2 dz}e^{-p_i^2 x}M_i(z) \tag{3.5}$$

where

$$\begin{aligned} M_i\left(z\right) \;&=\left\{\left(1+\frac{\left(\alpha_1-p_i^2\right)}{3}z^2+.0238\left(\alpha-p_i^2\right)^2z^4+.....\right)+N_1\left(2z^{\frac{1}{2}}+0.4(\alpha_1\right.\right.\\ &\left.\left.-p_i^2)z^{\frac{5}{2}}+.022\left(\alpha_1-p_i^2\right)^2z^{\frac{9}{2}}+......\right)\right\} \end{aligned} \tag{3.6}$$

Special Case II: In case of inter-conversion of primary pollutant to secondary pollutant due to chemical reaction, where we assume that rate of conversion as well as removal parameter are height dependent, we use theconcept of matrix form as Astarita et.al. (1979), Alam and Seinfeld (1981) to get uncoupled equation as

(3.7)

$$(u_1-ax)\left(\frac{z}{H}\right)^{\frac{1}{2}}\frac{\partial C_1}{\partial x}+\frac{2}{3}az\left(\frac{z}{H}\right)^{\frac{1}{2}}\frac{\partial C_1}{\partial z}=\frac{\partial}{\partial z}\left[K_H\left(\frac{z}{H}\right)^{\frac{1}{2}}\frac{\partial C_1}{\partial z}\right]-(\alpha_1+\alpha_2)\left(\frac{z}{H}\right)^{\frac{1}{2}}C_1$$

and
(3.8)

$$(u_1-ax)\left(\frac{z}{H}\right)^{\frac{1}{2}}\frac{\partial B_1}{\partial x}+\frac{2}{3}az\left(\frac{z}{H}\right)^{\frac{1}{2}}\frac{\partial B_1}{\partial z}=\frac{\partial}{\partial z}\left[K_H\left(\frac{z}{H}\right)^{\frac{1}{2}}\frac{\partial B_1}{\partial z}\right]-\alpha_3\left(\frac{z}{H}\right)^{\frac{1}{2}}B_1$$

$$\text{where}\qquad C_2(x,z)=B_1(x,z)-\frac{\alpha_1}{\alpha_1+\alpha_2-\alpha_3}C_1(x,z) \tag{3.9}$$

The boundary conditions for $B_1(x,z)$ are

$$B_1(x,z) = \frac{\alpha_1}{\alpha_1+\alpha_2-\alpha_3}\frac{Q\delta(z)}{u} \qquad \text{at } \ x=0 \tag{3.10}$$

$$K_z\frac{\partial B_1}{\partial z} = V_{d_2}B_1+\frac{\alpha_1}{\alpha_1+\alpha_2-\alpha_3}\left(V_{d_1}-V_{d_2}\right)C_1 \qquad \text{at } \ z=0 \tag{3.11}$$

$$K_z\frac{\partial B_1}{\partial z} = 0 \qquad \text{at } \ z=H \tag{3.12}$$

The solution of equation (3.7) as dimensionless concentration of primary pollutant is expressed as

$$C_1(x,z)=\sum_{i=1}^{\infty}\frac{N_i(0)}{\int_0^1 z^{\frac{1}{2}}N_i^2(z)\,dz}(1-\beta x)^{\frac{q_i^2}{\beta}}N_i(z) \tag{3.13}$$

where

$$\begin{aligned} N_i(z) \;&=\; \Bigg\{(1+\left(\frac{(\alpha_1+\alpha_2)-q_i^2}{3}\right)z^2+\frac{2}{21}\left(\frac{\beta}{3}+\left(\frac{(\alpha_1+\alpha_2)-q_i^2}{4}\right)\right)((\alpha_1+\alpha_2)\\ &\quad q_i^2)z^4+.....)+N_1\Bigg(2z^{\frac{1}{2}}+\frac{2}{5}\left(\frac{\beta}{3}+((\alpha_1+\alpha_2)-q_i^2)\right)z^{\frac{5}{2}}+\frac{16}{45}\Bigg(\frac{5\beta}{12}\\ &\quad +((\alpha_1+\alpha_2)-q_i^2))\left(\frac{\beta}{12}+((\alpha_1+\alpha_2)-q_i^2)\right)z^{\frac{9}{2}}+..\Bigg)\Bigg\} \end{aligned} \tag{3.14}$$

where $N_1=\frac{V_{d_1}H}{K_H}$

whereas the dimensionless concentration of secondary pollutant is expressed as

$$C_2(x,z)=B_1(x,z)-\frac{\alpha_1}{\alpha_1+\alpha_2-\alpha_3}C_1(x,z)\quad\text{where } \ B_1(x,z) \ \text{ is expressed as}$$

$$B_1(x,z)=\frac{\alpha_1}{\alpha_1+\alpha_2-\alpha_3}\sum_{k=1}^{\infty}\frac{P_k(0)}{\int_0^1 z^{\frac{1}{2}}P_k^2(z)\,dz}(1-\beta x)^{\frac{\eta_k^2}{\beta}}P_k(z) \tag{3.15}$$

where

$$\begin{aligned} P_k(z) \quad &= \left\{ \left(1 + \left(\frac{\alpha_3 - \eta_k^2}{3} \right) z^2 + \frac{2}{21} \left(\frac{\beta}{3} + \frac{(\alpha_3 - \eta_k^2)}{4} \right) \left(\alpha_3 - \eta_k^2 \right) z^4 + .. \right) \right. \\ &\quad + N_2 \left(2 z^{\frac{1}{2}} + \frac{2}{5} \left(\alpha_3 - \eta_k^2 \right) z^{\frac{5}{2}} + \frac{16}{45} \left(\frac{5\beta}{12} + \left(\alpha_3 - \eta_k^2 \right) \right) \right. \\ &\quad \left. \left. \left(\frac{\beta}{12} + \left(\alpha_3 - \eta_k^2 \right) \right) z^{\frac{9}{2}} + ... \right) \right\} \end{aligned} \tag{3.16}$$

where $N_2 = \frac{V_{d_2} H}{K_H}$

It may be noted that the solution (3.15) is obtained while considering that N_1 and N_2 are approximately equal.

4 Result Analysis:

The present study mainly focuses on the distribution of pollutant in the atmosphere in the presence of the large-scale wind as well as mesoscale wind. It is assumed that the centre of the heat island is located at the origin. As the chemical reaction is considered to vary with height, hence the investigation is made to understand the effect of such chemical reaction on the distribution of pollutant. Further in the special case we considered a comparative study on the effect of chemical reaction in the presence and absence of mesoscale wind, secondly we investigate another case of inter-conversion of primary to secondary pollutant in the presence of large-scale & mesoscale wind.We assumed that the pollutant is deposited on the ground with the deposition velocity V_d. As the expression of the concentration of pollutant in various cases is represented in the dimensionless form, so we assume the values of various parameters for computation, which are as follows $\alpha_1 = 20, \beta = 100, N_1 = .04$.

Under mesoscale wind the distribution of pollutant is considered to be along dimensionless downwind distance $0 \leq x \leq 0.01$ as well as along dimensionless vertical distance $0 \leq z \leq 1$.

The figure 1 and 2 exhibits the distribution of pollutant along vertical height as well as along downwind distance respectively. The dispersion of pollutant are considered under the effect of large-scale and mesoscale wind. The study reveals that the concentration of pollutant decreases with the increase of vertical height and tends to zero when $z = 0.8$. Further the concentration of pollutant decreases along downwind distance and tends to zero at $x = .008$. Also we found that the concentration of the pollutant exists upto significant level when the region is confined to $0 \leq x \leq .006$, $0 \leq z \leq 0.6$.In comparing the results of the concentration of pollutant as shown in figures 1 and 2 with the concentration of pollutant in the absence of mesoscale wind as shown in figure 3 and 4 it was found that the concentration of pollutant is higher when the mesoscale wind exist along with the large scale wind in the prescribed region. The reason is that the mesoscale wind traps the pollutant inside the domain and hence the concentration of pollutant exist upto significant level in the domain of urban heat island whereas in absence of mesoscale wind the pollutant is not trap in the domain and so disperse in the atmosphere and therefore the level of concentration of pollutant is less as compared to the mesoscale wind. Further in figure 4, the concentration of pollutant decreases with the increase of downwind distance but still existence at $x = 0.01$ in absence of mesoscale wind upto a significant level for $z \geq 0.6$.

While studying the inter-conversion of primary to secondary pollutant we considered the different rate of conversion as $\alpha_1 = 10, 20, 30$ and plotted the graphs in figure 5 and 6 for the concentration of primary and secondary pollutant respectively against vertical height at $x = 0.006$ in the presence of large-scale & mesoscale wind. The removal parameters for primary and secondary pollutant are taken as $\alpha_2 = \alpha_3 = 20$.The figure revealed that the concentration of primary pollutant decreases with the increase of vertical height and also with the increase of rate of conversion. Also the concentration of secondary pollutant increases with the increase of rate of conversion but decreases with the increase of downwind distance. While studying this it is noticed that the concentration of primary pollutant and secondary pollutant exist at a significant level at $z = 0.6$ and tends to zero at $z = 0.8$. In figure 7 and 8 the concentration of primary and secondary pollutant is plotted against downwind distance at $z = 0.4$ for different rate of conversion $\alpha_1 = 10, 20, 30$ in the presence of large-scale & mesoscale wind. In figure 7 the concentration of primary pollutant decreases with the increase of rate of conversion as well as with the increase of downwind distance, whereas the concentration of secondary pollutant in figure (8) increases with the increase of rate of conversion and also increases with the increase of downwind distance upto $x = 0.008$. This shows that the secondary pollutant persist upto greater downwind distance.

The figure 9 shows the concentration profile in a region where it can be seen that the concentration of pollutant decreases with the increase in the downwind distance and vertical height. In the figure, the concentration do exist upto significant level upto $z = 0.6$ and $x = 0.008$. Whereas in figure 10 the concentration profile is shown in absence of mesoscale wind which indicates that the concentration decreases with the increase in downwind distance and vertical height but still persist at $x = 0.1$ upto a significant level upto $z \geq 0.6$. Thus we can conclude that under the variable chemical reaction the concentration decreases along the dimensions x and z.

5 Conclusion:

A two-dimensional model is proposed to understand the distribution of pollutant in the atmosphere in the presence of large scale and mesoscale wind. It is assumed that the mesoscale wind is generated due to the urban heat island effect in the specified region. Further due to the variable chemical reaction and deposition velocity of the pollutant on the ground it becomes necessary to understand the behaviour of the concentration. The results so obtained showed that the mesoscale wind aggravate the concentration of pollutant. Also the concentration decreases with the increase in downwind direction and along vertical height as shown in the figure 9 and hence we may conclude that under variable chemical reaction the concentration decreases along all the dimensions. Further when inter-conversion process takes place under same circumstances then it is found that the concentration of primary pollutant decreases with increase of rate of conversion and concentration of secondary pollutant increases with the increase of rate of conversion, inspite of that the concentration of both pollutants exist upto significant level upto $z \geq 0.6$ and $x \geq 0.006$ in the specified domain of mesoscale wind.

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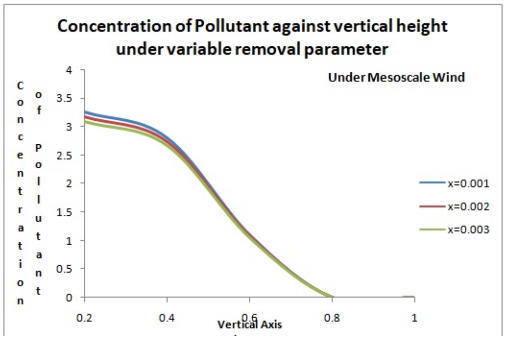

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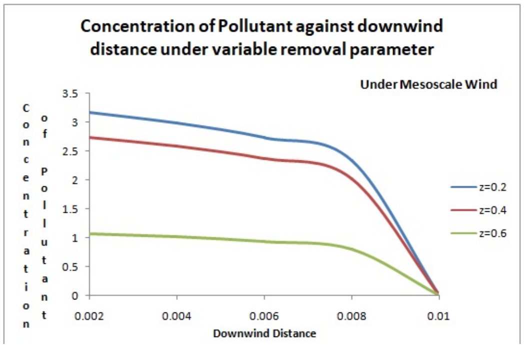

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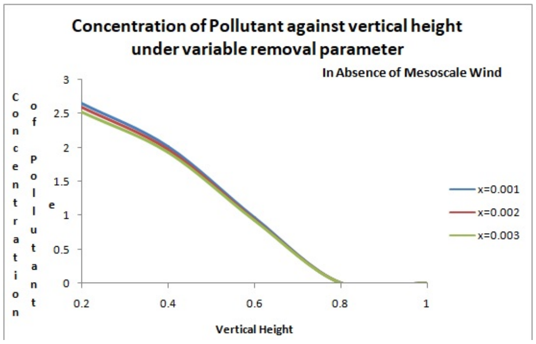

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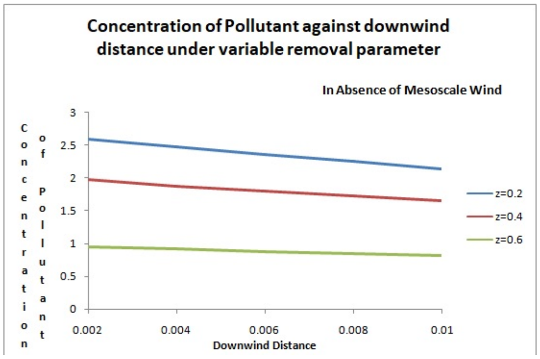

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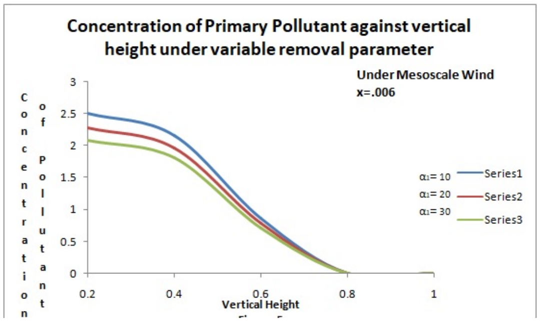

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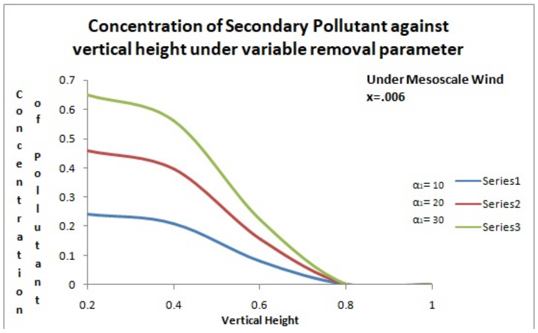

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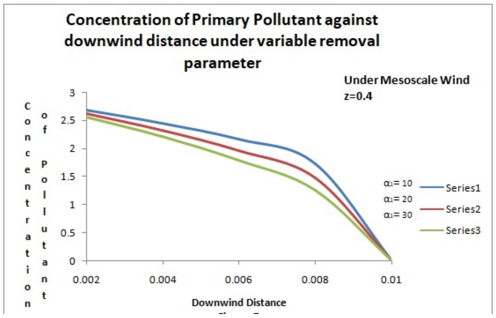

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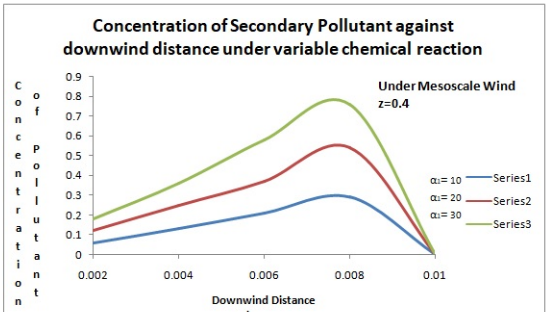

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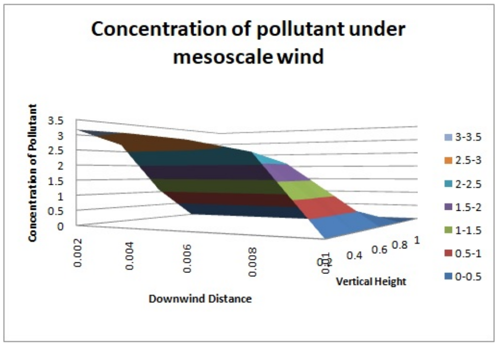

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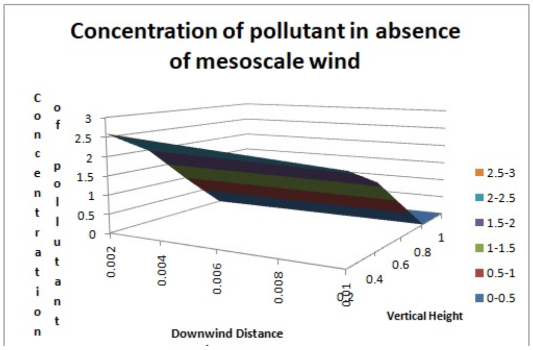

Fig. 10: