Certain Subclass of Univalent Functions Involving Modified Sigmoid Function Defined by Using Al-Oboudi Operator

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Abstract

By making use of an Al-Oboudi operator involving modified real sigmoid function, the authors defined a certain subclass of univalent functions denoted by $G_\gamma(\beta, \xi, \mu, \eta, \lambda)$. Various geometric properties of the class were established.

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1 Introduction

The study of subclasses of univalent function and their geometric properties is significant. Various authors such as [2], [3], [4], [5], [6], [8], [10] and [11] have successfully defined and investigated certain subclasses of univalent functions using differential operators while sigmoid function in univalent functions was also studied in [4], [5], [6], [7], [9] and [12].

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \qquad a_k \geq 0 \tag{1.1}$$

which are analytic in the unit disk $\mathcal{U} = \{z : |z| < 1\}$.
Let

$$\gamma(s) = \frac{2}{1+e^{-s}} = 1 + \frac{1}{2}s - \frac{1}{24}s^3 + \frac{1}{240}s^5 - \frac{17}{40320}s^7 + ... \qquad s \geq 0. \tag{1.2}$$

be a modified sigmoid function with $\gamma(0) = 1$
Let A_γ denote the class of function of the form

$$f_\gamma(z) = z + \sum_{k=2}^{\infty} \gamma(s) a_k z^k, \tag{1.3}$$

Note $A_1 \equiv A$

We define the identity function involving modified sigmoid function which is defined as follow

$$e_\gamma(z) = z \tag{1.4}$$

Definition 1. *: Let $n \in N_0 = N \cup \{0\}$ and $\lambda \geq 0$. Denoted by D^n_λ the Al-Oboudi operator [1] defined by,*

$$\begin{aligned} D^0_\lambda f_\gamma(z) &= f_\gamma(z) \\ D^1_\lambda f_\gamma(z) &= \gamma(s)\left[(1-\lambda) f_\gamma(z) + \gamma z f'_\gamma(z)\right] \\ D^n_\lambda f_\gamma(z) &= \gamma(s) z D_\lambda \left[D^{n-1}_\lambda f_\gamma(z)\right]. \end{aligned}$$

Therefore,

$$D^n f_\gamma(z) = \gamma^n(s) z + \sum_{k=2}^{\infty} \gamma^{n+1}(s) \left[1 + (k-1)\lambda\right]^n a_k z^k \tag{1.5}$$

$$Taking \lim_{s \to \infty} \gamma(s) = 1,$$

We have the Al-Oboudi Operator.

Remark 1. *When $\lambda = 1$ we get Salagean differential operator [10].*

Definition 2. *: A function $f_\gamma \in A_\gamma$ defined by (1.3) is said to belong to the class $G_\gamma(\beta, \xi, \mu, \eta, \lambda)$ if*

$$\left| \frac{\frac{D^{n+1}_\lambda f_\gamma(z)}{D^n_\lambda f_\gamma(z)} - \xi}{3\mu\left(\frac{D^{n+1}_\lambda f_\gamma(z)}{D^n_\lambda f_\gamma(z)}\right) - \beta\left(\frac{D^{n+1}_\lambda f_\gamma(z)}{D^n_\lambda f_\gamma(z)} - \xi\right)} \right| < \eta \tag{1.6}$$

where, $\eta > 0, 0 < \xi \leq 1, 0 < \beta \leq 1, 0 < \mu \leq 1, \lambda \geq 0, n \in N_0 = N \cup \{0\}$.

2 Main Results Coefficient Estimates

We begin with a sufficient coefficient estimatimation for functions to be in class $G_\gamma(\beta,\xi,\mu,\eta,\lambda)$.

Theorem 2.1. *Let $f_\gamma(z)$ defined by (1.3) belongs to the class $G_\gamma(\beta,\xi,\mu,\eta,\lambda)$ then*

$$\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n\left\{\begin{array}{l}\gamma(s)\left[1+(k-1)\lambda\right]-\xi\\ -\eta\begin{pmatrix}3\mu\gamma^2(s)\left[1+(k-1)\lambda\right]\\ -\beta\gamma^2(s)\left[1+(k-1)\lambda\right]+\beta\xi\gamma(s)\end{pmatrix}\end{array}\right\}a_k$$
$$\leq 3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s)).$$

where, $|z|<1,\eta>0,\gamma(s)=\frac{2}{1+e^{-s}},0<\xi\leq 1,0<\beta\leq 1,0<\mu\leq 1,$
$n\in N_0=N\cup\{0\}$.

Proof. Suppose, $f_\gamma\in G_\gamma(\beta,\xi,\mu,\eta,\lambda)$. Then, (1.6) holds. Thus

$$\frac{D_\lambda^{n+1}f_\gamma(z)}{D_\lambda^n f_\gamma(z)}-\xi=\frac{\gamma^n(s)[\gamma(s)-\xi]z+\sum_{k=2}^{\infty}\left\{\begin{array}{l}\gamma^{n+1}(s)\left[1+(k-1)\lambda\right]^n\times\\ \left[\gamma(s)(1+(k-1)\lambda)-\xi\right]\end{array}\right\}a_kz^k}{\gamma^n(s)z+\sum_{k=2}^{\infty}\gamma^{n+1}(s)\left[1+(k-1)\lambda\right]^n a_kz^k} \tag{2.1}$$

and

$$\begin{aligned}&3\mu\left(\frac{D_\lambda^{n+1}f_\gamma(z)}{D_\lambda^n f\gamma(z)}\right)-\beta\left(\frac{D_\lambda^{n+1}f_\gamma(z)}{D_\lambda^n f\gamma(z)}-\xi\right)\\
&=\frac{3\mu\gamma^{n+1}(s)z+\sum_{k=2}^{\infty}3\mu\gamma^{n+2}(s)\left[1+(k-1)\lambda\right]^{n+1}a_kz^k}{\gamma^n(s)z+\sum_{k=2}^{\infty}\gamma^{n+1}(s)\left[1+(k-1)\lambda\right]^n a_kz^k}\\
&-\frac{\left\{\begin{array}{l}\beta\gamma^{n+1}(s)z+\displaystyle\sum_{k=2}^{\infty}\beta\gamma^{n+2}(s)\left[1+(k-1)\lambda\right]^{n+1}a_kz^k\\ -\beta\xi\gamma^n(s)z-\displaystyle\sum_{k=2}^{\infty}\beta\xi\gamma^{n+1}(s)\left[1+(k-1)\lambda\right]^n a_kz^k\end{array}\right\}}{\gamma^n(s)z+\sum_{k=2}^{\infty}\gamma^{n+1}(s)\left[1+(k-1)\lambda\right]^n a_kz^k}\\
&=\frac{\left\{\begin{array}{l}\gamma^n(s)[3\mu\gamma(s)-\beta\gamma(s)+\beta\xi]z+\displaystyle\sum_{k=2}^{\infty}\gamma^n(s)[1+(k-1)\lambda]^n\\ \times\left[3\mu\gamma^2(1+(k-1)\lambda)-\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)\right]a_kz^k\end{array}\right\}}{\gamma^n(s)z+\sum_{k=2}^{\infty}\gamma^{n+1}(s)\left[1+(k-1)\lambda\right]^n a_kz^k}.\end{aligned} \tag{2.2}$$

Substituting (2.1) and (2.2) in (1.6), we have

$$\left|\gamma^n(s)[\gamma(s)-\xi]z+\sum_{k=2}^{\infty}\gamma^{n+1}(s)\left[1+(k-1)\lambda\right]^n\left[\gamma(s)(1+(k-1)\lambda)-\xi\right]a_kz^k\right|$$

$$<\left|\begin{array}{l}\eta\gamma^n(s)[3\mu\gamma(s)-\beta\gamma(s)+\beta\xi]z\\ +\sum_{k=2}^{\infty}\eta\gamma^n(s)[1+(k-1)\lambda]^n\begin{bmatrix}3\mu\gamma^2(s)(1+(k-1)\lambda)\\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)\end{bmatrix}a_kz^k\end{array}\right|$$

$As \quad |z|\to 1^+,$

$$\gamma^n(s)[\gamma(s)-\xi]+\sum_{k=2}^{\infty}\gamma^{n+1}(s)\left[1+(k-1)\lambda\right]^n\left[\gamma(s)(1+(k-1)\lambda)-\xi\right]a_k$$

$$\leq\gamma^n(s)\eta[3\mu\gamma-\beta\gamma+\beta\xi]$$

$$+\sum_{k=2}^{\infty}\eta\gamma^n(s)[1+(k-1)\lambda]^n\begin{bmatrix}3\mu\gamma^2(s)(1+(k-1)\lambda)\\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)\end{bmatrix}a_k$$

After simplification we get,

$$\sum_{k=2}^{\infty}\gamma^n(s)[1+(k-1)\lambda]^n\begin{Bmatrix}\gamma(s)[1+(k-1)\lambda]-\xi-\eta[3\mu\gamma^2(s)(1+(k-1)\lambda)\\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)]\end{Bmatrix}a_k$$

$$\leq\gamma^n(s)[\eta(3\mu\gamma(s)+\beta\xi-\beta\xi)+\xi-\gamma]$$

$$\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n\begin{Bmatrix}\gamma(s)[1+(k-1)\lambda]-\xi-\eta[3\mu\gamma^2(s)(1+(k-1)\lambda)\\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)]\end{Bmatrix}a_k$$

$$\leq 3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s)).$$

Which is sharp for

$$f_\gamma(z)=z+\frac{3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s))}{[1+(k-1)\lambda]^n\begin{Bmatrix}\gamma(s)[1+(k-1)\lambda]-\xi\\ -\eta[3\mu\gamma^2(s)(1+(k-1)\lambda)\\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)]\end{Bmatrix}}z^n.$$

□

Corollary 1. *A function $f\in G_\gamma(\beta,\xi,\mu,\eta,\lambda)$ if*

$$\sum_{k=2}^{\infty}a_k\leq\frac{3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s))}{[1+(k-1)\lambda]^n\begin{Bmatrix}\gamma(s)[1+(k-1)\lambda]-\xi\\ -\eta[3\mu\gamma^2(s)(1+(k-1)\lambda)\\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)]\end{Bmatrix}}z^n. \tag{2.3}$$

Corollary 2. *A function $f \in G_1(\beta, \xi, \mu, \eta, \lambda)$ if*

$$\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n \begin{Bmatrix} [1+(k-1)\lambda]-\xi-\eta[3\mu(1+(k-1)\lambda) \\ -\beta(1+(k-1)\lambda)+\beta\xi] \end{Bmatrix} \leq 3\eta\mu+(\beta\eta+1)(\xi-1).$$

which is sharp for

$$f_\gamma(z) = z + \frac{3\eta\mu+(\beta\eta+1)(\xi-1)}{[1+(k-1)\lambda]^n \begin{Bmatrix} [1+(k-1)\lambda]-\xi-\eta[3\mu(1+(k-1)\lambda) \\ -\beta(1+(k-1)\lambda)+\beta\xi] \end{Bmatrix}} z^n.$$

Corollary 3. *A function $f \in G_1(1, \xi, \mu, \eta, \lambda)$ if*

$$\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n \begin{Bmatrix} [1+(k-1)\lambda]-\xi-\eta[3\mu(1+(k-1)\lambda) \\ -(1+(k-1)\lambda)+\xi] \end{Bmatrix} \leq 3\eta\mu+(\eta+1)(\xi-1).$$

which is sharp for

$$f_\gamma(z) = z + \frac{3\eta\mu+(\eta+1)(\xi-1)}{[1+(k-1)\lambda]^n \begin{Bmatrix} [1+(k-1)\lambda]-\xi-\eta[3\mu(1+(k-1)\lambda) \\ -(1+(k-1)\lambda)+\xi] \end{Bmatrix}} z^n.$$

Corollary 4. *A function $f \in G_1(1, 1, \mu, \eta, \lambda)$ if*

$$\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n \begin{Bmatrix} [1+(k-1)\lambda]-1-\eta[3\mu(1+(k-1)\lambda) \\ -(1+(k-1)\lambda)+1] \end{Bmatrix} \leq 3\eta\mu.$$

which is sharp for

$$f_\gamma(z) = z + \frac{3\eta\mu}{[1+(k-1)\lambda]^n \begin{Bmatrix} [1+(k-1)\lambda]-1-\eta[3\mu(1+(k-1)\lambda) \\ -(1+(k-1)\lambda)+1] \end{Bmatrix}} z^n.$$

Corollary 5. *A function $f \in G_1(1, 1, 1, \eta, \lambda)$ if*

$$\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n \{[1+(k-1)\lambda]-1-\eta[2(1+(k-1)\lambda)+1]\} \leq 3\eta.$$

which is sharp for

$$f_\gamma(z) = z + \frac{3\eta}{[1+(k-1)\lambda]^n \{[1+(k-1)\lambda]-1-\eta[2(1+(k-1)\lambda)+1]\}} z^n.$$

3 Radii Properties

We obtain radii properties for function in the class $G_\gamma(\beta, \xi, \mu, \eta, \lambda)$.

Theorem 3.1. *Let the function $f_\gamma(z)$ defined by (1.3) in the class $G_\gamma(\beta, \xi, \mu, \eta, \lambda)$, then $f_\gamma(z)$ is starlike of order $\sigma(0 \leq \sigma < 1)$ in $|z| < r_1$, where*

$$r_1 = inf_k \left\{ \frac{\begin{bmatrix}(1-\sigma)\times \\ (1+(k-1)\lambda)^n\end{bmatrix} \begin{bmatrix}\gamma(s)(1+(k-1)\lambda) - \xi \\ -\eta(3\mu\gamma^2(s)(1+(k-1)\lambda) \\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s))\end{bmatrix}}{(k-\sigma)[3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s))]} \right\}^{\frac{1}{k-1}}, k \geq 2. \tag{3.1}$$

Proof. Let $\left|\frac{zf'_\gamma(z)}{f_\gamma(z)} - 1\right| < 1 - \sigma, \quad |z| < r_1$.
That is,

$$\left|\frac{zf'_\gamma(z)}{f_\gamma(z)} - 1\right| = \left|\frac{z - \sum_{k=2}^{\infty}\gamma(s)ka_kz^k - z + \sum_{k=2}^{\infty}\gamma(s)a_kz^k}{z - \sum_{k=2}^{\infty}\gamma(s)a_kz^k}\right|$$

It follows that,

$$\sum_{k=2}^{\infty}\gamma(s)\frac{(k-\sigma)|z|^{k-1}}{(1-\sigma)} \leq \frac{1}{a_k}$$

$$|z|^{k-1} \leq \frac{(1-\sigma)[1+(k-1)\lambda]^n \begin{bmatrix}\gamma(s)(1+(k-1)\lambda) - \xi \\ -\eta(3\mu\gamma^2(s)(1+(k-1)\lambda) \\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s))\end{bmatrix}}{(k-\sigma)[3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s))]}.$$

Thus,

$$r_1 = inf_k \left\{ \frac{\begin{bmatrix}(1-\sigma)\times \\ (1+(k-1)\lambda)^n\end{bmatrix} \begin{bmatrix}\gamma(s)(1+(k-1)\lambda) - \xi \\ -\eta(3\mu\gamma^2(s)(1+(k-1)\lambda) \\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s))\end{bmatrix}}{(k-\sigma)[3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s))]} \right\}^{\frac{1}{k-1}}, k \geq 2.$$

□

Theorem 3.2. *Let the function* $f_\gamma(z)$ *defined by (1.3) be in the class* $G_\gamma(\beta,\xi,\mu,\eta,\lambda)$ *then* $f_\gamma(z)$ *is convex of order* $\sigma (0 \le \sigma < 1)$ *in* $|z| < r_2$*, where*

$$r_2 = inf_k \left\{ \frac{(1-\sigma)[1+(k-1)\lambda]^n \begin{bmatrix} \gamma(s)(1+(k-1)\lambda) - \xi \\ -\eta(3\mu\gamma^2(s)(1+(k-1)\lambda) \\ -\beta\gamma^2(s)(1+(k-1)\lambda) + \beta\xi\gamma(s)) \end{bmatrix}}{k(k-\sigma)[3\eta\mu\gamma(s) + (\beta\eta+1)(\xi-\gamma(s))]} \right\}^{\frac{1}{k-1}} \tag{3.2}$$

Proof. The result holds following (3.1) since $f_\gamma(z)$ is convex if and only if $zf'_\gamma(z)$ is starlike. □

Theorem 3.3. *Let the function* $f_\gamma(z)$ *defined by (1.3) be in the class* $G_\gamma(\beta,\xi,\mu,\eta,\lambda)$ *then* $f_\gamma(z)$ *is close to convex of order* $\sigma (0 \le \sigma < 1)$ *in* $|z| < r_3$*, where*

$$r_3 = inf_k \left\{ \frac{\begin{bmatrix} (1-\sigma)\times \\ (1+(k-1)\lambda)^n \end{bmatrix} \begin{bmatrix} \gamma(s)(1+(k-1)\lambda) - \xi \\ -\eta(3\mu\gamma^2(s)(1+(k-1)\lambda) \\ -\beta\gamma^2(s)(1+(k-1)\lambda) + \beta\xi\gamma(s)) \end{bmatrix}}{k[3\eta\mu\gamma(s) + (\beta\eta+1)(\xi-\gamma(s))]} \right\}^{\frac{1}{k-1}}, k \ge 2. \tag{3.3}$$

Proof. We need to verify that $\left|f'_\gamma(z) - 1\right| = 1-\sigma, \quad (0 \le \sigma < 1)$ for $|z| < r_3$.
Thus,

$$\left|f'_\gamma(z) - 1\right| = \left|1 - \sum_{k=2}^{\infty} \gamma(s) k a_k z^{k-1} - 1\right| \le \sum_{k=2}^{\infty} \gamma(s) k a_k \left|z^{k-1}\right|.$$

Since

$$\left|f'_\gamma(z) - 1\right| \le \sum_{k=2}^{\infty} \gamma(s) k a_k \left|z^{k-1}\right| \le 1-\sigma.$$

If we devide bith sides by $(1-\sigma)$, then,

$$\sum_{k=2}^{\infty} \gamma(s) \left(\frac{k}{1-\sigma}\right) a_k \left|z^{k-1}\right| \le 1. \tag{3.4}$$

Following (3.3), we have that

$$\sum_{k=2}^{\infty} \frac{\gamma(s)k|z|^{k-1}}{(1-\sigma)} \le \frac{[1+(k-1)\lambda]^n \begin{bmatrix} \gamma(s)(1+(k-1)\lambda) - \xi \\ -\eta(3\mu\gamma^2(s)(1+(k-1)\lambda) \\ -\beta\gamma^2(s)(1+(k-1)\lambda) + \beta\xi\gamma(s)) \end{bmatrix}}{[3\eta\mu\gamma(s) - \gamma(s)(1-\xi)(\beta\eta-1)]}, k \ge 2.$$

which implies that

$$|z| \leq \left\{ \frac{(1-\sigma)[1+(k-1)\lambda]^n \begin{bmatrix} \gamma(s)(1+(k-1)\lambda)-\xi \\ -\eta(3\mu\gamma^2(s)(1+(k-1)\lambda) \\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)) \end{bmatrix}}{k[3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s))]} \right\}^{\frac{1}{k-1}}.$$

Hence,

$$r_3 = inf_k \left\{ \frac{\begin{bmatrix} (1-\sigma)\times \\ (1+(k-1)\lambda)^n \end{bmatrix} \begin{bmatrix} \gamma(s)(1+(k-1)\lambda)-\xi \\ -\eta(3\mu\gamma^2(s)(1+(k-1)\lambda) \\ -\beta\gamma^2(s)(1+(k-1)\lambda)+\beta\xi\gamma(s)) \end{bmatrix}}{k[3\eta\mu\gamma(s)+(\beta\eta+1)(\xi-\gamma(s))]} \right\}^{\frac{1}{k-1}}, k \geq 2.$$

□

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