Effect of stress jump coefficient on the creeping flow of micropolar fluid past through a porous sphere

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Abstract

In the present paper we have studied the creeping flow of viscous incompressible micropolar fluid past a porous sphere with permeability. Flow outside the sphere is governed by stokes equation and inside the sphere Brinkman equation is used. Analytical solution of the problem is obtained by using continuity of the velocity and normal stress and jump in tangential shear stress at the interface of micropolar fluid and porous sphere as boundary condition. Exact solution of the problem is obtained. Streamlines inside and outside the porous sphere, radial velocity and the drag force are shown in graphs for different values of the stress jump coefficient. The influence of stress jump coefficient on the flow has been discussed.

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1 Introduction:

The study of uniform flow of fluids around porous bodies involving a variety of geometries has been considered by many researchers in view of their applications in industry and engineering. Several studies of the flow past and within porous bodies are limited mainly to Newtonian fluids. Jang and Lee (1988) investigated Newtonian flow past a sphere in a tube of finite radius. Bhatt and Sacheti (1994) investigated the slow motion of a Newtonian fluid past a porous spherical shell. Grosan, et.al (2010) investigated a two-dimensional steady, viscous and incompressible flow past a permeable sphere embedded in another porous medium by using the Brinkman model. Rao, Sahu and Chhabra (2010) investigated flow of Newtonian and power-law fluids past an elliptical cylinder. Prakash and Raja (2011) examined the overall bed permeability of an assemblage of porous particles by using a hydrodynamic model popularly. Verma and Datta (2012) investigated the slow flow of an incompressible viscous fluid past a heterogeneous porous sphere with the radial variation of permeability by using the Brinkman model. Kumar and Munshi (2017) investigated the effect of blockage ratio i.e. ratio of diameter of cone and flow channel on the drag coefficients due to Newtonian fluid flow over cone.
Eringen (1964, 1966) introduce a subclass of fluids which he named micropolar fluids that ignores the deformation of the microelements but still allows for the particle micromotion to take place. Joseph and Tao (1964) examined the flow of an incompressible viscous fluid past a porous spherical particle by employing Darcy's law in the porous region and no-slip condition at the surface of the sphere they obtained that the drag on porous sphere is same as that of a rigid sphere with reduced radius. Chamkha, et.al (2002) investigated numerical

and analytical solutions of the developing laminar free convection of a micropolar fluid in a vertical parallel plate channel with asymmetric heating. Delhommelle and Evans (2002) investigated poiseuille flow of a micropolar fluid. Srinivasacharya and Rajyalakshmi (2004) examined the creeping flow of incompressible micropolar fluid past a porous sphere with permeability and obtained the drag force experienced by the sphere. They find that the drag force decreases as the coupling number decreases and decreases as the permeability parameter is increasing. Deo and Gupta (2008) investigated stokes flow for the micropolar fluid past a porous sphere. They used non homogeneous boundary conditions for the microrotation vector. They obtained the drag force experienced by the sphere and obtained the drag force decreases as the coupling number increases and decreases as the permeability parameter is increasing. Chen et.al (2011) investigated the fundamentals of micropolar fluid dynamics (MFD), and pro-poses a numerical scheme integrating Chorin's projection method and time-centred splitmethod (TCSM) for solving unsteady forms of MFD equation. Madasu and Gurdatta (2015) investigated the problem of slow steady rotation of a micropolar fluid sphere in a concentric spherical container filled with a viscous fluid. They obtained the hydrodynamic couple and wall correction factor exerted on the micropolar fluid sphere. They find that the wall correction factor increases monotonically with increase in the micropolarity parameter and the wall correction factor decreases monotonically with increasing spin parameter. Madasu and Gurdatta (2017) investigated the axisymmetric rotary oscillation of a micropolar fluid sphere in concentric spherical cavity filled with Newtonian viscous fluid. They used continuity of velocity components and stress together with the spin vorticity relation at the interface between fluid-fluid regions and obtained torque exerted on the micropolar fluid sphere analytically. They find that torque acting on the rotary oscillating sphere decreases as the viscosity ratio increases. The real torque coefficient increases with increase in micropolarity parameter. It is also found that the increase in spin parameter results in decrease of real torque coefficient and an increase of imaginary torque coefficient. Ramalakshmi and Shukla (2017) investigated an incompressible micropolar fluid flow through a porous sphere. They obtained drag exerted by the porous sphere. They find that the drag coefficient is decreasing as the permeability parameter is increasing.

In the present problem we have extended the work of Srinivasacharya et.al. (2004). We consider the creeping flow of viscous incompressible micropolar fluid past a porous sphere with permeability k. Flow outside the sphere is governed by stokes equation and inside the porous sphere Brinkman equation is used. Analytical solution of the problem is obtained by using continuity of the velocity and normal stress and jump in tangential shear stress at the interface of micropolar fluid and porous sphere as boundary condition.

2 Mathematical formulation:

In the present problem we consider a steady incompressible micropolar fluid flow past a porous sphere of radius $r = a$. We assume that the flow outside the porous sphere to be Stokesian and inside to be governed by Brinkman model. We assume that fluids have a uniform velocity U far away from the body along the axis of symmetry $\theta = 0$. Flow field is divided into two regions. Region I is the micropolar fluid region outside the porous sphere, i.e., $r^* > a$ and region II is the porous region, i.e., $r^* \leqslant a$. The flow in region I is governed by the steady flow of an incompressible micropolar fluid under Stokesian assumption with the absence of body force and body couple are given by Eringen (1964)

$$\nabla.\vec{q}^{(1)} = 0, \tag{2.1}$$

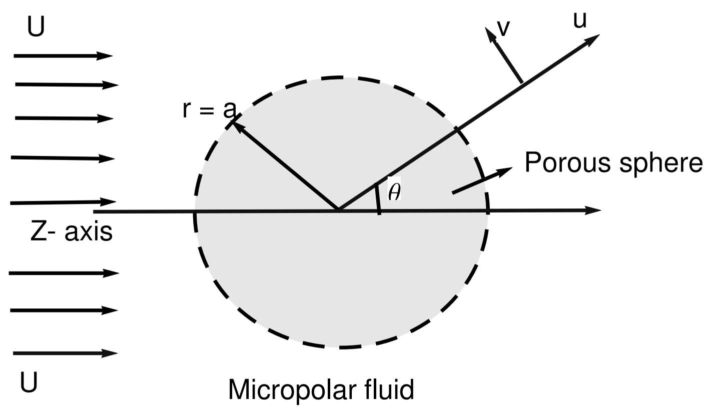

Fig. 1: Sketch of the problem

$$-\nabla p^{(1)} + \kappa\nabla\times\vec{\omega}^{(1)} - (\mu+\kappa)\nabla\times\nabla\times\vec{q}^{(1)} = 0, \tag{2.2}$$

$$-2\kappa\vec{\omega}^{(1)} + \kappa\nabla\times\vec{q}^{(1)} - \gamma\nabla\times\nabla\times\vec{\omega}^{(1)} + (\alpha+\beta+\gamma)\nabla(\nabla.\vec{\omega}^{(1)}) = 0, \tag{2.3}$$

where $\vec{q}^{(1)}$ is the velocity vector, $\vec{\omega}^{(1)}$ is the microrotation vector and $p^{(1)}$ is the fluid pressure outside the sphere (Region I). For the region inside the sphere (region II) the equations of the motion of the fluid in steady state in the porous medium based on Brinkman's model are given by

$$\nabla.\vec{q}^{(2)} = 0, \tag{2.4}$$

$$\frac{\mu}{k}\vec{q}^{(2)} + \nabla p^{(2)} - \kappa\nabla\times\vec{\omega}^{(2)} + (\mu+\kappa)\nabla\times\nabla\times\vec{q}^{(2)} = 0, \tag{2.5}$$

$$-2\kappa\vec{\omega}^{(2)} + \kappa\nabla\times\vec{q}^{(2)} - \gamma\nabla\times\nabla\times\vec{\omega}^{(2)} + (\alpha+\beta+\gamma)\nabla(\nabla.\vec{\omega}^{(2)}) = 0, \tag{2.6}$$

where k is the permeability of the porous medium, $\vec{q}^{(2)}$ is the velocity vector, $\vec{\omega}^{(2)}$ is the microrotation vector and $p^{(2)}$ is the fluid pressure inside the sphere (Region II). The material constants $\mu,\ \kappa,\ \alpha,\ \beta$ and γ satisfy the following inequalities Eringen (1964)

$$2\mu+\kappa \geqslant 0, \qquad \kappa \geqslant 0, \qquad 3\alpha+\beta+\gamma \geqslant 0 \ \text{ and } \ \gamma \geqslant |\beta|. \tag{2.7}$$

Since, the flow generated is axially symmetric, all the flow function are independent of ϕ. Hence the velocity and microrotation can be chosen in the spherical polar coordinates $(r,\ \theta,\ \phi)$ as

$$q^{(i)} = u^{(i)}(r,\theta)\hat{\mathbf{e}}_r + v^{(i)}(r,\theta)\hat{\mathbf{e}}_\theta \tag{2.8}$$

$$\omega^{(i)} = v_\phi^{(i)}(r,\theta)\hat{\mathbf{e}}_\theta. \tag{2.9}$$

and

Introducing the stream function through

$$u^{(i)} = -\frac{1}{r^2\sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta}, \qquad v^{(i)} = -\frac{1}{r\sin\theta}\frac{\partial\psi^{(i)}}{\partial r}. \tag{2.10}$$

We using the following non dimensional variables

$$r = a\tilde{r}, \qquad \psi^{(i)} = Ua^2\tilde{\psi}^{(i)}, \qquad p^{(i)} = \frac{\mu U}{a}\tilde{p}^{(i)}, \qquad v_\phi^{(i)} = \frac{U}{a}\tilde{v_\phi}^{(i)} \tag{2.11}$$

into the Eqs. (2.1) - (2.6) and dropping the tildes, we get the equations for the region outside the sphere as

$$-\frac{\partial p^{(1)}}{\partial r} + \left(\frac{N}{1-N}\right)\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(rsin\theta v_\phi^{(1)}\right) - \left(\frac{1}{1-N}\right)\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(E^2\psi^{(1)}\right) = 0, \tag{2.12}$$

$$-\frac{1}{r}\frac{\partial p^{(1)}}{\partial\theta} - \left(\frac{N}{1-N}\right)\frac{1}{rsin\theta}\frac{\partial}{\partial r}\left(rsin\theta v_\phi^{(1)}\right) + \left(\frac{1}{1-N}\right)\frac{1}{rsin\theta}\frac{\partial}{\partial r}\left(E^2\psi^{(1)}\right) = 0, \tag{2.13}$$

$$-2v_\phi^{(1)} + \frac{1}{rsin\theta}\frac{\partial}{\partial r}\left(E^2\psi^{(1)}\right) + \left(\frac{2-N}{m^2}\right)\left[\nabla^2 - \frac{1}{r^2 sin\theta}\right]v_\phi^{(1)} = 0, \tag{2.14}$$

and the equations for the region inside the sphere as

$$-\frac{\partial p^{(2)}}{\partial r} + \left(\frac{N}{1-N}\right)\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(rsin\theta v_\phi^{(2)}\right) - \left(\frac{1}{1-N}\right)\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(E^2\psi^{(2)}\right) + \sigma^2\frac{1}{r^2 sin\theta}\frac{\partial\psi^{(2)}}{\partial\theta} = 0, \tag{2.15}$$

$$-\frac{1}{r}\frac{\partial p^{(2)}}{\partial\theta} - \left(\frac{N}{1-N}\right)\frac{1}{rsin\theta}\frac{\partial}{\partial r}\left(rsin\theta v_\phi^{(2)}\right) + \left(\frac{1}{1-N}\right)\frac{1}{rsin\theta}\frac{\partial}{\partial r}\left(E^2\psi^{(2)}\right) = 0, \tag{2.16}$$

$$-2v_\phi^{(2)} + \frac{1}{rsin\theta}\frac{\partial}{\partial r}\left(E^2\psi^{(2)}\right) + \left(\frac{2-N}{m^2}\right)\left[\nabla^2 - \frac{1}{r^2 sin\theta}\right]v_\phi^{(2)} = 0, \tag{2.17}$$

where E^2 is Stokes stream operator, defined as

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right), \tag{2.18}$$

and $\sigma^2 = a^2/k$ is permeability variation parameter, $N = \kappa/(\mu+\kappa)$ is the coupling number $(0 \leqslant N < 1)$ and $m^2 = \frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}a^2$ is the micropolar parameter. The matching condition at the surface of porous sphere are

$$\begin{aligned} u^{(1)}(r,\theta) &= u^{(2)}(r,\theta) \qquad \text{at } r = a,\\ v^{(1)}(r,\theta) &= v^{(2)}(r,\theta) \qquad \text{at } r = a,\\ p^{(1)}(r,\theta) &= p^{(2)}(r,\theta) \qquad \text{at } r = a,\\ \tau^{(1)}_{r\theta}(r,\theta) &= \tau^{(2)}_{r\theta}(r,\theta) - \frac{a\eta}{\sqrt{k_o}}v_2 \qquad \text{at } r = a,\\ v^{(1)}_{\phi}(r,\theta) = 0 \qquad &\text{and} \qquad v^{(2)}_{\phi}(r,\theta) = 0 \qquad \text{at } r = a. \end{aligned}$$

The equivalent non dimensional conditions on the boundary $r = 1$ in terms of the stream function are

$$\begin{aligned} \psi^{(1)}(r,\theta) &= \psi^{(2)}(r,\theta), \qquad \psi^{(1)}_r(r,\theta) = \psi^{(2)}_r(r,\theta),\\ p^{(1)}(r,\theta) &= p^{(2)}(r,\theta), \qquad \psi^{(1)}_{rr}(r,\theta) = \psi^{(2)}_{rr}(r,\theta) - \eta\sigma\psi^{(2)}_r(r,\theta),\\ v^{(1)}_{\phi}(r,\theta) &= 0 \qquad \text{and} \qquad v^{(2)}_{\phi}(r,\theta) = 0 \end{aligned}$$

together with $\psi^{(1)} \to (1/2)r^2 sin^2\theta$ as $r \to \infty$ and $\psi^{(2)}$ is finite at $r = 0$.

3 Solution of the Problem:

Eliminating pressure $p^{(1)}$ from Eq. (2.12) and Eq. (2.13), we get

$$E^4\psi^{(1)} - NE^2(r\sin\theta v^{(1)}_{\phi}) = 0. \tag{3.1}$$

Substituting it in Eq. (2.14)

$$v^{(1)}_{\phi} = \frac{1}{2r\sin\theta}\left[E^2\psi^{(1)} + \frac{2-N}{Nm^2}E^4\psi^{(1)}\right]. \tag{3.2}$$

From Eq. (3.1) and Eq. (3.2), we get

$$E^4(E^2 - m^2)\psi^{(1)} = 0. \tag{3.3}$$

Eliminating pressure $p^{(2)}$ from Eq. (2.15) and Eq. (2.16), we get

$$E^4\psi^{(2)} - NE^2(r\sin\theta v^{(2)}_{\phi} - \sigma^2(1-N)E^2\psi^{(2)}) = 0. \tag{3.4}$$

By using Eq. (3.4) in Eq. (2.17) we get

$$v_{\phi}^{(2)} = \frac{1}{2r\sin\theta}\left[E^2\psi^{(2)} + \frac{2-N}{Nm^2}(E^4\psi^{(2)} - \sigma^2(1-N)E^2\psi^{(2)})\right]. \tag{3.5}$$

Eliminating $v_{\phi}^{(2)}$ from Eq. (3.4) and Eq. (3.5), we get

$$E^2(E^2-\alpha^2)(E^2-\beta^2)\psi^{(2)} = 0, \tag{3.6}$$

where $\alpha^2+\beta^2 = \sigma^2(1-N)+m^2$ and $\alpha^2\beta^2 = \sigma^2 m^2 \frac{2(1-N)}{2-N}$.

The stream function solution of Eqs. (3.3) and (3.6), which satisfies the uniform condition at infinity, $\psi^{(1)} \to (r^2\sin^2\theta)/2$ and requirement for spherical case are

$$\psi^{(1)}(r,\xi) = \left[r^2 + B_2 r^{-1} + D_2 r + E_2\sqrt{r}K_{3/2}(mr)\right]G_2(\xi), \tag{3.7}$$

$$\psi^{(2)}(r,\xi) = \left[A_2^* r^2 + D_2^*\sqrt{r}I_{3/2}(\alpha r) + F_2^*\sqrt{r}I_{3/2}(\beta r)\right]G_2(\xi). \tag{3.8}$$

Substituting the value of $\psi^{(1)}$ from Eq. (3.7) in Eq. (3.2) and $\psi^{(2)}$ from Eq. (3.8) in Eq. (3.5), we get microrotation components, respectively as

$$v_{\phi}^{(1)}(r,\xi) = \frac{1}{r\sin\theta}\left[-D_2 r^{-1} + \frac{m^2}{N}E_2\sqrt{r}K_{3/2}(mr)\right]G_2(\xi), \tag{3.9}$$

$$v_{\phi}^{(2)}(r,\xi) = \frac{1}{r\sin\theta}\left[A_\alpha D_2^*\sqrt{r}I_{3/2}(\alpha r) + A_\beta F_2^*\sqrt{r}I_{3/2}(\beta r)\right]G_2(\xi) \tag{3.10}$$

where $\xi = \cos\theta$, $K_{3/2}(mr)$ and $I_{3/2}(\alpha r)$ are modified Bessel functions of the first and second kind and $G_2(\xi)$ is Gegenbauer function of first kind. And

$$A_\alpha = \frac{\left[Nm^2 - (2-N)(1-N)\sigma^2\right]\alpha^2 + (2-N)\alpha^4}{2Nm^2},$$
$$A_\beta = \frac{\left[Nm^2 - (2-N)(1-N)\sigma^2\right]\beta^2 + (2-N)\beta^4}{2Nm^2}. \tag{3.11}$$

Using the boundary condition (2.19), we get the values of arbitrary constants $B_2,\ D_2,\ E_2,\ A_2^*,\ D_2^*$ and F_2^*, which are given in the Appendix. Using Eq. (3.7) in Eq. (2.10), we get the velocity of fluid in region I is given by

$$u^{(1)} = \frac{2\cos\theta}{r^2}\left[r^2 + B_2 r^{-1} + D_2 r + E_2\sqrt{r}K_{3/2}(mr)\right], \tag{3.12}$$

$$v^{(1)} = -\frac{\sin\theta}{r}\left[-\frac{B_2}{r^2} + D_2 + \frac{1}{2}mM_2\sqrt{r}(-K_{1/2}(mr) - K_{5/2}(mr)) + \frac{E_2 K_{3/2}(mr)}{2\sqrt{r}} + 2r\right] \tag{3.13}$$

and by using Eq. (3.8) in Eq. (2.10), we get velocity of fluid in the region II given by

$$u^{(2)} = \frac{2\cos\theta}{r^2}\left[A_2^* r^2 + D_2^*\sqrt{r}I_{3/2}(\alpha r) + F_2^*\sqrt{r}I_{3/2}(\beta r)\right], \tag{3.14}$$

$$v^{(2)} = -\frac{\sin\theta}{r}\Bigg[2A_2^* r + \frac{D_2^* I_{3/2}(r\alpha)}{2\sqrt{r}} + \frac{1}{2}\alpha D_2^*\sqrt{r}(I_{1/2}(r\alpha) + I_{5/2}(r\alpha)) + \frac{F_2^* I_{3/2}(r\beta)}{2\sqrt{r}} + \frac{1}{2}\beta F_2^*\sqrt{r}(I_{1/2}(r\beta) + I_{5/2}(r\beta))\Bigg]. \tag{3.15}$$

Using the expression for velocity and microrotation in Eq. (2.12) and (2.13) for outside region, we get the expression for the pressure for outside the region as

$$p^{(1)} = -\frac{2-N}{2(1-N)}D_2 r^{-2}P_1(\xi). \tag{3.16}$$

And using the expression for velocity and microrotation in Eq. (2.15) and (2.16) for inside region, we get the expression for the pressure for inside the region as

$$p^{(2)} = \frac{2-N}{2(1-N)}\frac{\alpha^2\beta^2}{2m^2}A_2^* r P_1(\xi) \tag{3.17}$$

where $P_1(\xi)$ is the Legendre polynomial.

3.1 Drag Force :

The drag force acting on the surface of porous sphere is given by

$$D = 2\pi a^2 \int_0^\pi \left(\tau_{rr}^{(1)}\cos\theta - \tau_{r\theta}^{(1)}\sin\theta\right)_{r=a}\sin\theta\, d\theta. \tag{3.18}$$

On evaluation of non dimensional stress components, we get

$$\tau_{rr}^{(1)} = f(r)\cos\theta \qquad \text{and} \qquad \tau_{r\theta}^{(1)} = g(r)\sin\theta \tag{3.19}$$

where

$$f(r) = (2\mu+\kappa)\frac{U}{a}\left[\frac{3}{r^4}B_2 + \frac{3}{2r^2}D_2 - \frac{1}{r^2}E_2\{\sqrt{r}K_{3/2}(mr)\}' + \frac{2}{r^3}E_2\sqrt{r}K_{3/2}(mr)\right], \tag{3.20}$$

$$g(r) = (2\mu+\kappa)\frac{U}{a}\left[\frac{3}{r^4}B_2 - \frac{1}{r^2}E_2\{\sqrt{r}K_{3/2}(mr)\}' + \frac{2}{r^3}E_2\sqrt{r}K_{3/2}(mr)\right]. \tag{3.21}$$

Substituting $\tau_{rr}^{(1)}$ and $\tau_{r\theta}^{(1)}$ from Eq. (3.19) in Eq. (3.18) and by using Eq. (3.20) and (3.21) , we get the Drag force on the porous sphere

$$D = 2\pi a^2 \int_0^\pi \left(f(1)\cos^2\theta - g(1)\sin^2\theta\right)_{r=a} \sin\theta \, d\theta.$$

After integration, we get

$$D = \frac{4}{3}\pi a^2 \left[f(1) - 2g(1)\right] = 2\pi(2\mu + \kappa) a U D_2 \tag{3.22}$$

where D_2 is given in Appendix. Hence, the non dimensional drag $D_N = D/(-2\pi\mu U a)$ is given by

$$D_N = -D_2 \frac{2 - N}{1 - N}. \tag{3.23}$$

When we take Stress jump Coefficient $\eta = 0$ and as the micropolar parameter $m \to \infty$ and $N \to 0$, (then $a^2 \to \sigma^2$, $\beta^2 \to \infty$, $A_\alpha \to \sigma^2/2$ and $A_\beta \to \infty$). After taking these limits the drag force simplifies to

$$D = -12\pi\mu a U \frac{\sigma^2(\sinh\sigma - \sigma\cosh\sigma)}{\sigma(3 + 2\sigma^2)\cosh\sigma - 3\sinh\sigma}. \tag{3.24}$$

This result was reported by Qin and Kaloni (1988) for slow viscous flow past a porous sphere. As $\sigma \to \infty$ the Eq. (3.24) reduces to

$$D = -6\pi\mu U a. \tag{3.25}$$

This is well known result for Newtonian flow past a solid sphere in unbounded medium.

4 Discussion:

Stream lines of the creeping flow of viscous incompressible micropolar fluid past a porous sphere are obtained by Eqs. (3.7) and (3.8) and are shown in Fig. (2). In this figure stream lines $\psi = 0.02$ and $\psi = 0.15$ are drawn for stress jump coefficient $\eta = -0.5$, 0 and 0.5 when $\sigma = 2.5$, micropolar parameter $m = 5$ and coupling number $N = 0.25$. We observe that the stress jump coefficient has remarkable effect on the flow. As the stress jump coefficient η decreases stream lines shifted away from the centre of sphere. Thus there is a decreases in fluid flow through porous region as stress jump coefficient decreases.

The radial velocity outside and within porous sphere is given by Eqs. (3.13) and (3.15), respectively. Fig. (3) present the variation of the radial velocity of fluid u with r for different values of stress jump coefficient $\eta = -0.5$, 0 and 0.5 when $\theta = \pi/4$, $\sigma = 2.5$, micropolar parameter $m = 5$ and coupling number $N = 0.25$. We observe that the velocity decreases as the stress jump coefficient η decreases from 0.5 to - 0.5. Thus there is a decreases in fluid flow through porous region as stress jump coefficient decreases.

Variation of drag force on the porous sphere with σ for different values of stress jump coefficient $\eta = -0.5$, 0 and 0.1 when micropolar parameter $m = 20$ and coupling number $N = 0.5$ shown by the fig. (4) figure reveal that as the permeability parameter σ increases(i.e. permeability of the porous region decreases) the drag force increases. We also observe that as stress jump coefficient η decreases drag force increases.

5 Conclusion:

In this work an exact solution for the creeping flow of viscous incompressible micropolar fluid past a porous sphere is obtained. Brinkman equation is used for the flow inside the porous sphere and Stokes equation is used in the region outside the porous sphere. At the fluid/porous interface continuity of velocity and normal stress and jump in tangential shear stress has been used. Expressions for the fluid velocity and drag force on the sphere are obtained. Obtained results are exhibited graphically. It has been found that stress jump coefficient has significant effect on flow velocity and drag force. It is found that increase in stress jump coefficient η caused decrease in drag force on the porous sphere. In the limiting case the obtained results reduced to the classical results. Obtained results are useful for the flow of micropolar fluids past porous particles.

Appendix

$$\begin{aligned}
B_2 &= A_\alpha I_q(\alpha)(\beta I_{q-1}(\beta)(K_q(m)(2\eta m^4\sigma+\alpha^2\beta^2m^2(\eta\sigma-N+2)+3\alpha^2\beta^2\eta N\sigma)\\
&+ \alpha^2\beta^2\eta m N\sigma K_{q-1}(m))+I_q(\beta)(\alpha^2\beta^2(-m)N\left(\beta^2+\eta\sigma\right)K_{q-1}(m)-K_q(m)\\
&\times \left(6\eta m^4\sigma+\alpha^2\beta^2m^2\left(\beta^2+\eta\sigma-3N+6\right)+3\alpha^2\beta^2N\left(\beta^2+\eta\sigma\right)\right)))+A_\beta I_q(\beta)\\
&\times (I_q(\alpha)(K_q(m)\left(6\eta m^4\sigma+\alpha^2\beta^2m^2\left(\alpha^2+\eta\sigma-3N+6\right)+3\alpha^2\beta^2N\left(\alpha^2+\eta\sigma\right)\right)\\
&+ \alpha^2\beta^2mN\left(\alpha^2+\eta\sigma\right)K_{q-1}(m))+\alpha I_{q-1}(\alpha)(\alpha^2\beta^2\eta(-m)N\sigma K_{q-1}(m)-K_q(m)\\
&\times \left(2\eta m^4\sigma+\alpha^2\beta^2m^2(\eta\sigma-N+2)+3\alpha^2\beta^2\eta N\sigma\right)))/m\nabla\\
D_2 &= -(3\alpha^2\beta^2mK_q(m)(A_\beta I_q(\beta)\left(\left(\alpha^2+\eta\sigma\right)I_q(\alpha)-\alpha\eta\sigma I_{q-1}(\alpha)\right)+A_\alpha I_q(\alpha)\\
&\times \left(\beta\eta\sigma I_{q-1}(\beta)-\left(\beta^2+\eta\sigma\right)I_q(\beta)\right)))/\nabla\\
M_2 &= -(3\alpha^2\beta^2N(A_\beta I_q(\beta)\left(\left(\alpha^2+\eta\sigma\right)I_q(\alpha)-\alpha\eta\sigma I_{q-1}(\alpha)\right)+A_\alpha I_q(\alpha)(\beta\eta\sigma I_{q-1}(\beta)\\
&- \left(\beta^2+\eta\sigma\right)I_q(\beta))))/m\nabla\\
A_2^* &= (3m^3K_q(m)(A_\beta I_q(\beta)\left(\left(\alpha^2+\eta\sigma\right)I_q(\alpha)-\alpha\eta\sigma I_{q-1}(\alpha)\right)+A_\alpha I_q(\alpha)(\beta\eta\sigma I_{q-1}(\beta)
\end{aligned}$$

$$\begin{aligned}
&- \left(\beta^2+\eta\sigma\right)I_q(\beta))))/\nabla\\
F_2^* &= -\left(3mA_\alpha K_q(m)I_q(\alpha)\left(2\eta m^2\sigma+\alpha^2\beta^2(2-N)\right)\right)/\nabla\\
D_2^* &= 3mA_\beta K_q(m)I_q(\beta)\left(2\eta m^2\sigma+\alpha^2\beta^2(2-N)\right)/\nabla\\
\nabla &= A_\alpha I_q(\alpha)((\left(\beta^2+\eta\sigma\right)I_q(\beta)\left(\alpha^2\beta^2(NK_{q-1}(m)-2mK_q(m))-3m^3K_q(m)\right)\\
&+ I_{q-1}(\beta)\left(\beta mK_q(m)\left(\eta m^2\sigma+\alpha^2\beta^2(2\eta\sigma+N-2)\right)-\alpha^2\beta^3\eta N\sigma K_{q-1}(m)\right))\\
&+ A_\beta I_q(\beta)((\left(\alpha^2+\eta\sigma\right)I_q(\alpha)\left(3m^3K_q(m)+\alpha^2\beta^2(2mK_q(m)-NK_{q-1}(m))\right)\\
&+ I_{q-1}(\alpha)\left(\alpha^3\beta^2\eta N\sigma K_{q-1}(m)-\alpha mK_q(m)\left(\eta m^2\sigma+\alpha^2\beta^2(2\eta\sigma+N-2)\right)\right)))
\end{aligned}$$

Where $q=3/2$

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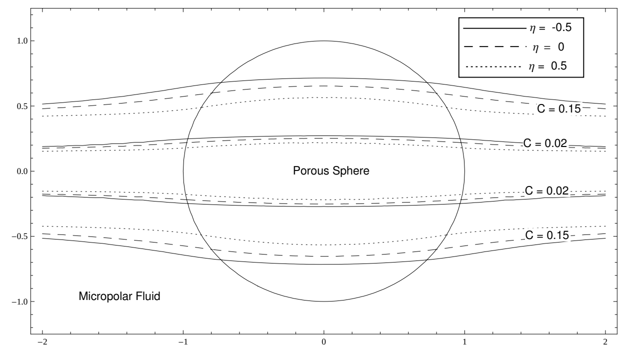

Fig. 2: Stream lines $\psi = C$ of the flow for $C = 0.02$ and 0.15 for different values stress jump coefficient when $\sigma = 2.5$, $m = 5$ and $N = 0.5$.

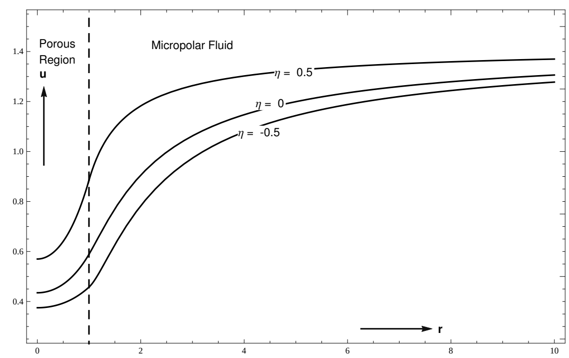

Fig. 3: Variation of radial velocity u_1 (u for $r \geqslant 1$) and u_2 (u for $r \leqslant 1$) with r for different values stress jump coefficient when $\sigma = 2.5$, $m = 5$, $N = 0.5$ and $\theta = \pi/4$.

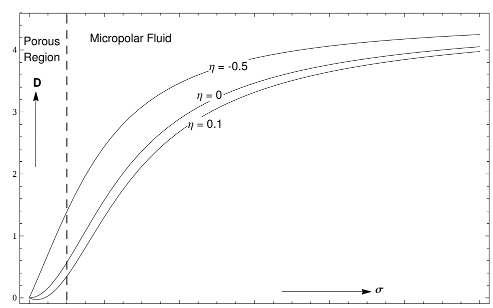

Fig. 4: Variation of Drag Force D on porous sphere with permeability parameter σ for different values of stress jump coefficient when $m = 20$ and $N = 0.5$.