New Ranking Function Introduced To Solve Fully Fuzzy Linear Fractional Programming Problem

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Abstract

The goal of this research is to provide a new trapezoidal fuzzy number ranking function. for solving Fully Fuzzy Linear Fractional Programming Problems (FFLFPP) with trapezoidal fuzzy integers as the objective function and constraints. The proposed method is based on crisp linear fractional programming and the simplex method. In this approach first, we transformed FFLFPP into Crisp Linear Programming Problem with the help of a new proposed ranking function and then the resulting problem is converted into LPP. A numerical example is presented to illustrate the proposed approach.

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1 Introduction

Linear fractional programming (LFP) problem is one of the most important techniques in operation research. A number of real-world issues can be turned into a linear fractional programming problem. In real-life problems, there may exist uncertainty about the parameters. In such a situation the parameters of linear programming problems may be represented as fuzzy numbers. Bellman and Zadeh [1] gave the idea of fuzzy optimization. H. R. Maleki et al. [4-6] introduced an efficient method for solving linear programming problems with fuzzy variables using a ranking function. Lotfi et al. [7] developed a system for achieving the estimated solution of fully fuzzy linear programming problems. Amit Kumar et al. [9] and Stanojevic et al.[16] suggested a method for solving the FFLP problems with equality constraints occurring in real-life situations, using the concept of crisp linear programming and ranking function. Chen and Hsieh [10] used Graded Mean Integration

Representation for representing a generalized fuzzy number. Charnes et al. [11], [13] gave the idea to replace any linear fractional programming with at most two straightforward linear programming problems. Hassan et al. [12] introduce a computer-oriented technique for solving Linear Fractional Programming (LFP) problem by converting it into a single Linear Programming (LP) problem.

In this paper, a Fully Fuzzy Linear Fractional Programming Problems has been proposed with trapezoidal fuzzy numbers. In which the objective function and Constraints are converted into Linear Programming Problem with the help of introduced a new ranking function.

This paper is subdivided into four sections - In section 2, we have given the basic concept of fuzzy set, trapezoidal fuzzy number with its arithmetic properties followed by the general form of Fractional Linear Programming and Fully Fuzzy Fractional Linear Programming Problems. The proposed new ranking function is given in section 3 while in section 4, we elaborated the algorithm to solve Fully Fuzzy Linear Fractional Programming Problems with the help of the proposed new ranking function. For a better understanding of the concept, a numerical example is given in section 5 and the conclusion is listed in section 6. The present work proposed a new ranking function of trapezoidal fuzzy numbers for solving Fully Fuzzy Linear Fractional Programming Problems (FFLFPP) with the help of a crisp linear fractional programming problem and the simplex method.

2 Some Basic Concept

Definition: $\bar{A} = [\{x, \mu_{\bar{A}}(x)\} : x \in X]$, where $\bar{A}$ is the fuzzy set and $\mu_{\bar{A}}(x)$is a membership function.

A fuzzy set $\bar{A}$is defined on the universal set of real number R (member function $\mu_{\bar{A}}\,; R \to [0,1]$) if its membership function has the following properties

[i] $\bar{A}$ is normal $\exists\ \tilde{x} \in R$ such that $\mu_{\bar{A}}(\tilde{x}) = 1$.

[ii]$\bar{A}$ is convex fuzzy set $\mu_{\bar{A}}\left[\lambda y_1 + (1-\lambda)\, y_2\right] \geq \min\left\{\mu_{\bar{A}}(y_1), \mu_{\bar{A}}(y_2)\right\} : y_{1,}y_2 \in R\ forevery\ \lambda \in [0,1]$.

[iii] $\mu_{\bar{A}}$ is upper continuous and Support of $\bar{A}$is bounded.

Definition: Trapezoidal fuzzy number $\bar{A}$ is a fuzzy set defined on R and is represented by $\bar{A} = (b_{1,}b_2, b_{3,}b_4)$; $b_1 \leq b_2 \leq b_3 \leq b_4$

$$\mu_{\bar{A}}(x) = \begin{cases} r\left(\frac{b_1 - x}{b_1 - b_2}\right), & b_1 \leq x \leq b_2 \\ r, & b_2 \leq x \leq b_3 \\ r\left(\frac{b_4 - x}{b_4 - b_3}\right), & b_3 \leq x \leq b_4 \\ 0, & otherwise \end{cases}$$

Let $\tilde{E} = (e_1, e_2, e_3, e_4)\ and\ \tilde{G} = (g_1, g_2, g_3, g_4)\ be\ two\ trapezoidal fuzzy\ numbers$, Where $e_1, e_2, e_3, e_4, g_1, g_2, g_3, g_4 \in R$. Then the arithmetic operations are defined by

$$\begin{aligned}
&[i]\quad \tilde{E} + \tilde{G} = (e_1 + g_1, e_2 + g_2, e_3 + g_3, e_4 + g_4)\\
&[ii]\quad \tilde{E} - \tilde{G} = (e_1 - g_1, e_2 - g_2, e_3 - g_3, e_4 - g_4)\\
&[iii]\quad \tilde{E}.\tilde{G} = (e_1.g_1, e_2.g_2, e_3.g_3, e_4.g_4)\\
&[iv]\quad \frac{\tilde{E}}{\tilde{G}} = \left(\frac{e_1}{g_1}, \frac{e_2}{g_2}, \frac{e_3}{g_3}, \frac{e_4}{g_4}\right)
\end{aligned}$$

The general form of Fractional Linear Programming Problems

$$Max\ f(x) = \frac{\sum_{j=1}^{l} R_j{}^T x_j + p}{\sum_{j=1}^{l} S_j{}^T x_j + q}$$

$$Subject\ to \quad \sum A_{ij}x_j \leq B_i; \quad i = 1........m; \quad j = 1.......l$$

$$x_j \geq 0,$$

$$where \text{ R, S, A and B are } crisp\ numbers.$$

The general form of Fully Fuzzy Fractional Linear Programming Problems

$$Max\ f(x) = \frac{\sum_{j=1}^{l} \tilde{R}_j{}^T x_j + p}{\sum_{j=1}^{l} \tilde{S}_j{}^T x_j + q}$$

$$subject\ to \ \sum \tilde{A}_{ij}x_j \leq \tilde{B}_i; \quad i = ...m; \quad j = 1........l$$

$$x_j \geq 0$$

$$where\ \tilde{R} = (r_1, r_2, r_3, r_4) \quad \tilde{S} = (s_1, s_2, s_3, s_4)$$
$$\tilde{A} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) \quad \tilde{B} = (\beta_1, \beta_2, \beta_3, \beta_4)\ are\ trapezoidal\ fuzzy\ numbers.$$

3 Proposed New Ranking Function

The ranking function is a suitable technique for comparing two fuzzy numbers. The ranking function is defined as $R : F(R) \rightarrow R$.
Now, assume that $\tilde{r}$ *and* $\tilde{s}$ are to be trapezoidal fuzzy numbers then we define order as F(R) as follows

$$\tilde{r} \geq \tilde{s}\ if\ and\ only\ if\ R(\tilde{r}) \geq R(\tilde{s})$$

$$\tilde{r} \leq \tilde{s}\ if\ and\ only\ if\ R(\tilde{r}) \leq R(\tilde{s})$$

$$\tilde{r} = \tilde{s}\ if\ and\ only\ if\ R(\tilde{r}) = R(\tilde{s})$$

$$are\ \tilde{r}\ and\ \tilde{s}\ are\ in\ F(R).$$

Let$\theta \in (0, 1)$

$$r\left(\frac{b_1 - x}{b_1 - b_2}\right) = \theta \quad \Rightarrow x = b_1 - \frac{\theta}{r}(b_1 - b_2) = glb\bar{A}(\theta)$$

$$r\left(\frac{b_4 - x}{b_4 - b_3}\right) = \theta \quad \Rightarrow x = b_4 - \frac{\theta}{r}(b_4 - b_3) = lub\bar{A}(\theta)$$

By Graded Mean Integration[10] Represent of $\bar{A}$

$$R(\bar{A}) = \frac{\frac{1}{2}\int_0^r \theta^3 \left\{glb\bar{A}(\theta) + lub\bar{A}(\theta)\right\} d\theta}{\int_0^r \theta^3 d\theta}$$

$$R(\bar{A}) = \frac{\frac{1}{2}\int_0^r \theta^3 \left\{b_1 - \frac{\theta}{r}(b_1 - b_2) + b_4 - \frac{\theta}{r}(b_4 - b_3)\right\} d\theta}{\int_0^r \theta^3 d\theta}$$

$$R(\bar{A}) = \frac{\frac{1}{2}\int_0^r \left\{\theta^3 b_1 - \frac{\theta^4}{r}(b_1 - b_2) + \theta^3 b_4 - \frac{\theta^4}{r}(b_4 - b_3)\right\} d\theta}{\int_0^r \theta^3 d\theta}$$

$$R(\bar{A}) = \frac{\frac{1}{2}\left\{\frac{\theta^4}{4}b_1 - \theta^5 \frac{(b_1 - b_2)}{5r} + \frac{\theta^4}{4}b_4 - \frac{\theta^5}{5r}(b_4 - b_3)\right\}_0^r}{\left(\frac{\theta^4}{4}\right)_0^r}$$

$$R(\bar{A}) = \frac{(b_1 + 4b_2 + 4b_3 + b_4)}{10}$$

4 Proposed Algorithm to solve FFLFP Problems

Step-1 Choose any FFLFP Problems
Step-2 Convert it into Crisp Fractional Linear Programming Problem using Proposed New Ranking Function and GMIR method
Step-3 Convert Crisp Fractional Linear Programming Problem into Standard Linear Programming Problem using Transformation Techniques by Charnes and Cooper.
Step-4 Finally Obtained optimal solution by Simplex Method

5 Numerical Example

Consider the following FFLFP problem example (taken problem from Moumita and De 2014).

$$Max\, u = \frac{(4,7,10,12)\,t_1 + (8,10,14,15)\,t_2 + (2.5,4,7.5,11.5)\,t_3 + (2,3,4,6)}{(10,14,20,22)\,t_1 + (20,23,27.5,29)\,t_2 + (18,20,25,28)\,t_3 + (5,10,18,20)}$$

Subject to

$$[10,17,19,25]\,t_1 + [14,16,22,24]\,t_2 + [20,25,27,30]\,t_3 \leq [100,35,27,101.8]$$

$$[0.25,0.04,04,0.12]\,t_1 + [0.15,0.04,0.04,0.12]\,t_2 + [0.15,0.04,0.04,0.12]\,t_3 \leq [1.3,0.9,0,1.5]$$

$$[4,6,10,13]\,t_1 + [0,5,11,16]\,t_2 + [8,11,15,20]\,t_3 \leq [80,21.9,22,95]$$

Non-negative condition

$$t_1, t_2, t_3 \geq 0$$

Now using the proposed new Ranking Function $R(\bar{A}) = \frac{(b_1+4b_2+4b_3+b_4)}{10}$ and GMIR method [10]. We have converted the above problem into a crisp fractional linear programming problem, so the given problem is now written as a crisp fractional linear programming problem -

$$Max\ u\ = \frac{8.4t_1 + 11.9t_2 + 6t_3 + 3.6}{16.8t_1 + 25.3t_2 + 22.6t_3 + 13.7}$$

Subject to

$$17.9t_1 + 19t_2 + 25.8t_3 \leq 44.96$$

$$0.07t_1 + 0.06t_2 + 0.06t_3 \leq 0.92$$

$$8.1t_1 + 7.5t_2 + 12.8t_3 \leq 35$$

Non-negative restriction

$$t_1, t_{2,} t_3 \geq 0$$

Now, we reduce the above crisp fractional linear programming problem to the following Linear Programming Problem [11-13]

$R = (8.4, 11.9, 6)$ $S = (16.8, 25.3, 22.6)$ $A_1 = (17.9, 19, 25.8)$ $A_2 = (0.07, 0.06, 0.06)$ $B_1 = 44.96$ $B_2 = 0.92B_3 = 3$

$$Max\ H(y) = \frac{3.98y_1 + 5.25y_2 + 0.06y_3 + 0.26}{1}$$

Subject to

$$1000.58y_1 + 1397.78y_2 + 1369.56y_3 \leq 44.96$$

$$16.35y_1 + 24y_2 + 21.53y_3 \leq 0.92$$

$$698.97y_1 + 988.25y_2 + 966.36y_3 \leq 35$$

Non-negative restriction

$$y_1, y_{2,} y_3 \geq 0$$

The obtained optimal solutions are as follows,

$$y_1 = 0.0449, y_2 = 0, y_3 = 0$$

Using the value of $Y = (y_1, y_2, y_3)$we can find the value of $T = (t_1, t_2, t_3)$using

$$(t_1, t_2, t_3) = \frac{(y_1, y_2, y_3)\, q}{1 - S\,(y_1, y_2, y_3)}; \quad from\ Mohammad\ Babul\ Hasanand\ Sumi\ Acharjee$$

$$= \frac{(0.0449, 0.0)\, 13.7}{1 - (16.8, 25.3, 22.6)\,(0.0449, 0.0)}$$

The fuzzy optimal solution is given by $t_1 = 2.503, t_2 = 0\&t_3 = 0$
The case from Moumita, D., and De, P. K. (2014)[15] has been solved using a new trapezoidal fuzzy number ranking function for addressing Fully Fuzzy Linear Fractional Programming Problems (FFLFPP) with trapezoidal fuzzy numbers as the objective function and constraints. The fuzzy optimal Value of the given problem is$F(u) =$ 0.43. Which is exactly as given in the original problem.

6 Conclusion

The suggested technique has solved the problem of optimising Max u with three variables that are constrained, as referred to by Moumita, D., and De, P. K. (2014). Using the proposed new ranking function and the GMIR method, it was first converted into a crisp fractional linear programming problem, which was then transformed into a linear programming problem using transformation techniques. The values of variables y1 = 0:0449; y2 = 0; and y3 = 0 have been acquired and the optimal solution of the problem taken is obtained F(u) = 0:43 .
In this work, we have proposed a new ranking function of trapezoidal fuzzy numbers and showed an efficient approach to solve FFLFPP in which first, we have converted the original problem into a Crisp Linear Fractional Programming Problem with the help of the proposed ranking function. After that reduce it from crisp LFPP into Linear Programming Problem and obtained optimal solution by Simplex method. The numerical example and its result show clearly the usefulness of the proposed method.

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