New Congruence Method towards Assignment Problem

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Abstract

In this paper, we have develop the new congruence method for assignment problem in minimum steps which gives the result in minimum stipulated time as compare to other method to solve assignment problem. Also as an application we have compare the result with Hungarian method.

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1 Introduction

Though the theory of Transportation problems generally evolved during the world war II, origin of its roots are right from the 400 B. C. or from 3500 B. C. after wheel was invented in the Middle East of Asia ([4],[8]). On the other hand the origin of an assignment problem were discovered by the great mathematician Carl Gustav Jacobi in 19^{th} century. Whenever one think of an assignment problem the first case come into our mind ([4],[7],[8]) is transportation problem in which the objective depends upon available resources which were depending upon machines with different efficiency of performing job. The assignment problem tells us that ([4],[8]) How should the assignment be made so as to optimize the given objective? this is because the assignment problem has varying degree of efficiency for performing different activities. Therefore cost, profit or time of performing different activities is different ([4],[8]).
To overcome this difficulty it is interesting to modify the given transportation problem as number theoretic approach using the congruence relation. We know that, the congruence relation $a \equiv b(modm)$ is an equivalence relation [3] which tells us that $m \mid (b-a) \Leftrightarrow a \equiv b(modm)$.
The paper mainly consists of three parts. In first part some basic definitions were given while in second part the algorithm for proposed new method were given. In the third part, this new method along with numerical example were explained. In the fourth part, we have compared the result with Hungarian method along with conclusion.

2 Basic Definition

1) **Transportation:**
Let there be 'm' origins $O_1, O_2, ..., O_m$ having $a_i (a_i > 0\ , i = 1, 2, ..., m)$ units of avail-

ability respectively and n destinations $D_1, D_2, ..., D_n$ having $b_j (b_j > 0\ , j = 1, 2, ..., n)$ units of requirements. If $C_{ij}(x_1, x_2, x_3, x_4, x_5)$ are the cost of transporting one unit of the commodity from i^{th} origin to j^{th} destination and X_{ij} be the units of transporting from i^{th} to j^{th} destination.The objective is to determine X_{ij} which minimizes the total transporting cost(Z) satisfying all the availability constraints and the requirement constraints [4]. Where the variable are given as,
x_1 = Loading Charges
x_2 = Vehicle Operator Charges
x_3 = Road transportation cost including Tolls and Taxes
x_4 = Maintenance cost of vehicle
x_5 = Unloading charges
where, $x_1, x_2, x_4, x_5 > 0$ and $x_3 \geq 0$.

2) **Mathematical Formulation of Assignment Problem:**
It can be stated as [6]; minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$ Subject to the constraints
$\sum_{j=1^n} x_{ij} = 1, i = 1, 2, 3, ..., n$ (Resource Availability)
$\sum_{i=1^n} x_{ij} = 1, j = 1, 2, 3, ..., n$ (Activity Requirement)
and $X_{ij} = 0 or 1$ for all i and j.

3) **Methods to solve Assignment Problem:**
Depending upon the given conditions there are mainly four methods which are mentioned below [4]:
A] Enumeration method
B] Simplex method
C] Transportation method
D] Hungarian method
Among which the Hungarian method were commonly used. The time complexity of the original algorithm is $O(n^4)$.

4) **Algebraic Operations Research:**
The algebraic operations research can be thought of as algebraic approach towards solving problems in operations research [8].

3 Algorithm of Proposed Method

The new method can be summarized into following steps applied for assignment problem using algebraic approach.

I) Examine whether the given assignment problem is of the form *nxn* square matrix. If not, add dummy row or dummy column whose all entries are 0.

II) Write the penalties over each rows by taking i^{th} row $[\sum_{j=1}^{n} C_{ij}](modulom)$ and write the penalties over each column by taking $[\sum_{i=1}^{n} C_{ij}](modulom)$ respectively; where m is the minimum positive / maximum positive value of cost C_{ij} for the respective rows and columns depending upon the minimization / maximization problem respectively.

III) Select the row or column with the highest penalty and allocate as much as possible in the cell that has least cost in the selected rows or column and satisfies the given condition. If there is tie in the values of penalties, one can take any one of them where the minimum allocation can be made.

IV) Any row or column with zero supply or demand should not be used in computing future penalties.

V) Repeat steps from II] to IV] until the available jobs at various sources and demand at various machines are satisfied.

4 Numerical Example

A) Consider the following example to find out minimizing the given assignment problem

	I	**II**	**III**	**IV**
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	17	26	24	10

Solution:

	I	**II**	**III**	**IV**	**Penalty**	**Penalty**	**Penalty**	**Penalty**
A	18	26	[17]	11	−5	2	1	17
B	[13]	28	14	26	3	1	1	−−
C	38	[19]	18	15	0	−−	−−	−−
D	17	26	24	[10]	−1	3	−−	−−
Penalty	−3	4	3	2				
Penalty	−2	−−	−1	3				
Penalty	5	−−	3	−−				
Penalty	−−	−−	17	−−				

So the solution of given assignment problem is

	A	**B**	**C**	**D**
−−	*III*	*I*	*II*	*IV*
Units	17	13	19	10

Total Units = 59 Units. The same solution can be comapared with the Hungarian method [4].

B) Solve the following assignment problem to find maximum total expected sale.

	I	**II**	**III**	**IV**
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12

Solution:

	I	II	III	IV	Penalty	Penalty	Penalty	Penalty
A	[42]	35	28	21	0	−−	−−	−−
B	30	[25]	20	15	0	0	−−	−−
C	30	25	[20]	15	0	0	5	−−
D	24	20	16	[12]	0	0	4	12
Penalty	6	5	4	3				
Penalty	−−	10	8	6				
Penalty	−−	−−	4	3				
Penalty	−−	−−	−−	12				

So the solution of given assignment problem is

	A	B	C	D
−−	*I*	*II*	*III*	*IV*
Units	42	25	20	12

Total Units = 99 Units,which can be comapared with the Hungarian method [4].

C) Maximize the given assignment problem.

	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Solution:

	A	B	C	D	E	Penalty	Penalty	Penalty	Penalty	Penalty
1	32	38	40	28	[40]	10	−−	−−	−−	−−
2	[40]	24	28	21	360	2	8	−−	−−	−−
3	41	27	[33]	30	37	−6	−4	9	−−	−−
4	22	[38]	41	36	36	−3	−5	−7	−2	38
5	29	33	40	[35]	39	2	−8	−9	2	−−
Penalty	−10	−8	−14	−3	8					
Penalty	0	2	2	−4	−−					
Penalty	−−	−10	−9	−11	−−					
Penalty	−−	−9	−18	−11	−−					
Penalty	−−	−4	−−	−−	−−					
Penalty	−−	38	−−	−−	−−					

So the solution of given assignment problem is

	1	2	3	4	5
−−	*E*	*A*	*C*	*B*	*D*
Units	40	40	33	38	35

Total Units = 186 Units which can be comapared with the Hungarian method [4].

D) A department has 04 tasks to be performed and 03 subordinates. the subordinates differ in efficiency. Find the allocation towards the task to each men to minimize the total time required in man - hours.

	1	**2**	**3**
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

Solution:

	1	**2**	**3**	**4**	**Penalty**	**Penalty**	**Penalty**	**Penalty**
I	[9]	26	15	0	-4	-3	-3	9
II	13	27	[6]	0	-4	-1	1	$--$
III	35	[20]	15	0	-5	$--$	$--$	$--$
IV	18	30	20	[0]	-4	2	$--$	$--$
Penalty	-6	3	2	0				
Penalty	-4	$--$	2	0				
Penalty	4	$--$	3	$--$				
Penalty	9	$--$	$--$	$--$				

So the solution of given assignment problem is

	I	**II**	**III**	**IV**
$--$	1	3	2	4
Units	9	6	20	0

Total Units = 35 Hours which can be comapared with the Hungarian method [4].

5 Conclusion

In this paper,the algebraic approach using congruence relation is to find the solution towards the assignment problem is very much simple as compare to other assignment method espacially Hungarian method. The Hungarian method takes too much time as the number of rows and columns were increased while in the above proposed mrethod we have to take row / column addition and apply simple congruence relation to find the remainder and then penalty.
As in Hungarian method; when the assignment problem is unbalanced one have to add dummy row / column with zero cost same can be done in this method.But, the method we have introduced reduces the time of calculation i.e. minimum time complexity as compare to other methods to solve assignment problems especially Hungarian Method.

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