Graceful Labeling for Some snake related Graphs

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Abstract

Point of this paper is to demonstrate a new graph which is called barycentric subdivision of cycle C_n snake BSC_n. We got a new graph which is called barycentric subdivision of cycle C_n snake BSC_n. We derived that the barycentric subdivision of cycle C_n snake BSC_n and barycentric subdivision of alternate cycle C_n snake $ABSC_n$ are graceful. We also proved that quadrilateral (Q_n) and barycentric subdivision of alternate cycle C_n snake (BSC_n) are graceful.

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1 Introduction

The idea of graceful labeling was presented by Rosa [7] in 1967 and for numbering in graph was characterized by S.W.Golomb [5]. Numerous researchers have considered gracefulness of graphs, refer Gallian survey [4]. A decent number of papers are found with assortment of utilizations in coding theory, radar communication, cryptography etc. A profundity insights regarding utilizations of graph labeling is found in Bloom and Golomb [2]. We acknowledge all documentations and phrasing from Harary [6]. We review a few definitions which are use in this paper.

A function f is called graceful labeling of a graph $G = (V, E)$ if $f : V \to \{0, 1, ..., q\}$ is injective and the induce function $f^* : E \to \{1, ..., q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u, v) \in E(G)$. A graph G is called graceful graph if it admits a graceful labeling.

A quadrilateral snake Q_n is obtained from a path $u_1, u_2, ..., u_n$ by joined u_i, u_{i+1} to new vertices v_i and w_i respectively and adding edges v_i w_i for i=1, 2, ..., n-1. That is every edge of a path is replaced by a cycle C_4.

An alternate quadrilateral snake $A(QS_n)$ is obtained from a path $u_1, u_2, ..., u_n$ by joined

u_i, u_{i+1} (alternatively) to new vertices v_i and w_i respectively and adding edges $v_i\ w_i$ where $(1 \leq i \leq n-1)$ for even n and $(1 \leq i \leq n-2)$ for odd n. That is every alternate edge of a path is replaced by a cycle C_4.

Let G = (V, E) be a graph if every edge of graph G is subdivided, then the resulting graph is called barycentric subdivision of graph G. In other words barycentric subdivision is the graph obtained by inserting a vertex of degree 2 into every edge of original graph. The barycentric subdivision of any graph G is denoted by S(G). It is easy to observe that $|VS(G)| = |V(G)| + |E(G)|$ and $|ES(G)| = 2|E(G)|$.

1.1 The barycentric subdivision of cycle C_n snake graph BSC_n:

The barycentric subdivision of cycle C_n snake graph BSC_n is obtained from the path P_n through replacing every edge of a path by barycentric subdivision of cycle C_n, $n \equiv 0, 2$ (mod 4).

An alternate barycentric subdivision of cycle C_n snake $ABSC_n$ is every alternate edge of path is replaced by barycentric subdivision of cycle C_n, $n \equiv 0, 2$ (mod 4).

In this paper, we presented gracefulness of a barycentric subdivision of cycle C_n snake BSC_n, a barycentric subdivision of alternate cycle C_n snake $ABSC_n$, a quadrilateral (Q_n) and a barycentric subdivision of alternate cycle C_n snake (BSC_n). For detail overview of graph labeling, we allude Gallian [4].

2 Main Results:

2.1 Theorem

The barycentric subdivision of cycle C_n snake graph BSC_n is a graceful graph, where $n \equiv 0, 2$ (mod 4).

***Proof*:** Let vertices of $BSC_n = \{c_1, c_2, ..., c_n\}$ and edges of $(BSC_n) = \{e_1, e_2, ..., e_{n-1}\}$. To build BSC_n,

***Case-1*:** For n = 4.

Here p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices, $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and edges q = $\{e_1, e_2, ...\}$ of BSC_4 .

$f : v \rightarrow \{0, 1, ...q\}$, where q (no. of edges for graph G) = 8i.

$|V(BSC_4)| = |c_j| + |d_j|$ = 7i + 1, $|E(BSC_4)|$ = 8i.

For vertices of BSC_4

c_1 = 8i $\quad$ c_2 = 8i - 1

c_3 = 8i - 2 $\quad$ c_4 = 8i - 3

.

c_j = 8i - (j-1).

$(\forall\ j = 1, 2, ..., \forall\ i = 1, 2, ...)$.

(j = labeling in i^{th} cycle, i = number of cycles.)

$|c_j|$ = 4 + 3(i-1), $|d_j|$ = 4i, $|e_j|$ = 8i.

For edges

e_1 = 8i $\quad$ e_2 = 8i - 1

e_3 = 8i - 2 e_4 = 8i - 3

.

$e_{(8i)} = 1$.

Where

$e_{(8i-7)} = d_{(4i-3)}\ c_{(3i-2)}$ $e_{(8i-6)} = d_{(4i-3)}\ c_{(3i-1)}$

$e_{(8i-5)} = d_{(4i-2)}\ c_{(3i-1)}$ $e_{(8i-4)} = d_{(4i-2)}\ c_{(3i)}$

$e_{(8i-3)} = d_{(4i)}\ c_{(3i-2)}$ $e_{(8i-2)} = d_{(4i-1)}\ c_{(3i)}$

$e_{(8i-1)} = d_{(4i-1)}\ c_{(3i+1)}$ $e_{(8i)} = d_{(4i)}\ c_{(3i+1)}$

And

$d_{(4i-3)}$ = 5i - 5 $d_{(4i-2)}$ = 5i - 4

$d_{(4i-1)}$ = 5i - 2 $d_{(4i)}$ = 5i - 1.

Define induced edge labeling function by $f^* : E \to \{1, ..., q\}$ defined as $f^*(e) = |f(u)-f(v)|$ is bijective for every edge e = (ei, ej) in BSC_4. Therefore barycentric subdivision of cycle C_n snake graph BSC_4 is recognition graceful labeling. Henceforth given graph is graceful.

2.2 Illustration

The barycentric subdivision of cycle C_n snake graph BSC_n shown includes n = 4, i = 3 (the number of snakes), and has a graceful labeling.

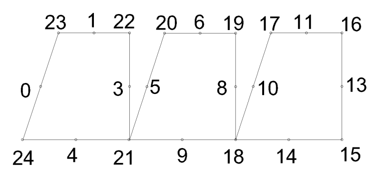

figure 1: Graceful labeling of barycentric subdivision of cycle C_n snake graphs BSC_n with $p = 22$ and $q = 24$.

***Case-2*:** For n = 6

Here p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices, $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and q = $\{e_1, e_2, ...\}$ are edges of BSC_6 .

$f : v \to \{0, 1, ...q\}$ where q (no.of edges of graph) = 12i.

$|V(BSC_6)| = |c_j| + |d_j|$ = 11i + 1, $|E(BSC_6)|$ = 12i.

For vertices

c_1 = 12i c_2 = 12i - 1

c_3 = 12i - 2 c_4 = 12i - 3

.

c_j = 12i - (j-1).

$(\forall\ j = 1, 2, ..., \forall\ i = 1, 2, ...)$.

(j = labeling in i^{th} cycle, i = number of cycles in a graph.)

$|c_j|$ = 6 + 5(i-1), $|d_j|$ = 6i, $|e_j|$ = 12i.

For edges

$e_1 = 12i$ $e_2 = 12i - 1$
$e_3 = 12i - 2$ $e_4 = 12i - 3$
.
$e_{(12i)} = 1.$
Where
$e_{(12i-11)} = d_{(6i-5)}\ c_{(5i-4)}$ $d_{(6i-5)} = 7i - 7$
$e_{(12i-10)} = d_{(6i-5)}\ c_{(5i-3)}$ $d_{(6i-4)} = 7i - 6$
$e_{(12i-9)} = d_{(6i-4)}\ c_{(5i-3)}$ $d_{(6i-3)} = 7i - 5$
$e_{(12i-8)} = d_{(6i-4)}\ c_{(5i-2)}$ $d_{(6i-2)} = 7i - 3$
$e_{(12i-7)} = d_{(6i-3)}\ c_{(5i-2)}$ $d_{(6i-1)} = 7i - 2$
$e_{(12i-6)} = d_{(6i-3)}\ c_{(5i-1)}$ $d_{(6i)} = 7i - 1.$
$e_{(12i-5)} = d_{(6i)}\ c_{(5i-4)}$
$e_{(12i-4)} = d_{(6i-2)}\ c_{(5i-1)}$
$e_{(12i-3)} = d_{(6i-2)}\ c_{(5i)}$
$e_{(12i-2)} = d_{(6i-1)}\ c_{(5i)}$
$e_{(12i-1)} = d_{(6i-1)}\ c_{(5i+1)}$
$e_{(12i)} = d_{(6i)}\ c_{(5i+1)}$

Define induced edge labeling function by $f^* : E \to \{1, ..., q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge e = (ei, ej) in BSC_6. Therefore barycentric subdivision of cycle C_n snake graph BSC_6 is recognition graceful labeling. Henceforth given graph is graceful.

***Case-3*:** In General

Similarly we can draw BSC_n for C_8, C_{10}, C_{12}, ...

Here p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices and $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and edges q = $\{e_1, e_2, ...\}$ of BSC_n, where $n \equiv 0, 2 \pmod 4$.
$f : v \to \{0, 1, ...q\}$ where q = (4j + 4)i.
$|V(BSC_n)| = |c_j| + |d_j|$ = (4j + 3)i + 1, according to c_{2j+2} and $|E(BSC_n)|$ = (4j + 4)i.
For vertices
$c_1 = (4j + 4)i$ $c_2 = (4j + 4)i - 1$
$c_3 = (4j + 4)i - 2$ $c_4 = (4j + 4)i - 3$
.
$c_k = (4j + 4)i - (k - 1).$

($\forall\ j = 1, 2, ..., \forall\ i = 1, 2, ..., \forall\ k = 1, 2, ...$).
(j = labeling in i^{th} cycle, i = number of cycles and k is position according to c_{2j+2})
$|c_k|$ = (2j + 2) + (2j + 1)(i - 1), $|d_k|$ = (2j + 2)i, $|e_k|$ = (4j + 4)i.
For edges
$e_1 = (4j + 4)i$ $e_2 = (4j + 4)i - 1$
$e_3 = (4j + 4)i - 2$ $e_4 = (4j + 4)i - 3$
.
$e_{((4j+4)i)} = 1.$
Where
$e_{((4j+4)i-(4j+3))} = d_{(2j+2)i-(2j+1)}\ c_{(2j+1)i-(2j)}$
$e_{((4j+4)i-(4j+2))} = d_{(2j+2)i-(2j+1)}\ c_{(2j+1)i-(2j-1)}$

$e_{((4j+4)i-(4j+1))} = d_{(2j+2)i-(2j)}\ c_{(2j+1)i-(2j-1)}$
$e_{((4j+4)i-(4j))} = d_{(2j+2)i-(2j)}\ c_{(2j+1)i-(2j-2)}$
.
$e_{((4j+4)i-(2j+1))} = d_{(2j+2)i}\ c_{(2j+1)i-(2j)}$
$e_{((4j+4)i-(2j))} = d_{(2j+2)i-(j)}\ c_{(2j+1)i-(j-1)}$
.
$e_{((4j+4)i} = d_{(2j+2)i}\ c_{(2j+1)i+1}$

And
$d_{((2j+2)i-(2j+1))}$ = (2j + 3)i - (2j + 3)
$d_{((2j+2)i-(2j))}$ = (2j + 3)i - (2j + 2)
.
$d_{((2j+2)i-(k))}$ = (2j + 3)i - (k + 1)
$d_{((2j+2)i-(k-1))}$ = (2j + 3)i - (k)
.
$d_{((2j+2)i)}$ = (2j + 3)i - (1)

Define induced edge labeling function by $f^* : E \rightarrow \{1, ..., q\}$ defined as $f^*(e) = |f(u)-f(v)|$ is bijective for every edge e = (ei, ej) in BSC_n. Therefore barycentric subdivision of cycle C_n snake graph BSC_n is recognition graceful labeling. Henceforth given graph is graceful.

2.3 Theorem

Barycentric subdivision of an alternate cycle C_n snake graph $ABSC_n$ is a graceful graph, where $n \equiv 0, 2 \pmod 4$.
***Proof*:** Here vertices p = $\{c_1, c_2, ..., c_n\}$ and edges q = $\{e_1, e_2, ..., e_{n-1}\}$ for barycentric subdivision of an alternate cycle C_n snake graph $ABSC_n$. To build $ABSC_n$,

***Case-1*:** For n = 4

For $ABSC_4$ p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices, $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and q = $\{e_1, e_2, ...\}$ are edges taken.
$f : v \rightarrow \{0, 1, ...q\}$ where q (no. of edges of graph) = 8i + (2i - 2).
$|V(ABSC_4)| = |c_j| + |d_j|$ = 8i + (i - 1), $|E(BSC_4)|$ = 8i + (2i - 2).
For vertices
c_1 = 8i + (2i - 2) c_2 = 8i + (2i - 3)
c_3 = 8i + (2i - 4) c_4 = 8i + (2i - 5)
.
c_j = 8i - (2i - (j+1)).

$(\forall\ j = 1, 2, ..., \forall\ i = 1, 2, ...)$.
(j = labeling in i^{th} cycle, i = number of cycles.)
$|c_j|$ = 4i, $|d_j|$ = 5i - 1, $|e_j|$ = 8i + (2i - 2).
For edges
e_1 = 8i + (2i - 2) e_2 = 8i + (2i - 3)
.
$e_{((8i)+(2i-2))} = 1$.
Where

$e_{(10i-9)} = d_{(5i-4)}\ c_{(4i-3)}$ $d_{(5i-4)}$ = 6i - 6
$e_{(10i-8)} = d_{(5i-4)}\ c_{(4i-2)}$ $d_{(5i-3)}$ = 6i - 5
$e_{(10i-7)} = d_{(5i-3)}\ c_{(4i-2)}$ $d_{(5i-2)}$ = 6i - 3
$e_{(10i-6)} = d_{(5i-3)}\ c_{(4i-1)}$ $d_{(5i-1)}$ = 6i - 2
$e_{(10i-5)} = d_{(5i-1)}\ c_{(4i-3)}$ $d_{(5i)}$ = 6i - 1
$e_{(10i-4)} = d_{(5i-2)}\ c_{(4i-1)}$
$e_{(10i-3)} = d_{(5i-2)}\ c_{(4i)}$
$e_{(10i-2)} = d_{(5i-1)}\ c_{(4i)}$
$e_{(10i-1)} = d_{(5i)}\ c_{(4i)}$
$e_{(10i)} = d_{(5i)}\ c_{(4i+1)}$

Define induced edge labeling function by $f^* : E \to \{1, ..., q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge e = (ei, ej) in $ABSC_4$. Therefore barycentric subdivision of an alternate cycle C_n snake graph $ABSC_4$ is recognition graceful labeling. Henceforth given graph is graceful.

2.4 Illustration

Barycentric subdivision of an alternate cycle C_n snake graph $ABSC_n$ shown consisting n = 4, i = 3 (no. of snakes) with graceful labeling.

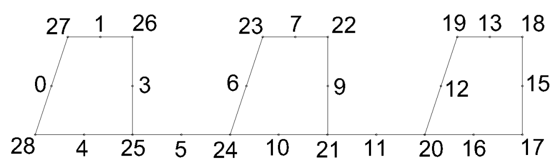

figure 2: Graceful labeling of barycentric subdivision of an alternate cycle C_n snake graph $ABSC_n$ with $p = 26$ and $q = 28$.

***Case-2*:** For n = 6

For $ABSC_6$, p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices, $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and q = $\{e_1, e_2, ...\}$ are edges taken.
$f : v \to \{0, 1, ...q\}$ where q (no. of edges) = 12i + (2i - 2).
No.of Vertices $|V(ABSC_6)| = |c_j| + |d_j|$ = 12i + (i - 1), No. of Edges $|E(BSC_6)|$ = 12i + (2i - 2).
For vertices
c_1 = 12i + (2i - 2) c_2 = 12i + (2i - 1).
c_3 = 12i + (2i) c_4 = 12i + (2i - (-1)).
.
c_j = 12i - (2i - (j + 1)).

$(\forall\ j = 1, 2, ...\forall\ i = 1, 2, ...)$.
(j = labeling in i^{th} cycle, i = number of cycles.)
$|c_j|$ = 6i, $|d_j|$ = 7i - 1, $|e_j|$ = 12i + (2i - 2) .
For edges
e_1 = 12i + (2i - 2) e_2 = 12i + (2i - 1).
.
$e_{(12i+(2i-2))} = 1$.
Where

$e_{(14i-13)} = d_{(7i-6)}\ c_{(6i-5)}$ $d_{(7i-6)}$ = 8i - 8
$e_{(14i-12)} = d_{(7i-6)}\ c_{(6i-4)}$ $d_{(7i-5)}$ = 8i - 7
$e_{(14i-11)} = d_{(7i-5)}\ c_{(6i-4)}$ $d_{(7i-4)}$ = 8i - 6
$e_{(14i-10)} = d_{(7i-5)}\ c_{(6i-3)}$ $d_{(7i-3)}$ = 8i - 5
$e_{(14i-9)} = d_{(7i-4)}\ c_{(6i-3)}$ $d_{(7i-2)}$ = 8i - 3
$e_{(14i-8)} = d_{(7i-4)}\ c_{(6i-2)}$ $d_{(7i-1)}$ = 8i - 2
$e_{(14i-7)} = d_{(7i-1)}\ c_{(6i-5)}$ $d_{(7i)}$ = 8i - 1
$e_{(14i-6)} = d_{(7i-3)}\ c_{(6i-2)}$
$e_{(14i-5)} = d_{(7i-3)}\ c_{(6i-1)}$
$e_{(14i-4)} = d_{(7i-2)}\ c_{(6i-1)}$
$e_{(14i-3)} = d_{(7i-2)}\ c_{(6i)}$
$e_{(14i-2)} = d_{(7i-1)}\ c_{(6i)}$
$e_{(14i-1)} = d_{(7i)}\ c_{(6i)}$
$e_{(14i)} = d_{(7i)}\ c_{(6i+1)}$

Define induced edge labeling function by $f^* : E \to \{1, ..., q\}$ defined as $f^*(e) = |f(u)-f(v)|$ is bijective for every edge e = (ei, ej) in $ABSC_6$. Therefore barycentric subdivision of an alternate cycle C_n snake graph $ABSC_6$ is recognition graceful labeling. Henceforth given graph is graceful.

***Case-3*:** In General

Similarly we can draw $ABSC_n$ for C_8, C_{10}, C_{12},....
p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices, $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and q = $\{e_1, e_2, ...\}$ are edges for $ABSC_n$, where $n \equiv 0, 2$ (mod 4)..
$f : v \to \{0, 1, ...q\}$ where q (no. of edges of graph) = (4j + 4)i + (2i - 2).
$|V(ABSC_n)| = |c_j| + |d_j|$ = (4j + 4)i + (i - 1), $|E(ABSC_n)|$ = (4j + 4)i + (2i - 2).
For vertices
c_1 = (4j + 4)i + (2i - 2)
c_2 = (4j + 4)i + (2i - 1)
c_3 = (4j + 4)i + (2i)
.
c_k = (4j + 4)i + (2i - (k + 1))

$(\forall\ j = 1, 2, ..., \forall\ i = 1, 2, ...)$.
(j = labeling in i^{th} cycle, i = number of cycles and k is position according to c_{2j+2})

$|c_j|$ = (2j + 2)i, $|d_j|$ = (2j + 3)i - 1, $|e_j|$ = (4j + 4)i + (2i - 2).
For edges

e_1 = (4j + 4)i + (2i - 2)
e_2 = (4j + 4)i + (2i - 3)
.
$e_{((4j+4)i)+(2i-2)} = 1.$
Where
$e_{((4j+6)i-(4j+5))} = d_{(2j+3)i-(2j+2)}\ c_{(2j+2)i-(2j+1)}$
$e_{((4j+6)i-(4j+4))} = d_{(2j+3)i-(2j+2)}\ c_{(2j+2)i-(2j)}$
$e_{((4j+6)i-(4j+3))} = d_{(2j+3)i-(2j+1)}\ c_{(2j+1)i-(2j)}$
.
$e_{((4j+6)i-(2j+4))} = d_{(2j+3)i-(j+2)}\ c_{(2j+2)i-(j)}$
$e_{((4j+6)i-(2j+3))} = d_{(2j+3)i-(1)}\ c_{(2j+2)i-(2j+1)}$
$e_{((4j+6)i-(2j+2))} = d_{(2j+3)i-(j+1)}\ c_{(2j+2)i-(j)}$
.
$e_{((4j+6)i} = d_{(2j+3)i}\ c_{(2j+2)i+1}$
And
$d_{((2j+3)i-(2j+2))}$ = (2j + 4)i - (2j + 4)
$d_{((2j+3)i-(2j+1))}$ = (2j + 4)i - (2j + 3)

.
$d_{((2j+2)i-(k+1))}$ = (2j + 4)i - (k + 2)
$d_{((2j+2)i-(k))}$ = (2j + 4)i - (k + 1)
.
$d_{(2j+3)i}$ = (2j + 4)i - (1)

Define induced edge labeling function by $f^* : E \to \{1, ..., q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge e = (ei, ej) in $ABSC_n$. Therefore barycentric subdivision of an alternate cycle C_n snake graph $ABSC_n$ is recognition graceful labeling. Henceforth given graph is graceful.

2.5 Theorem

Quadrilateral Q_n and barycentric subdivision of an alternate cycle C_n snake graph BSC_n is a graceful graph, where $n \equiv 0, 2 \pmod 4$.
***Proof*:** Let quadrilateral Q_n and barycentric subdivision of an alternate cycle C_n snake graph BSC_n denoted by G. Vertices of G are $\{c_1, c_2, ..., c_n\}$ and edges of G are $\{e_1, e_2, ..., e_{n-1}\}$. To build graph G,

***Case-1*:** For n = 4

Here p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices, $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and q = $\{e_1, e_2, ...\}$ edges for given graph.
$f : v \to \{0, 1, ...q\}$ where q (no. of edges) = $\frac{i}{2}[4 + 2n] + (i - 1)$.

(i = total no. of cycle in snake , n = no. of vertices in BSC_n, where $n \equiv 0, 2$ (mod 4)).
$|V(G)| = |c_j| + |d_j| = \frac{i}{2}[4 + 2n]$, $|E(G)| = \frac{i}{2}[4 + 2n] + (i - 1)$.
For vertices

$c_{(16j-14)}$ = (15j - 15) $c_{(16j-12)}$ = (15j - 13)
$c_{(16j-7)}$ = (15j -7) $c_{(16j-5)}$ = (15j - 5)
$c_{(16j-3)}$ = (15j -4) $c_{(16j-2)}$ = (15j - 3)
$c_{(16j-1)}$ = (15j - 2) $c_{(16j)}$ = (15j - 1)
And
$c_{(16j-15)} = \frac{i}{2}[4+2n]+[(i-(13j-12)]$
$c_{(16j-13)} = \frac{i}{2}[4+2n]+[(i-(13j-11)]$
$c_{(16j-11)} = \frac{i}{2}[4+2n]+[(i-(13j-10)]$
$c_{(16j-10)} = \frac{i}{2}[4+2n]+[(i-(13j-9)]$
$c_{(16j-9)} = \frac{i}{2}[4+2n]+[(i-(13j-8)]$
$c_{(16j-8)} = \frac{i}{2}[4+2n]+[(i-(13j-7)]$
$c_{(16j-6)} = \frac{i}{2}[4+2n]+[(i-(13j-6)]$
$c_{(16j-4)} = \frac{i}{2}[4+2n]+[(i-(13j-5)]$

$(\forall\ j=1,2,....,\forall\ i=1,2,...,\forall\ n=1,2,......)$.
(j = labeling in C_n, i = number of cycles in snake, n = no. of vertices in BSC_n.)
$|c_j|$ = 4i, $|d_j|$ = 2i, $|e_j| = \frac{i}{2}[4+2n]+(i-1)$.

For edges
$e_{(28j-27)} = \frac{i}{2}[4+2n]+[(i-(13j-12))]$
$e_{(28j-26)} = \frac{i}{2}[4+2n]+[(i-(13j-11))]$
$e_{(28j-25)} = \frac{i}{2}[4+2n]+[(i-(13j-10))]$
⋮
$e_{(28j)} = \frac{i}{2}[4+2n]+[(i-(13j-(-15)))]$
Where
$e_{(28j-27)} = c_{(16j-15)}\ c_{(16j-14)}$
$e_{(28j-26)} = c_{(16j-14)}\ c_{(16j-13)}$
$e_{(28j-25)} = c_{(16j-15)}\ c_{(16j-12)}$
$e_{(28j-24)} = c_{(16j-13)}\ c_{(16j-12)}$
$e_{(28j-23)} = c_{(16j-12)}\ c_{(16j-11)}$
$e_{(28j-22)} = c_{(16j-11)}\ d_{(8j-7)}$
$e_{(28j-21)} = c_{(16j-10)}\ d_{(8j-7)}$
$e_{(28j-20)} = c_{(16j-10)}\ d_{(8j-6)}$
$e_{(28j-19)} = c_{(16j-9)}\ d_{(8j-6)}$
$e_{(28j-18)} = c_{(16j-11)}\ d_{(8j-4)}$
$e_{(28j-17)} = c_{(16j-9)}\ d_{(8j-5)}$
$e_{(28j-16)} = c_{(16j-8)}\ d_{(8j-5)}$
$e_{(28j-15)} = c_{(16j-8)}\ d_{(8j-4)}$
$e_{(28j-14)} = c_{(16j-8)}\ c_{(16j-7)}$
$e_{(28j-13)} = c_{(16j-7)}\ c_{(16j-6)}$
$e_{(28j-12)} = c_{(16j-6)}\ c_{(16j-5)}$
$e_{(28j-11)} = c_{(16j-7)}\ c_{(16j-4)}$
$e_{(28j-10)} = c_{(16j-5)}\ c_{(16j-4)}$
$e_{(28j-9)} = c_{(16j-4)}\ c_{(16j-3)}$

.

$e_{(28j)} \quad = c_{(16j)}\ c_{(16j+1)}$

And

$d_{(8j-7)} = 15j - 12 \quad d_{(8j-6)} = 15j - 11$

$d_{(8j-5)} = 15j - 10 \quad d_{(8j-4)} = 15j - 8$

$d_{(8j-3)} = \frac{i}{2}[4+2n] + [(i - (13j - 4))]$

$d_{(8j-2)} = \frac{i}{2}[4+2n] + [(i - (13j - 3))]$

$d_{(8j-1)} = \frac{i}{2}[4+2n] + [(i - (13j - 1))]$

$d_{(8j)} \quad = \frac{i}{2}[4+2n] + [(i - (13j))]$

Define induced edge labeling function by $f^* : E \rightarrow \{1, ..., q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge e = (ei, ej) in G. Therefore quadrilateral Q_n and barycentric subdivision of an alternate cycle C_n snake graph BSC_4 is recognition graceful labeling. Henceforth given graph is graceful.

2.6 Illustration

For graph G shown consisting n = 4, i = 4 (no. of snakes) with graceful labeling.

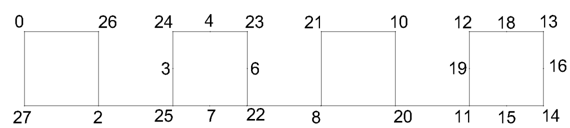

figure 3: Graceful labeling of graph G with $p = 24$ and $q = 27$.

***Case-2*:** For n = 6

Here p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices, $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and q = $\{e_1, e_2, ...\}$ are edges for G.

$f : v \rightarrow \{0, 1, ...q\}$ where q (no. of edges of graph) = $\frac{i}{2}[4+2n] + (i-1)$.

(i = total no. of cycle in snake, n = no. of vertices in G, where $n \equiv 0, 2 \pmod 4$).

$|V(G)| = |c_j| + |d_j| = \frac{i}{2}[4+2n]$, $|E(G)| = \frac{i}{2}[4+2n] + (i-1)$.

For vertices

$c_{(20j-18)}$ = (19j - 19) $\quad c_{(20j-16)}$ = (19j - 17)

$c_{(20j-9)}$ = (19j - 9) $\quad c_{(20j-7)}$ = (19j - 7)

$c_{(20j-5)}$ = (19j - 6) $\quad c_{(20j-4)}$ = (19j - 5)

$c_{(20j-3)}$ = (19j - 4) $\quad c_{(20j)}$ = (19j - 1)

.

And

$c_{(20j-19)} = \frac{i}{2}[4+2n] + [(i - (17j - 16)]$

$c_{(20j-17)} = \frac{i}{2}[4+2n] + [(i-(17j-15)]$
$c_{(20j-15)} = \frac{i}{2}[4+2n] + [(i-(17j-14)]$
$c_{(20j-14)} = \frac{i}{2}[4+2n] + [(i-(17j-13)]$
$c_{(20j-13)} = \frac{i}{2}[4+2n] + [(i-(17j-12)]$
$c_{(20j-10)} = \frac{i}{2}[4+2n] + [(i-(17j-9)]$
$c_{(20j-8)} = \frac{i}{2}[4+2n] + [(i-(17j-8)]$
$c_{(20j-6)} = \frac{i}{2}[4+2n] + [(i-(17j-7)]$

$(\forall\ j = 1, 2, ..., \forall\ i = 1, 2, ..., \forall\ n = 1, 2, ...)$.

(j = labeling in C_n, i = number of cycles in snake, n = no. of vertices in G.

$|c_j| =$ 5i, $|d_j| =$ 3i, $|e_j| = \frac{i}{2}[4+2n] + (i-1)$.
For edges
$e_{(36j-35)} = \frac{i}{2}[4+2n] + [(i-(17j-16))]$
$e_{(36j-34)} = \frac{i}{2}[4+2n] + [(i-(17j-15))]$
$e_{(36j-33)} = \frac{i}{2}[4+2n] + [(i-(17j-14))]$
$\vdots$
$e_{(36j)} = \frac{i}{2}[4+2n] + [(i-(17j-(-19)))]$
Where
$e_{(36j-35)} = c_{(20j-19)}\ c_{(20j-18)}$
$e_{(36j-34)} = c_{(20j-18)}\ c_{(20j-17)}$
$e_{(36j-33)} = c_{(20j-19)}\ c_{(20j-16)}$
$e_{(36j-32)} = c_{(20j-17)}\ c_{(20j-16)}$
$e_{(36j-31)} = c_{(20j-16)}\ c_{(20j-15)}$
$e_{(36j-30)} = c_{(20j-15)}\ d_{(12j-11)}$
$e_{(36j-29)} = c_{(20j-14)}\ d_{(12j-11)}$

.

$e_{(36j-24)} = c_{(20j-15)}\ d_{(12j-6)}$
$e_{(36j-23)} = c_{(20j-12)}\ d_{(12j-8)}$
.
$e_{(36j-18)} = c_{(20j-10)}\ c_{(20j-9)}$
.
$e_{(36j)} = c_{(20j-10)}\ c_{(20j+1)}$
And
$d_{(12j-11)} =$ 19j - 16 $\quad d_{(12j-10)} =$ 19j - 15
$d_{(12j-9)} =$ 19j - 14 $\quad d_{(12j-8)} =$ 19j - 12
$d_{(12j-7)} =$ 19j - 11 $\quad d_{(12j-6)} =$ 19j - 10
And
$d_{(12j-5)} = \frac{i}{2}[4+2n] + [(i-(17j-6))]$
$d_{(12j-4)} = \frac{i}{2}[4+2n] + [(i-(17j-5))]$
$d_{(12j-3)} = \frac{i}{2}[4+2n] + [(i-(17j-4))]$
$d_{(12j-2)} = \frac{i}{2}[4+2n] + [(i-(17j-2))]$
$d_{(12j-1)} = \frac{i}{2}[4+2n] + [(i-(17j-1))]$

$d_{(12j)} \quad = \frac{i}{2}[4+2n]+[(i-(17j))]$

Define induced edge labeling function by $f^*: E \rightarrow \{1,...,q\}$ defined as $f^*(e) = |f(u)-f(v)|$ is bijective for every edge e = (ei, ej) in G. Therefore quadrilateral Q_n and barycentric subdivision of an alternate cycle C_n snake graph BSC_6 is recognition graceful labeling. Henceforth given graph is graceful.

***Case-3*:** In General

Similarly we can draw graph G for C_8, C_{10}, C_{12}, ...

Here p = $\{c_1, c_2, c_3, c_4, ...\}$ are vertices and $\{d_1, d_2, d_3, d_4, ...\}$ are vertices inserted due to barycentric subdivision and q = $\{e_1, e_2, ...\}$ are edges for BSC_n, where $n \equiv 0, 2 \pmod 4$.
$f : v \rightarrow \{0, 1, ...q\}$ where q (no. of edges of graph) = $\frac{i}{2}[4+2n]+(i-1)$.
$|V(G)| = |c_j| + |d_j| = \frac{i}{2}[4+2n]$
$|E(G)| = \frac{i}{2}[4+2n]+(i-1)$.
For vertices
$c_{(4k+12)j-(4k+10)}$ = (4k + 11)j - (4k + 11)
$c_{(4k+12)j-(4k+8)}$ = (4k + 11)j - (4k + 9)
$c_{(4k+12)j-(2k+5)}$ = (4k + 11)j - (2k + 5)
$c_{(4k+12)j-(2k+3)}$ = (4k + 11)j - (2k + 3)
$c_{(4k+12)j-(2k+1)}$ = (4k + 11)j - (2k + 2)
$c_{(4k+12)j-(2k)}$ = (4k + 11)j - (2k + 1)
$c_{(4k+12)j-(2k-1)}$ = (4k + 11)j - (2k)

.

$c_{(4k+12)j-(2k-2k)}$ = (4k + 11)j - (1)
And
$c_{(4k+12j)-(4k+11)} = \frac{i}{2}[4+2n][i-((4k+9)j-(4k+8))]$
$c_{(4k+12j)-(4k+9)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(4k+7))]$
$c_{(4k+12j)-(4k+7)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(4k+6))]$
$c_{(4k+12j)-(4k+6)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(4k+5))]$
$c_{(4k+12j)-(4k+5)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(4k+4))]$
.
$c_{(4k+12j)-(2k+6)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(2k+5))]$
$c_{(4k+12j)-(2k+4)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(2k+4))]$
$c_{(4k+12j)-(2k+2)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(2k+3))]$

($\forall\ j = 1, 2, ...,\forall\ i = 1, 2, ...,\forall\ k = 1, 2, ...$).
(j = labeling in i^{th} cycle, i = number of cycles, k = labeling in j)

$|c_j|$ = (k+3)i, $|d_j|$ = (k+1)i, $|e_j| = \frac{i}{2}[4+2n]+(i-1)$
For edges

$$e_{(8k+20)j-(8k+19)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(4k+8))]$$
$$e_{(8k+20)j-(8k+18)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(4k+7))]$$
$$e_{(8k+20)j-(8k+17)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(4k+6))]$$
$$\cdot$$
$$e_{(8k+20)j-(8k-8k)} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(-(4k+11)))]$$

Where

$$e_{((8k+20)j-(8k+19))} = c_{(4k+12)j-(4k+11)}c_{(4k+12)j-(4k+10)}$$
$$e_{((8k+20)j-(8k+18))} = c_{(4k+12)j-(4k+10)}c_{(4k+12)j-(4k+9)}$$
$$e_{((8k+20)j-(8k+17))} = c_{(4k+12)j-(4k+11)}c_{(4k+12)j-(4k+8)}$$
$$e_{((8k+20)j-(8k+16))} = c_{(4k+12)j-(4k+9)}c_{(4k+12)j-(4k+8)}$$
$$e_{((8k+20)j-(8k+15))} = c_{(4k+12)j-(4k+8)}c_{(4k+12)j-(4k+7)}$$
$$\cdot$$
$$e_{((8k+20)j-(8k+14))} = c_{(4k+12)j-(4k+7)}d_{(4k+4)j-(4k+3)}$$
$$e_{((8k+20)j-(8k+13))} = c_{(4k+12)j-(4k+6)}d_{(4k+4)j-(4k+3)}$$
$$\cdot$$
$$e_{((8k+20)j-(6k+12))} = c_{(4k+12)j-(4k+7)}d_{(4k+4)j-(2k+2)}$$
$$e_{((8k+20)j-(6k+11))} = c_{(4k+12)j-(3k+6)}d_{(4k+4)j-(3k+2)}$$
$$\cdot$$
$$e_{((8k+20)j-(4k+10))} = c_{(4k+12)j-(2k+6)}c_{(4k+12)j-(2k+5)}$$
$$\cdot$$
$$e_{((8k+20)j)} = c_{(4k+12)j}c_{(4k+12)j+1}$$

And

$$d_{((4k+4)j-(4k+3))} = (4k+11)j-(4k+8)$$
$$d_{((4k+4)j-(4k+2))} = (4k+11)j-(4k+7)$$
$$\cdot$$
$$d_{((4k+4)j-(3k+2))} = (4k+11)j-(3k+6)$$
$$d_{((4k+4)j-(3k+1))} = (4k+11)j-(3k+5)$$
$$\cdot$$
$$d_{((4k+4)j-(2k+2))} = (4k+11)j-(2k+6)$$

And

$$d_{((4k+4)j-(2k+1))} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(2k+2))]$$
$$d_{((4k+4)j-(2k))} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(2k+1))]$$
$$\cdot$$
$$d_{((4k+4)j-(k))} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(k))]$$
$$d_{((4k+4)j-(k-1))} = \frac{i}{2}[4+2n]+[i-((4k+9)j-(k-1))]$$
$$d_{(4k+4)j} = \frac{i}{2}[4+2n]+[i-((4k+9)j)]$$

Define induced edge labeling function by $f^* : E \rightarrow \{1, ..., q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge e = (ei, ej) in G. Therefore quadrilateral Q_n and barycentric subdivision of an alternate cycle C_n snake graph BSC_n is recognition graceful labeling. Henceforth given graph is graceful.

2.7 Concluding Remark

The current work has contributed some new outcomes. We examined gracefulness of barycentric subdivision of cycle C_n snake BSC_n, barycentric subdivision of an alternate cycle C_n snake $ABSC_n$, quadrilateral Q_n and barycentric subdivision of an alternate cycle C_n snake graph. The labeling pattern is shown through representations to more readily comprehend the determined outcomes.

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