Application of a Soft graph in decision making using adjacency matrix of soft graph

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Abstract

In the present paper we give an application of soft graph in decision making using the adjacency matrix of soft graph. For this we have developed an algorithm and we give an example to illustrate.

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1 Introduction

In 1999, Molodtsov [3] introduced the concept of soft set, which can be seen a new mathematical tool for dealing with uncertainty. In soft set theory, the problem of setting the membership function does not arise, which makes the theory easily applied to many different fields. Most interesting application of soft set by parameterzation reduction was given by Degang Chen, E.C.C Tsang, Daniel S yeung, Xizhao Wang [4], also an application of soft sets in online shopping by normal parameter reduction algorithm was discussed by Xiuqin Ma, Hongwu Qin [15] and an application of soft sets in a decision making problem is discussed by P.K. Maji, A.R.Roy and R.Biswas [12]. Further, Rajesh K Thumbakara [13] and Bobin George have given a notion of soft graph which plays a crucial role in the subject. They have defined soft graph, soft graph homomorphism, soft graph isomorphism, soft complete graphs etc. In addition to this Muhammad Akram, Saira Nawaz [10]-[11] have also defined another aspect of soft graph and some operations on soft graphs are also studied.J.D.Thenge, B.S.Reddy and R.S. Jain have introduced the concept of adjacency and incidence matrix.

2 Preliminaries

Definition 2.1. ***Soft Set****[3]*
Let U be an universe and E be a set of parameters. Let $\mathcal{P}(U)$ denote the power set of U and C be a non-empty subset of E. A pair (H, C) is called a soft set over U, where H is a mapping given by $H : C \to \mathcal{P}(U)$.
In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\epsilon \in C$, $H(\epsilon)$ may be considered as the set of ϵ-elements of the soft set (H, C) or as the set of ϵ-approximate elements of the soft set.

Definition 2.2. ***Soft subset****[3]*
For two soft sets (F, A) and (G, B) over a common universe U, (F, A) is a soft subset of (G, B) if

1. *$A \subset B$*
2. *For all $\epsilon \in A$, $F(\epsilon)$ and $G(\epsilon)$ are identical approximations.*

we write $(F, A) \subset (G, B)$.
(F, A) is said to be soft super set of (G, B), if (G, B) is soft subset of (F, A).

Definition 2.3. ***Soft graph*** *[11]*
A 4-tuple $G^* = (G, S, T, A)$ is called a soft graph if it satisfies the following conditions,

1. *$G = (V, E)$ is a simple graph.*
2. *A is non empty set of parameters.*
3. *(S, A) is a soft set over V.*
4. *(T, A) is soft set over E.*
5. *$(S(a), T(a))$ is a subgraph of G $\forall$ $a \in A$.*

The subgraph $(S(a), T(a))$ is denoted by $F(a)$ for convenience. A soft graph can also be represented by,

$$G^* = (G, S, T, A) = \{F(x), x \in A\}$$

In this paper we denote this soft graph as (F, A).

Definition 2.4. ***Degree of a vertex in soft graph*** *[7]*
Let $G = (V, E)$ be a simple connected graph, C be any non-empty subset of V. If set valued function $S : C \to \mathcal{P}(V)$ is defined as $S(x)=\{y \in V | d(x, y) \leq 1\}$ and set valued function $T : C \to \mathcal{P}(E)$ is defined as $T(x)=\{xu \in E | u \in S(x)\}$, then (F, C) is a soft graph of G where $F(x) = (S(x), T(x))$. The degree of a vertex $v \in V$ is defined as $max\{deg_{F(v_i)}(v), \forall v_i \in C\}$. It is denoted by $deg_{(F,C)}(v)$ or $d_{(F,C)}(v)$.

Definition 2.5. ***Adjacent vertices in soft graph****[7]*
Let $G = (V, E)$ be a simple connected graph such that $C \subseteq V$, a set valued function $S : C \to \mathcal{P}(V)$ is defined as $S(x)=\{y \in V | d(x, y) \leq 1\}$ and a set valued function $T : C \to \mathcal{P}(E)$ is defined as $T(x)=\{xu \in E | u \in S(x)\}$. Thus (F, C) be a soft graph of G where $F(x) = (S(x), T(x))$. Any two vertices v_i and v_j in V are said to be adjacent with respect to soft graph (F, C) if,

1. $\{v_i, v_j\} \subseteq F(v_i) \cap F(v_j)$; *if* $v_i, v_j \in A$ *and* $i \neq j$
2. $v_i \in F(v_j)$; *if* $v_i \notin A$ *and* $v_j \in A$.

If both the vertices v_i, v_j *are not in* C *then are said to be not adjacent.*

Consider the graph $G = (V, E)$ *as shown in figure*

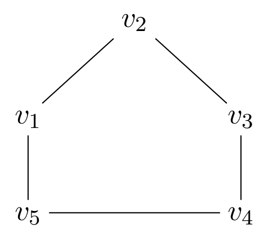


Fig. 1: G

Let $C = \{v_1, v_4\}$ *and define* $S(x)=\{z \in V | d(x, z) \leq 1\}$, $T(x)=\{xu \in E | u \in S(x)\}$.
Then $S(v_1) = \{v_1, v_2, v_5\}$, $S(v_4) = \{v_3, v_4, v_5\}$, $T(v_1) = \{v_1v_2, v_1v_5\}$, $T(v_4) = \{v_4v_3, v_4v_5\}$.
Denote $F(x) = (S(x), T(x)), \forall\, x \in C$

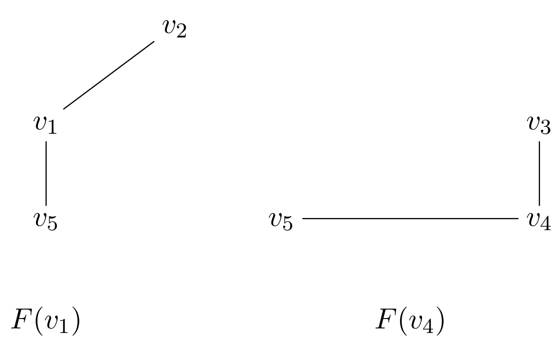


Fig. 2: $F(v_1)$ and $F(v_4)$

By above definition we find the adjacent vertices with respect to soft graph

1. *The vertices* v_1 *and* v_2 *are adjacent since* $v_2 \in F(v_1)$ *as* $v_1 \in C$ *and* $v_2 \notin C$.
2. *The vertices* v_1 *and* v_5 *are adjacent since* $v_5 \in F(v_1)$ *as* $v_1 \in C$ *and* $v_5 \notin C$.
3. *The vertices* v_3 *and* v_4 *are adjacent since* $v_3 \in F(v_4)$ *as* $v_4 \in C$ *and* $v_3 \notin C$.
4. *The vertices* v_1 *and* v_4 *are not adjacent since* $\{v_1, v_4\} \notin F(v_1) \cap F(v_4)$ *and* $v_1, v_4 \in C$.
5. *The vertices* v_2 *and* v_3 *are not adjacent since* $v_2 \notin C$ *and* $v_3 \notin C$.

Definition 2.6. ***Adjacency matrix of a soft graph***
Let $G = (V, E)$ be a simple connected graph, $C \subseteq V$ and (F, C) be a soft graph of G where set valued function $S : C \to \mathcal{P}(V)$ is defined as $S(x)=\{y \in V | d(x, y) \leq 1\}$, a set valued function $T : C \to \mathcal{P}(E)$ is defined as $T(x)=\{xu \in E | u \in S(x)\}$ and $F(x) = (S(x), T(x))$. Let $A = \bigcup_{v \in C} S(v) = \{v_1, v_2, ...v_n\}$. The adjacency matrix of the soft graph (F, C) is a square matrix of order $n \times n$ denoted as $\mathcal{A}(F, C) = (c_{ij})$, $(i, j)^{th}$ entry c_{ij} is given by

$$c_{ij} = \begin{cases} 1, & \text{if } v_i \text{ is adjacent to } v_j \\ 0, & \text{if } v_i \text{ is not adjacent to } v_j, \quad i, j = 1, 2, 3, ..., n. \end{cases}$$

Consider the graph $G = (V, E)$ as shown in figure

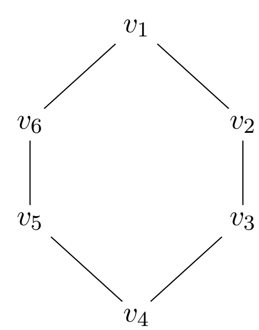
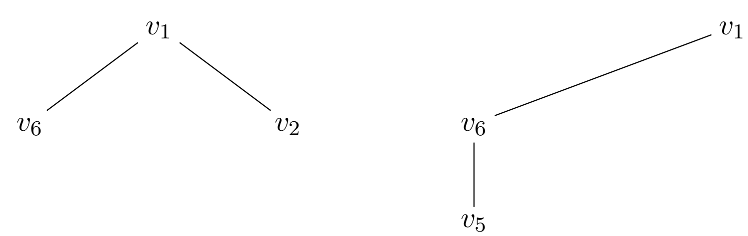


Fig. 3: $G = (V, E)$

Let $C = \{v_1, v_6\}$ and $S(x)=\{z \in V | d(x, z) \leq 1\}$, $T(x)=\{xu \in E | u \in S(x)\}$.
Then $S(v_1) = \{v_1, v_2, v_6\}$, $S(v_6) = \{v_1, v_5, v_6\}$, $T(v_1) = \{v_1v_2, v_1v_6\}$, $T(v_6) = \{v_6v_1, v_6v_5\}$
Denote $F(x) = (S(x), T(x))$, $\forall\, x \in C$.
Here $A = \bigcup_{v_i \in C} S(v_i) = \{v_1, v_2, v_5, v_6\}$.



$F(v_1)$ $F(v_6)$

Fig. 4: $F(v_1)$ and $F(v_6)$

The adjacency matrix of soft graph (F, C) is given by,

$$\mathcal{A}(F,C) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_5 & v_6 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

3 Application of soft graph in decision making

In this section we propose an algorithm for reduction of parameters using adjcency matrix of a soft graph. We give an application of this algorithm to decision making problem.

3.1 Algorithm

Consider the product set $\{m_1, m_2, ..., m_n\}$ with parameters $\{p_1, p_2, ..., p_k\}$, then to select a best product having these parameters among the available, we give the following algorithm using adjacency matrix of soft graph.

- Form a bipartite graph of the given problem $G = (V, E)$.
- Construct a soft graph (F, A) by taking A= $\{m_1, m_2, ..., m_n\}$ using $S(x) = \{z \in V : d(x, z) \leq 1\}$ and $T(x)$= $\{xu \in E : u \in S(x)\}$ and $F(x)$=$(S(x), T(x))$
- Write the adjacency matrix of the given soft graph (F, A) and consider the entries (m_i, p_j)
- If either (m_i, p_j) =1 or (m_i, p_j) =0, $\forall\ i = 1, 2, ..., n$ then remove parameter p_j.
- Find the row total in the modified adjacency matrix of soft graph in last column.
- Find a product m_i with maximum row total.
- The product with maximum row total will be an optimal choice.

3.2 Application

Suppose a person Mr.X wants to buy a mobile among the 5 mobile phones say m_1, m_2, m_3, m_4, m_5 which depends on 4 parameters/properties p_1, p_2, p_3, p_4. The mobile m_1 has two properties p_1, p_2, mobile m_2 has three properties p_1, p_2, p_4, mobile m_3 has p_2 property, mobile m_4 has p_2 property and lastly m_5 has p_2, p_3 property. What should be an optimal choice of mobile?
First draw graph of given problem as follows:

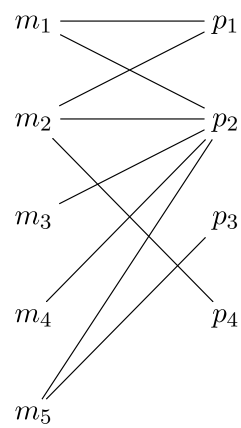


Fig. 5: $G = (V, E)$

V=$\{m_1, m_2, m_3, m_4, m_5, p_1, p_2, p_3, p_4\}$
Select A=$\{m_1, m_2, m_3, m_4, m_5\}$, define $S(x) = \{z \in V : d(x, z) \leq 1\}$, $T(x)$= $\{xu \in E : u \in S(x)\}$ and $F(x)$=$(S(x), T(x))$
$F(m_1) = \{m_1, p_1, p_2\}$, $F(m_2) = \{m_2, p_1, p_2, p_4\}$, $F(m_3) = \{m_3, p_2\}$, $F(m_4) = \{m_4, p_2\}$, $F(m_5) = \{m_5, p_2, p_3\}$.
Thus (F, A) is a soft graph.
The adjacency matrix of above soft graph is given by:

	p_1	p_2	p_3	p_4	m_1	m_2	m_3	m_4	m_5	Row total=Degree of a vertex
m_1	1	1	0	0	0	0	0	0	0	2
m_2	1	1	0	1	0	0	0	0	0	3
m_3	0	1	0	0	0	0	0	0	0	1
m_4	0	1	0	0	0	0	0	0	0	1
m_5	0	1	1	0	0	0	0	0	0	2
p_1	0	0	0	0	1	1	0	0	0	2
p_2	0	0	0	0	1	1	1	1	1	5
p_3	0	0	0	0	0	0	0	0	1	1
p_4	0	0	0	0	0	0	1	0	0	1

The row total obtained in last column of adjacency matrix is the degree of vertex in A with respect to soft graph (F, A).
Here $(m_i, p_2) = 1$ for all $i = 1, 2, 3, 4, 5$.
So, we drop second column and rewrite the adjacency matrix:

	p_1	p_3	p_4	m_1	m_2	m_3	m_4	m_5	Row total
m_1	1	0	0	0	0	0	0	0	1
m_2	1	0	1	0	0	0	0	0	2
m_3	0	0	0	0	0	0	0	0	0
m_4	0	0	0	0	0	0	0	0	0
m_5	0	1	0	0	0	0	0	0	1
p_1	0	0	0	1	1	0	0	0	2
p_2	0	0	0	1	1	1	1	1	5
p_3	0	0	0	0	0	0	0	1	1
p_4	0	0	0	0	0	1	0	0	1

In last column observe the row total corresponding to m_1, m_2, m_3, m_4, m_5, the row total corresponding to m_2 is maximum. Hence m_2 will be an optimal choice.

4 Conclusion

Soft graph is a new area of research in mathematics. In the present paper we have given an application of soft graph in decision making using the notion of adjacency matrix of soft graph by designing an algorithm.

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