Inflationary Bianchi type-II spacetime with exponential potential

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Abstract

In the present study, I have calculated LRS Bianchi Type II inflationary spacetime under the influence of effective potential $V(\phi) = exp(-\lambda\phi)$, $\phi > 0$, where ϕ represents Higg's field. To obtain the precise solution average scale factor $\widetilde{a}(t)$ is advised as $\widetilde{a}^3 = (R^2)S = exp[\lambda\phi(t)]$. The model becomes isotropic and shear-free in a special case. Also, the model exhibits no singularity at the initial stage. I have discussed the physical and kinematic behavior of the model using some dynamical parameters.

Subject Classification:[2020]PACS number: 04.20.Jb, 98.80.Cq
Keywords: Bianchi type-II model, Inflationary universe, Exponential potential, Higg's field.

1 Introduction

The word 'Inflation' indicates the enormously fast extension of the Universe. The scenario of inflationary Universe primarily developed in inquiring about the first-order phase transitions. Gliner [1] proposed that the universe which is governed by vacuum should be consistent with the De Sitter model with positive energy density. Zeldovich [2] extended this idea later on. In the framework of gauge theories, Linde [3] suggested that the exact evaluation of phase transitions was close to the present inflationary situation. The cosmological consequences of the first-order phase transition were studied by Kolb and Wolfram [4] and they found that the rate of expansion of the Universe was exponential in place of a simple power law. Initially, Starobinsky [5] has given the idea about the inflationary cosmological model then independently Guth [6] has introduced his world-view changing work on Inflation for solving the problem of excessive production of magnetic monopoles using grand unified theories and found it was due to a fast extension of the Universe. This notion rapidly became a remedy for the horizon: isotropy, flatness: entropy, and other cosmological problems. Kazanas [7] then explained that the results of phase transitions of first-order can significantly change the law of extension of the early universe and under the appropriate assumptions the exponential expansion might be existed long enough for the mass of the associated Higgs boson and could justify the said isotropy of the Universe. Guth [8] acknowledged that if the rate of nucleation is not large enough, such a phase transition may never complete. The spontaneous symmetry breaking and its associated phase transition is the survival of a certain zero rotation field $\phi(x)$, called the Higgs field. The Higgs field has a potential energy function denoted by $V(\phi)$. Stein-Schabes [9] has explained that inflation occurs when Higg's field (ϕ) progresses slowly and the effective potential $V(\phi)$ shows a flat region, but the universe expands exponentially because of the energy of the vacuum field. Several authors viz. Benisty et al. [10], Bali and Jain [11], Naidu et al. [12], Bali [13] discussed the inflationary situation of the universe using different forms of the scalar field (ϕ) in general relativity. For unfolding the development of the universe in its early stages and for resolution and specification of the wide-ranged formation of the universe, Bianchi Type II spacetimes possess its significant role. Several researchers have considered LRS Bianchi Type II spacetimes as Asseo and Sol [14] highlighted its cosmological significance, Hajj-Boutros [15] have considered it for a perfect fluid distribution of matter which is precisely solved by a generating technique, Amirhashchi et al. [16] studied it for inflationary models using stiff fluid for vanishing Cosmological Term, Venkateswarlu and Reddy [17] presented its accurate solutions for stiff fluid concerning an electromagnetic field theory. In LRS Bianchi type II spacetime with flat potential using a massless scalar field in an inflationary situation, Singh and Kumar [18], Bali and Poonia [19] observed the evolution of anisotropy during the inflationary era keeping in mind the proposal made by Rothman and Ellis [20], Ram and Priyanka [21], Singh et al. [22], Bali and Swati [23] have studied this situation for time-variant Λ. Bari et al [24] observed inflation for exponential gravitation, Odinstov et al. [25] considered dynamics of inflation for a certain class of gravity. Recently some researchers studied inflation taken various conditions as [26-29]. In this paper, I have considered a scalar potential as an exponential function of Higg's field (ϕ), which is taken as $V(\phi) = exp(-\lambda\phi)$, $\phi > 0$, for the study of an inflationary framework in spatially homogeneous and anisotropic Bianchi Type-II spacetime and tried to describe that inflationary framework can be proposed for an anisotropic and homogeneous metric with exponential potential and the model attains isotropy in a special case.

2. Field and Metric Equations

The metric for LRS Bianchi type-II spacetime is taken as

$$ds^2 = \eta_\alpha \beta \theta^\alpha \theta^\beta, \tag{1.1}$$

$$\theta^2 = S[dy + (-x)dz] \tag{1.2}$$

Using equation (1.2), metric (1.1) takes the form as

$$ds^2 = -dt^2 + R^2 dx^2 + S^2(dy - xdz)^2 + R^2 dz^2 \tag{1.3}$$

Here, R and S depend upon cosmic time t. The Lagrangian is that of gravity minimally coupled to ϕ with effective potential $V(\phi)$ is taken as

$$L = \int \left\{\sqrt{-g}\right\} \left(R - V(\phi) - \frac{1}{2} g_{ij} \partial_i \phi \delta^j \phi\right) d^4x \tag{1.4}$$

We can get Einstein Field equations from the variation of L concerning the massless scalar fields, as

$$R_{ij} - \frac{1}{2} g_{ij} R = -8\pi T_{ij} \tag{1.5}$$

Here, it is assumed that gravitational units $G = c = 1$. T_{ij} are the energy-momentum tensors, which are given as

$$T_{ij} = \phi_{,i}\phi_{,j} - \left(\frac{1}{2}\phi_{,\rho}\phi^{,\rho} + V(\phi)\right) g_{ij} \tag{1.6}$$

$$\frac{1}{\sqrt{-g}}\left[\sqrt{-g}\phi_{,i}\right]_{,i} = -\frac{dV}{d\phi} \tag{1.7}$$

Where v^i the flow vector, g^{ij} the metric tensor and here

$$\phi_{,i} = \frac{\partial\phi}{\partial x^i}, \quad \phi^{,\rho} = g^{\rho l}\frac{\partial\phi}{\partial x^l} \tag{1.8}$$

Now using equations (1.6)-(1.8) with metric (1.3), Einstein field equations (1.5) reduced as

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 - \frac{3}{4}\frac{S^2}{R^4} = -8\pi\left[\frac{\dot{\phi}^2}{2} - V(\phi)\right] \tag{1.9}$$

$$2\frac{\dot{R}}{R}\frac{\dot{S}}{S} + \left(\frac{\dot{R}}{R}\right)^2 - \frac{1}{4}\frac{S^2}{R^4} = 8\pi\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right] \tag{1.10}$$

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{1}{4}\frac{S^2}{R^4} = -8\pi\left[\frac{\dot{\phi}^2}{2} - V(\phi)\right] \tag{1.11}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{S}}{S} + 2\frac{\dot{R}}{R}\right) = -\frac{dV}{d\phi} \tag{1.12}$$

3. The solution of Field Equations (Inflationary model)

To attain the substantially relevant outcomes, we assume effective potential $V(\phi)$and average scale factor $(\widetilde{a})$ for Bianchi type II space-time (1.3) as

$$V = e^{-\lambda\phi}, \lambda > 0 \tag{1.13}$$

$$\widetilde{a}^3 = R^2 S = e^{\lambda\phi(t)} \tag{1.14}$$

Now using Eq. (1.13) in Eq. (1.12), we have

$$\ddot{\phi} + \lambda\dot{\phi}^2 = \lambda e^{-\lambda\phi} \tag{1.15}$$

Equation (1.15) can be solved as

$$\frac{d}{d\phi}f^2 + 2\lambda f^2 = 2\lambda e^{-\lambda\phi} \tag{1.16}$$

where$\dot{\phi} = f(\phi)$, $\ddot{\phi} = ff'$, $f' = \frac{df}{d\phi}$. From eq.(1.16) we have

$$f^2 = 2e^{-\lambda\phi} + \ell e^{-2\lambda\phi} \tag{1.17}$$

Here, $\ell$is a constant of integration. From Equation (1.17), we get

$$e^{\lambda\phi} = \alpha t^2 + \beta t + \gamma \tag{1.18}$$

Here$\alpha = \lambda^2/2, \beta = \lambda\sqrt{2}\beta', \gamma = \beta'^2 - \ell/2$, $\beta'$is a constant of integration. Thus using (1.18) in (1.14), we lead to

$$R^2 S = \alpha t^2 + \beta t + \gamma \tag{1.19}$$

From the above equation, we get

$$\frac{\dot{S}}{S} + 2\frac{\dot{R}}{R} = \lambda\dot{\phi} = \frac{2\alpha t + \beta}{\alpha t^2 + \beta t + \gamma} \tag{1.20}$$

Now using equations (1.10), (1.11), (1.13), (1.15) and (1.20) we get

$$\frac{\ddot{R}}{R} + \lambda\dot{\phi}\frac{\dot{R}}{R} - \left(\frac{\dot{R}}{R}\right)^2 = aV(\phi) \tag{1.21}$$

Where, $a = (\lambda^2 - 16\pi)$. From eq. (1.21) we have

$$\frac{\dot{R}}{R} = \frac{at + b}{\alpha t^2 + \beta t + \gamma} \tag{1.22}$$

Here, b is a constant of integration. Solving Eq. (1.22) we have

$$R = N\left(\alpha t^2 + \beta t + \gamma\right)^{\frac{a}{2\alpha}} \left(\frac{t - m + d}{t + m + d}\right)^{M/2m\alpha} \tag{1.23}$$

Using eq. (1.14) and (1.23), we have

$$S = \frac{1}{N^2}\left(\alpha t^2 + \beta t + \gamma\right)^{1-\frac{a}{\alpha}} \left(\frac{t + m + d}{t - m + d}\right)^{M/m\alpha} \tag{1.24}$$

Where, $d = \frac{\beta}{2\alpha}, m^2 = d^2 - \frac{\gamma}{\alpha}, M = b - ad$, N is a constant of integration. Hence the metric (1.3) takes the form

$$ds^2 = -dt^2 + N^2 \left(\alpha t^2 + \beta t + \gamma\right)^{\frac{a}{\alpha}} \left(\frac{t-m+d}{t+m+d}\right)^{\frac{M}{m\alpha}} (dx^2 + \tag{1.25}$$

$$+\frac{1}{N^4} \left(\alpha t^2 + \beta t + \gamma\right)^{2\left(1-\frac{a}{\alpha}\right)} \left(\frac{t+m+d}{t-m+d}\right)^{\frac{2M}{m\alpha}} (dy - x\,dz)^2 \tag{1.26}$$

At t=0, we see that the model (1.25) is singularity-free, and in general, the model represents the anisotropic universe.

4. Some physical features of the model

One can observe that model (1.25) along with equations (1.23) and (1.24) represents the LRS Bianchi type-II metric in the existence of exponential potential $V(\phi)$. Some dynamical parameters are discussed here to examine the existence of our model.

From eq. (1.18), we get the Higg's Field as

$$\phi = \frac{1}{\lambda} \log(\alpha t^2 + \beta t + \gamma) \tag{1.27}$$

Using eq. (1.18) in (1.13), we get the effective potential as

$$V(\phi) = \frac{1}{\alpha t^2 + \beta t + \gamma} \tag{1.28}$$

Using (1.18) in (1.14), the average scale factor $\widetilde{a}(t)$ is taken as

$$\widetilde{\mathrm{a}}(\mathrm{t}) = \left(\alpha t^2 + \beta t + \gamma\right)^{1/3} \tag{1.29}$$

and Spatial volume (τ) of the model is

$$\tau = \widetilde{\mathrm{a}}^3 = SR^2 = (\alpha t^2 + \beta t + \gamma) \tag{1.30}$$

Using eq. (1.20), H, the average Hubble parameter and θ, the expansion can be given as

$$H = \frac{1}{3}\left(\frac{\dot{S}}{S} + 2\frac{\dot{R}}{R}\right) = \frac{1}{3}\frac{(2\alpha t + \beta)}{(\alpha t^2 + \beta t + \gamma)} \tag{1.31}$$

$$\theta = 3H = \left(\frac{\dot{S}}{S} + 2\frac{\dot{R}}{R}\right) = \frac{(2\alpha t + \beta)}{(\alpha t^2 + \beta t + \gamma)} \tag{1.32}$$

The shear (σ) is

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{R}}{R} - \frac{\dot{S}}{S}\right) = \frac{1}{\sqrt{3}}\left(\frac{(3a-2\alpha)t + (3b-\beta)}{\alpha t^2 + \beta t + \gamma}\right) \tag{1.33}$$

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} \left(\frac{(3a - 2\alpha)t + (3b - \beta)}{2\alpha t + \beta} \right)$$

Average anisotropy parameter is

$$A_h = 2 \left(\frac{(3a - 2\alpha)t + (3b - \beta)}{2\alpha t + \beta} \right)^2 \tag{1.34}$$

And the deceleration parameter is

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{1}{2} + \frac{3(\beta^2 - 4\alpha\gamma)}{2(2\alpha t + \beta)^2} \tag{1.35}$$

5. Conclusion

The model illustrates a Bianchi type II anisotropic model, in the existence of an engaging massive scalar field ϕ, in general relativity. The model exhibits a continuous extension starting from determinate volume and this leads to inflation. Also, a power-law relation exists here, between scalar field ϕ and average scale factor $\widetilde{a}$. The kinematic parameters $\tau, \widetilde{a}, A_h, \phi$ provide us determinate values at $t = 0$. The model discussed here, has not shown initial singularity. The spatial volume τ rises exponentially with time t and tends to infinity as t tends to infinity. Therefore our model is an inflationary model. If t tends to infinity then scalar potential $V(\phi)$ tends to zero, it shows that at large values of t, the model exhibits a flat potential. The inflationary scenario predicts that the spatial geometry of our universe must be flat. For cosmic time t, the deceleration parameter q is negative, when $(2\alpha\ t + \beta)^2 + 3\beta^2 < 4\alpha\ \gamma$, hence we will have an expanding and accelerating phase of the universe, and q is positive, if $(2\alpha\ t + \beta)^2 + 3\beta^2 > 4\alpha\gamma$, which represents a decelerating phase of the universe. If $(2\alpha t + \beta)^2 + 3\beta^2 = 4\alpha\gamma$, then $q = 0$, in this situation every galaxy moves at a constant speed. The average Hubble parameter, expansion, and shear tensor attain constant values at the initial epoch. We find out here that $\frac{\sigma}{\theta} \neq 0$ which indicates that the model taken here is anisotropic, in general. Especially when, $a \to \frac{2\alpha}{3}$ $a \to \frac{2\alpha}{3}, b \to \frac{\beta}{3}$, then $\frac{\sigma}{\theta} \to 0$ at these values the model becomes isotropic and shear free. It is interesting to note that ϕ increases slowly and it tends to infinity when$t \to \infty$ (Stein-Schabes [10]). It shows an asymptotic behavior initially. If we assume $\beta = 0$, $\gamma = 0$, then the model represents FLRW model where average scale factor $\widetilde{a} \propto t^{2/3}$, the Hubble parameter $(H) \propto 1/t$, $\theta \propto 1/t$ and deceleration parameter $(q) =$.

With the help of the dynamical parameters, we discussed here the physical behavior of the model and observed that our model represents an inflationary, anisotropic universe and this inflationary situation can solve the flatness problem because space was stretched out so much that it became a geometrically flat universe after the inflationary epoch. Also, the model attains isotropy in a special case. The decelerating and accelerating phases of the universe attained here matches with recently observed astronomical observations.

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