An investigation on non-Darcy nanofluid flow due to the interaction of inclined magnetic field and nonlinear radiation

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Abstract

This study presents the mathematical computation for non-Darcy nanofluid flow through a permeable surface which is a vertical surface interacting with a magnetic field. Present study of nanofluid is very essential due to industrial applications for which inclusion of magnetic field, the effect of heat generation/absorption enhances the fluid character. The nonlinear coupled partial differential equations are converted into ordinary differential equations with help of dimensionless functions and further solved by employing approximate analytical method (Variational Iteration method)

Keywords: Nanofluid; magnetic field; nonlinear radiation, Non-Darcy nanofluid.

1 Introduction

As we are aware of the thermos-physical characteristics of nanofluids for which many industries are using nanofluids for heat transfer purposes. Choi [1] was the first person who put forward a discussion on nanofluid in 1995. Recently Elgazery [2] has considered a problem of nanofluid by taking four different types of nanoparticles and water as base fluid. As a result of this study, the author has concluded the cooling and heating effects of nanofluids on velocity and temperature profiles. Another brief experiment has been conducted by Pattnaik et al. [3] for Al2O3-Ethylene Glycol C2H6O2 Nanofluid which is based on the KKL model. Verma et al. [4] have studied numerically a steady 2-D viscous Cu-water and Ag-Water nanofluid in a porous medium (Darcian). A mathematical model and calculation to analyze simultaneously 3 models for rate of heat transfer, viscous flow including other effects has been studied in this problem. Kumaran et al. [5] have taken into account a theoretical investigation on nanofluid, specially authors have concentrated on thermal conductance, radiation and chemical reaction. Recently Jena et al. [6-7] have studied two different investigations on nanofluids by employing numerical and approximate analytical methods. Through these studies authors have emphasized on magnetic parameter, porous matrix, chemical reaction parameter and heat source parameter, Joule dissipation, heat source/sink. A different kind of 2 carbon nanotube nanoparticles study has been carried out by Berrehal et al. [8]. This different model has been tackled by authors using homotopy perturbation method. Recently Beg et al. [9-10] have simulated different couple-stress nanofluids in the cited studies.

1. Mathematical procedure

Let us consider a 2-dimensional flow of conducting nano fluid past an expanding surface in this study. A free convection stagnation flow through a permeable medium has been presenting this study.An inclined magnetic field of strengthB_0,velocity of the sheet is $u_w$being applied and depending on the flow behavior it is wise to neglect the assumed external forces and pressure gradient. Assuming the wall temperature and concentration have dominating nature in comparison to the ambient state. The governing equations obtained by referring to Elgazery [2]are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1.1}$$

$$\rho_{n_f}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{nf}\frac{\partial^2 u}{\partial y^2}+(\rho\beta)_{nf}\,g\,(T-T_\infty)-\sigma_{nf}B_0^2\sin^2\gamma u \tag{1.2}$$

$$(\rho c_p)_{nf}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_{nf}\frac{\partial^2 T}{\partial y^2}+Q\,(T-T_\infty)+\sigma_{nf}B_0^2\sin^2\gamma u^2 \tag{1.3}$$

The boundary conditions are defined as

$$\begin{array}{ll} u=u_w=\frac{ax}{1-ct}, v=v_w=-\left(\frac{a\nu_f}{1-ct}\right)^{\frac{1}{2}}f_w, T=T_w-T_\infty=\frac{bx}{(1-ct)^2}, & \text{at} \;\; y=0 \\ u=0, \qquad T=T_\infty, & \text{as} \;\; y\to\infty \end{array} \tag{1.4}$$

By incorporating the physical nanofluid properties in terms of fluid and nanoparticles the below values are expressed as follows:

(1.5)

$$\rho_{n_f}=(1-\phi)\,\rho_f+\phi\rho_s, \mu_{nf}=\mu_f(0.904)e^{14.8\phi}, k_{nf}=k_f(1+1.72\phi),\; (\rho c_p)_{nf}=(1-\phi)\,(\rho c_p)_f+\phi(\rho c_p)_s,$$
$$(\rho\beta)_{nf}=(1-\phi)\,(\rho\beta)_f+\phi\,(\rho\beta)_s\,,\;\sigma_{nf}=\sigma_f\left(1+\frac{3(\sigma-1)\phi}{(\sigma+2)-(\sigma-1)\phi}\right),\sigma=\frac{\sigma_s}{\sigma_f}.$$

Using following dimensionless similarity variables such as dimensionless stream function and dimensionless temperature as given below:

$$\eta=\left(\frac{a}{\nu_f\,(1-ct)}\right)^{\frac{1}{2}}y, \psi(x,y,t)=\left(\frac{a\nu_f}{(1-ct)}\right)^{\frac{1}{2}}xf(\eta), \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{1.6}$$

Stream function $\psi(x, y)$satisfied with,
$u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$can be of the form,

$$u = \frac{a\nu_f}{1-ct} x f'(\eta), v = -\left(\frac{a\nu_f}{1-ct}\right)^{\frac{1}{2}} f(\eta). \tag{1.7}$$

Applying (5-7), the momentum Eq. and the energy Eq. can be rewritten as,

$$(0.904)e^{14.8\phi} f''' + A_3 \left\{ f f'' - f'^2 - S\left(f' + \frac{\eta}{2} f''\right)\right\} + A_1 \lambda\theta - A_2 M \sin^2 \gamma f' = 0 \tag{1.8}$$

$$(1 + 1.72\phi)\theta'' + \mathrm{Ec}\,\mathrm{Pr}\, A_2 M \sin^2 \gamma f'^2 + \mathrm{Pr}\left(f\theta' - A_4 \frac{\eta}{2} S\theta' + F\theta\right) = 0 \tag{1.9}$$

The transformed boundary conditions are

$$\left.\begin{array}{l} f(0) = f_w, f'(0) = 1, \theta(0) = 1, \\ f'(\infty) = 0, \theta(\infty) = 0 \end{array}\right\} \tag{1.10}$$

where the dimensionless parameters are defined as,
(1.11)

$$\left.\begin{array}{l} M\,(\text{magnetic parameter}) = \frac{\sigma_f \nu_f B_0^2}{\rho_f b}, \lambda\,(\text{thermal buoyancy parameter}) = \frac{\beta_f b g (T_w - T_\infty)}{a^2}, \\ Ec\,(\text{Eckert number}) = \frac{a^2 x}{b(\rho c_p)_f}, \mathrm{Pr}\,(\text{Prandtl number}) = \frac{\mu_f (\rho c_p)_f}{k_f}, \\ S\,(\text{unsteadiness parameter}) = \frac{c}{a}, F\,(\text{Heat generation/absorption}) = \frac{Q(1-ct)\rho_f k_f}{a(\rho c_p)_f}, \\ A_1 = \frac{(\rho\beta)_{nf}}{(\rho\beta)_f}, A_2 = \frac{\sigma_{nf}}{\sigma_f}, A_3 = \frac{\rho_{nf}}{\rho_f}, A_4 = \frac{(\rho c_p)_{nf}}{(\rho c_p)_f} \end{array}\right\}.$$

Methodology
If we consider below non-linear system

$$Su + Tu = F(x) \tag{1.12}$$

Where $S$and $T$are linear and nonlinear operators respectively.
Then $u_{n+1}(x) = u_n(x) + \int_0^x \lambda \left[S u_n(\varsigma) + T\overline{u_n}(\varsigma) - F(\varsigma)\right] d\varsigma$
The above equation is called Variational Iteration method VIM.
Where $\lambda$is the Lagrange's multiplier, $\overline{u_n}$is the restricted variation.

Results and discussion
The time-dependent nanofluid flow through a permeable vertical surface is considered in the present investigation in conjunction with the free convection effect and the influence of the inclined magnetic field. However, the present flow phenomena are equipped with

the kerosene-based nanofluid with the inclusion of metal such as Cu and oxide like Al_2O_3. The transformed governing equations are characterized by certain parameters that are formed due to the inclusion of thermophysical properties i.e., viscosity, the conductivity of nanofluid considering various models presented earlier. Approximate analytical technique such as Variational Iteration method is proposed for the complex nonlinear problem and the performance of the physical parameters are presented through graphs. The physical properties of nanoparticles along with the base liquid kerosene for the fixed temperature of 298^0K are presented in Table.1.

Table 1: Nanoparticles as well as base fluid properties

Properties	Cu	Al_2O_3	**Kerosene**
C_p **(J/Kg K)**	**385**	**765**	**2090**
ρ **(Kg/m^3)**	**8933**	**3970**	**783**
κ **(W/mK)**	**400**	**40**	**0.145**
σ**(S/m)**	$\mathbf{5.96}\times 10^7$	$\mathbf{35}\times 10^6$	**0.149**
β**(S^{-1})**	$\mathbf{3.05}\times 10^{-6}$	$\mathbf{0.85}\times 10^{-5}$	$\mathbf{298.2}\times 10^{-6}$

The resulting discussions present the effects of various parameters showing the above phenomena. Fig.1 and Fig.2 present the velocity and temperature distribution exhibiting the effect of nanoparticle volume fraction(ϕ), Unsteadiness parameter(S), Magnetic parameter(M) , and the behavior of inclined angle(γ).In both profiles,it has been tested for pure fluid ($\phi = 0$) as well as for nanofluid ($\phi = 0.1, 0.2$). Due to free convection near the surface, the fluid velocity and fluid temperature accelerate upward and for both, the profiles give a similar result. But Figure (b) in both Figs. 1 and 2 illustrate the impact of unsteadiness parameter(S)result reverse effect on both the profiles. The fluid velocity gradually decreases but temperature gradually increases. The same reverse effect is encountered in the case of Magnetic parameter(M) and inclined angle(γ). Fluid velocity decreases for both the parameters whereas fluid temperature increases with increasing parametric value. Fig.3 renders the effect of $f_w$and λon velocity profile whereas the effect of Prand Fon the temperature profile. It is demonstrated that the efficiency of the diffusers in conjunction with the high compression ratio caused by the suction, early separation occurs and the fluid velocity decelerates. The reverse effect is encountered in the case ofλ. Prandtl number and heat generation/absorption reflects the same characteristic that both parameters decrease the temperature profile.

Concluding Remarks

The behavior of the distinguished parameters is presented and elaborated clearly. Further, the major concluding statements are presented below:

1. The augmentation in the volume concentration enriches the fluid temperature significantly.

2. An enhanced unsteadiness factor decreases the velocity boundary layer to control the fluid velocity whereas the reverse impact is encountered for the fluid temperature.

3. Nanoparticle volume fraction, Unsteadiness parameter, Magnetic parameter, and inclined angleare the enhancingfactor for heat transfer.

4. Prandtl number and heat generation/absorption are decelerating factors for the temperature profile.

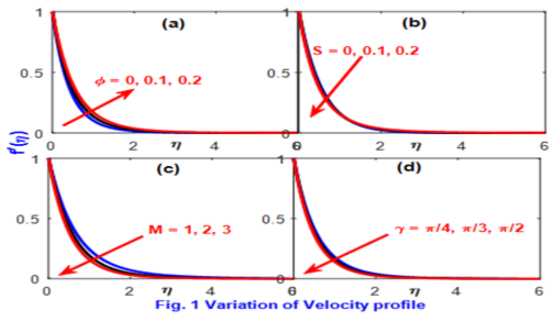

Fig. 1

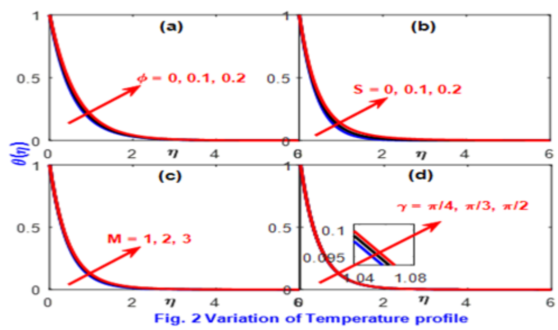

Fig. 2

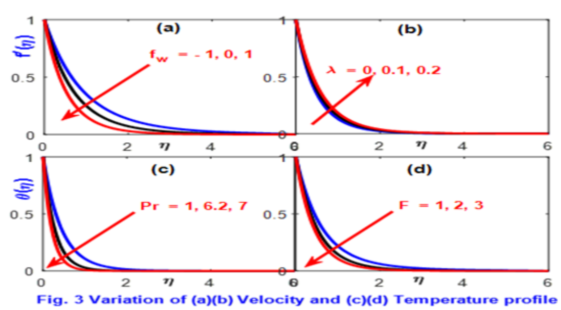

Fig. 3

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