Flow of modified Bingham fluid in a cylindrical cavity enclosed by porous annulus

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Abstract

The present work concerns the flow of a modified Bingham fluid through a cylindrical cavity covered by porous annulus in which flow of Newtonian liquid is considered.The Bingham fluid and Newtonian liquid are assumed to be immiscible. The flow of Bingham fluid in cavity region and Newtonian liquid in the porous region are fully developed, steady, laminar and this flow occurs due to a constant pressure gradient. Here, we have obtained analytical expressions of velocities, flow rate and wall shear stress for both modified Bingham fluid and Newtonian liquid. Effects of different flow parameters on the velocity profiles, flow rate and wall shear stress are also discussed graphically.

Subject Classification:[2020] 76A05, 76D99.
Keywords: Modified Bingham fluid, Newtonian liquid, Yield stress, flow rate.

1 Introduction

Several theories and models have been developed to explain the behavior of non-Newtonian fluids. In 1916, Bingham [1] was the first to propose the mathematical form of such material that behaves as a rigid body at low stress and flows as viscous fluid at high stress. These materials which required a certain threshold stress to start flow are usually known as Bingham plastics or Bingham fluids. This threshold stress is known as yield stress. Bingham model is able to describe the yielded and unyielded regions of flow of Bingham fluid. This is the basic model that deals with the flow of mud in drilling engineering and slurries. Bingham model has great significance because it explains flow behaviours of various fluids upto a great extent. In various cases, researchers have obtained analytical solutions for different flow fields (Couette flow and Poiseuille flow). But, it is difficult to numerically model the fluid flow by using Bingham model due to incapability in finding yield stress surface. The study of Bingham fluid is important due to its vast applications in various fields, such as food and dairy processing,hydrology, oil explorations, etc. Some very common examples of Bingham fluids are paints, slurries, pastes, and food substances like margarine, mayonnaise, ketchup, etc.

Depra and Scarpi [2] studied the flow of a Bingham fluid through a pipe. Murthy *et al.* [3] investigated the analytical solution for the flow of immiscible Bingham fluid and Newtonian liquid through a horizontal channel. Chemloul [4] obtained the analytical solution for the flow of a Bingham fluid through a canonical tube. Nandagopal *et al.* [5] studied the channel flow of the Bingahm fluid bounded by permeable beds in which suction and injection was permissible. Misra and Adhikary [6] studied the the flow of Bingham fluid in a porous bed with applied external magnetic field. Sengupta and Sirshendu [7] discussed the problem of Couette-Poiseuille channel flow of a Bingham fluid across a porous layer. Sekhar *et al.* [8] studied the effect of temperature during the flow of a Bingham fluid between parallel plates. Peristaltic flow of Bingham fluid through an inclined pipe was studied by Babu [9]. Srilatha *et al.* [10] numerically investigated the unsteady MHD flow of a Bingham fluid under consideration of Hall effect of heat transfer.

Reseachers are solved various flow problems related to channel/cylindrical geometry, Such real fluid problems have great importance in real world. Deo [11] studied the axisymmetric Stokes flow past a swarm of porous cylindrical shells. Ansari and Deo [12] observed influence of magnetic field on two immiscible viscous fluids through channel filled with porous medium. Deo *et al.*[13] investigated effect of magnetic field on micropolar fluid through cylinder such that impermeable core of cylinder covered with porous layer. Maurya [14] observed Stokes flow of micropolar fluid through a porous cylinder.

In 1987, Papanastasiou [15] was the first to introduce a modified version of the Bingham model, which is known as Bingham-Papanastasiou model. He proposed an exponential regularization in the governing equation, by defining a parameter m^*, which has dimension in time and controls the exponential growth of stress. The formulation which is used in Bingham-Papanastasiou model holds uniformly in both yielded and unyielded regions. Unsteady flow of Bingham fluid through porous medium was studied by Attia *et al.* [16]. Baioumy *et al.* [18] observed the flow of Bingham fluid at the inlet region of pipe. Mehmood *et al.* [17] investigated flow of modified Bingham fluid through a channel which contain obstacles. Ewis [19] discussed analytical solution for modified Bingham fluid flow through parallel plate channel subjected to Forchheimer medium.

In this work, the problem of steady, laminar and fully developed flow of Bingham fluid through a cylindrical cavity and the flow of Newtonian liquid through the porous cylidrical annulus is considered. The Bingham fluid and Newtonian fluid are assumed to be immiscible and outer surface of the annulus is taken impervious. Bingham-Papanastasiou model is used to find the governing equations of motion for the Bingham fluid. Analytical expressions for velocity profiles, volumetric flow rate and wall shear stress are obtained and results are discussed graphically.

2 Mathematical formulation

Here, we have assumed fully developed laminar flow of modified Bingham fluid and Newtonian liquid in the cylindrical shell, such that mixing of the fluid and liquid is not permissible. The outer surface of the cylindrical annulus is taken impervious. The radius of the inner cylindrical cavity through which flow of modified Bingham fluid takes place is r_1^*. The radius of outer cylindrical shell is r_2^*. So, the thickness of porous shell region is $(r_2^* - r_1^*)$ (Figure 1). The Newtonian liquid flow takes place through a homogeneous porous medium with permeability k^*. Let region I be defined as the region which permits the flow of modified Bingham fluid and region II as the porous region which allows the flow of Newtonian liquid. It is assumed that V^* is the entering velocity of fluids in both regions I and II. Assume that $\vec{u_i^*} = \{0, 0, v_i^*(r)\}$ where, $i = 1, 2$ represents velocity vectors for both fluid and porous regions, respectively. Due to these assumptions, velocity vectors satisfy the continuity equation in both regions. The motion is governed by the following equations:
Region I: For non-porous region $(0 \leq r^* \leq r_1^*)$, the governing momentum equation of motion is

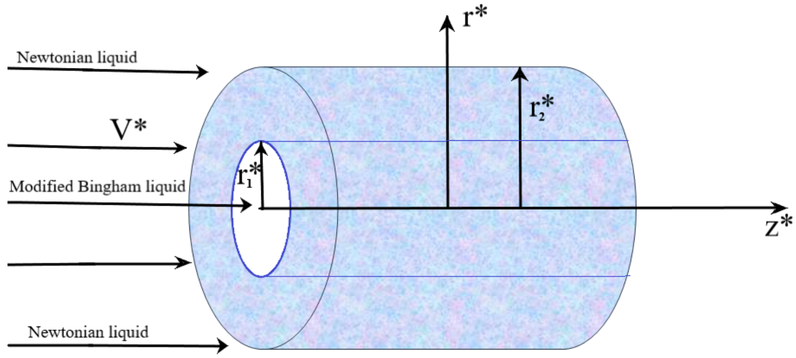

Fig. 1: Geometry of the problem

(2.1) $$\frac{1}{r^*}\frac{d(r^*\tau^{1^*}_{r^*z^*})}{dr^*} = \frac{dp^*}{dz^*},$$

where, $\tau^{1^*}_{r^*z^*}$ and p^* denote shearing stress component and pressure, respectively. Papanastasiou explained the constitutive relations for modified Bingham fluid as

(2.2) $$\tau^*_{ij} = -p^*\delta_{ij} + \eta(S)D^*_{ij},$$

and

(2.3) $$\eta(S) = \mu^*_1 + \tau^*_y\left(\frac{1-exp(-m^*\frac{S^{1/2}}{2})}{S^{1/2}}\right),$$

where, m^*, τ^*_y and μ^*_1 are material constant, yield stress and viscosity of the modified Bingham fluid, respectively. Also, τ^*_{ij} and D^*_{ij} denote shear stress tensor and deformation rate tensor, respectively and S is defined by

(2.4) $$S = 2\{trace(D^2) - (traceD)^2\}.$$

For the problem, simple shearing stress is defined as follows:

(2.5) $$\tau^{1^*}_{r^*z^*} = \left[\mu^*_1 + \tau^*_y\left(\frac{1-exp(-m^*\frac{dv^*_1}{dr^*})}{\frac{dv^*_1}{dr^*}}\right)\right]\left(\frac{dv^*_1}{dr^*}\right).$$

Region II: For the porous region ($r^*_1 \leq r^* \leq r^*_2$), the governing Brinkman's equation is

(2.6) $$-\frac{\mu^*_2}{k^*}v^*_2 + \frac{\mu^*_2}{r^*}\frac{d}{dr^*}\left(r^*\frac{dv^*_2}{dr^*}\right) = \frac{dp^*}{dz^*},$$

where, μ^*_2 and v^*_2 are viscosity and velocity component of the Newtonian liquid, respectively.

3 Dimensionless governing equations of motion

To non-dimensionalise the governing equations of motion, we are introducing some non-dimensional variables as:

$$\begin{aligned} r &= \frac{r^*}{r_1^*}, z = \frac{z^*}{r_1^*}, \sigma = \frac{r_2^*}{r_1^*}, k = \frac{k^*}{r_1^{*2}}, p = \frac{p^*}{\mu_1^* V^*/r_1^*}, m = \frac{m^*}{r_1^*/V^*}, \\ Q &= \frac{Q}{r_1^{*2} V^*}, \beta = \frac{\mu_2^*}{\mu_1^*}, \quad \tau_{rz}^1 = \frac{\tau_{r^*z^*}^{1^*}}{\tau_y^*}, \tau_y^* = \mu_1^* V^*/r_1^*, v_i = \frac{v_i^*}{V^*}, i = 1, 2. \end{aligned} \tag{3.1}$$

Using these non-dimensional variables that described in the above equation (3.1), governing equations of motion in both Bingham fluid and porous regions can be expressed as follows:
Region I: For the Bingham fluid region ($0 \le r \le 1$)

$$\frac{1}{r}\frac{d(r\tau_{rz}^1)}{dr} = \frac{dp}{dz} \tag{3.2}$$

and

$$\tau_{rz}^1 = \left[1 + \left(\frac{1 - exp(-m\frac{dv_1}{dr})}{\frac{dv_1}{dr}}\right)\right]\left(\frac{dv_1}{dr}\right). \tag{3.3}$$

Region II: For the porous region ($1 \le r \le \sigma$)

$$-\frac{v_2}{k} + \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_2}{dr}\right) = \frac{1}{\beta}\frac{dp}{dz}. \tag{3.4}$$

4 Boundary conditions

The solution of the problem will be obtained under the following boundary conditions: **(a) No-slip boundary condition**
It is observed experimentally that when a viscous liquid comes in the direct contact of rigid solid/impervious surface, then at liquid-solid interface, velocity of the viscous liquid relative to rigid boundary is zero. This is known as no-slip boundary condition. Here, we will use no-slip boundary condition at the outer impervious boundary of the cylindrical annulus as follows:

$$v_2^* = 0 \quad \text{at} \quad r^* = r_2^*. \tag{4.1}$$

(b) Regularity condition
As no physical quantity like shearing stress, velocity etc. will attain infinity during regular flow. Sometimes we get mathematical expression of physical quantities which tend to infinity at any point, then some conditions are applied to retain physical quantities to be finite are known as regularity conditions. Here, regularity condition is used as given below:

$$\tau_{r^*z^*}^{1^*} \text{ is finite at } \; r^* = 0. \tag{4.2}$$

(c) Interface conditions
Here, we will use following boundary conditions at the porous-fluid interface ($r^* = r_1^*$):
(i) Continuity of velocity condition

$$v_1^* = v_2^* \quad \text{at} \quad r^* = r_1^*. \tag{4.3}$$

(ii) Continuity of stress condition

$$\tau_{r^*z^*}^{1^*} = \tau_{r^*z^*}^{2^*} \quad \text{at} \quad r^* = r_1^*, \tag{4.4}$$

where, $\tau_{r^*z^*}^{2^*}$ denotes the shearing stress in region II.
The boundary conditions (4.1)-(4.4) in non-dimensional variables take the form

$$v_2 = 0 \quad \text{at} \quad r = \sigma. \qquad \text{(No-slip boundary condition)} \tag{4.5}$$

$$\tau_{rz}^1 \text{ is finite at } \; r = 0. \qquad \text{(Regularity condition)} \tag{4.6}$$

$$v_1 = v_2 \quad \text{at} \quad r = 1. \qquad \text{(velocity continuity condition)} \tag{4.7}$$

$$\tau_{rz}^1 = \tau_{rz}^2 \quad \text{at} \quad r = 1. \qquad \text{(stress continuity condition)} \tag{4.8}$$

5 Analytical solutions

In this section, we have obtained analytical expressions of velocities for both regions I and II by finding the solution of equations (3.2) and (3.4). The values of arbitrary constants are also determined by applying boundary condtions.
Expressions for velocity of modified Bingham fluid and Newtonian liquid
Region I: For the fluid region ($0 \le r \le 1$)
Assume that $\frac{dp}{dz} = 2P =$ constant, then the equation (3.2) becomes as

$$\frac{1}{r}\frac{d(r\tau_{rz}^1)}{dr} = 2P. \tag{5.1}$$

On integration,we obtain

$$\tau_{rz}^1 = Pr + \frac{C}{r}, \quad C \text{ is constant of integration.} \tag{5.2}$$

Due to regularity condition (4.6), C must be zero. Hence,

$$\tau_{rz}^1 = Pr. \tag{5.3}$$

Substituting the expression of shearing stress τ_{rz}^1 from equation (3.3) in the equation (5.3), we get

$$\frac{dv_1}{dr} + \left(1 - exp(-m\frac{dv_1}{dr})\right) = Pr,$$

or, With the help of **MATHEMATICA** software, we have evaluated the expression of velocity component of the modified Bingham fluid flowing in the cylindrical cavity as given below,

$$v_1(r) = -\frac{W(m\ e^{(-m(Pr-1))})}{m^2P} - \frac{W(m\ \ e^{(-m(Pr-1))})^2}{2m^2P} - r + \frac{1}{2}Pr^2 + A_1, \tag{5.4}$$

where W(.) denotes the principle Lambert function or product logarithm function.
Region II: For the porous region $(1 \leq r \leq \sigma)$
Since, we have assumed earlier that $\frac{dp}{dz} = 2P =$ constant. Hence, the equation (3.4) assumes the form

$$-\frac{v_2}{k} + \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_2}{dr}\right) = \frac{2P}{\beta}. \tag{5.5}$$

Solving the equation (5.5), we find the expression of velocity component of the Newtonian liquid in porous region as mentioned below:

$$v_2(r) = -\frac{2kP}{\beta} + C_1 I_0\left(\frac{r}{\sqrt{k}}\right) + C_2 K_0\left(\frac{r}{\sqrt{k}}\right). \tag{5.6}$$

Here, $I_0(.)$ and $K_0(.)$ are zeroth order modified Bessel functions of first and second kinds, respectively.
The values of arbitrary constants A_1, C_1 and C_2 are determined by using boundary conditions $(4.5), (4.7)$ and(4.8). These values are

(5.7)

$$\begin{aligned}
&A_1 = 1/(2m^2P)((exp(-mP - W(me^{m(1-P)}))m(I_1\left(\frac{1}{\sqrt{k}}\right)(4exp(mP + W(me^{m(1-P)}))\\
&kmP^2K_0\left(\frac{1}{\sqrt{k}}\right) - (-2exp(m)\beta + exp(mP + W(me^{m(1-P)}))mP(4kP + (-2+P)\beta))\\
&K_0\left(\frac{\sigma}{\sqrt{k}}\right)) + 2exp(mP + W(me^{(m-mP)}))\sqrt{k}mP^2 I_0\left(\frac{1}{\sqrt{k}}\right)\left(K_0\left(\frac{\sigma}{\sqrt{k}}\right) + 2\sqrt{k}K_1\left(\frac{1}{\sqrt{k}}\right)\right)\\
&I_0\left(\frac{\sigma}{\sqrt{k}}\right)(2exp(mP + W(me^{m(1-P)}))\sqrt{k}mP^2K_0\left(\frac{1}{\sqrt{k}}\right) + (-2exp(m)\beta + exp(mP + W(me^{m(1-P)}))\\
&mP(4kP + (-2+P)\beta))K_1\left(\frac{1}{\sqrt{k}}\right))))/\left(\beta\left(I_1\left(\frac{1}{\sqrt{k}}\right)K_0\left(\frac{\sigma}{\sqrt{k}}\right) + I_0\left(\frac{\sigma}{\sqrt{k}}\right)K_1\left(\frac{1}{\sqrt{k}}\right)\right)\right)\\
&\qquad + W(me^{m(1-P)})^2).
\end{aligned}$$

(5.8)

$$C_1 = \left(\sqrt{k}PK_0\left(\frac{\sigma}{\sqrt{k}}\right) + 2kPK_1\left(\frac{1}{\sqrt{k}}\right)\right)/\left(\beta I_1\left(\frac{1}{\sqrt{k}}\right)K_0\left(\frac{\sigma}{\sqrt{k}}\right) + \beta I_0\left(\frac{\sigma}{\sqrt{k}}\right)K_1\left(\frac{1}{\sqrt{k}}\right)\right).$$

(5.9)

$$C_2 = \left(-\sqrt{k}PI_0\left(\frac{\sigma}{\sqrt{k}}\right) + 2kPI_1\left(\frac{1}{\sqrt{k}}\right)\right)/\left(\beta I_1\left(\frac{1}{\sqrt{k}}\right)K_0\left(\frac{\sigma}{\sqrt{k}}\right) + \beta I_0\left(\frac{\sigma}{\sqrt{k}}\right)K_1\left(\frac{1}{\sqrt{k}}\right)\right).$$

6 Volumetric flow rate

The volumetric flow rate Q^* thrugh the cross-section of the cylindrical shell can be evaluated by applying the formula as

$$Q^* = 2\pi \left(\int_0^{r_1^*} r^* v_1^* dr^* + \int_{r_1^*}^{r_2^*} r^* v_2^* dr^* \right). \tag{6.1}$$

In terms of non-dimensional variables, we have

$$Q = 2\pi \left(\int_0^{1} r v_1 dr + \int_{1}^{\sigma} r v_2 dr \right). \tag{6.2}$$

Substituting the expressions of velocities v_1 and v_2 from equations (5.4) and (5.6) in the equation (6.2) and then integrating, we get an expression for the volumetric flow rate as

$$\begin{aligned} Q = {} & (\pi/(36m^4P^3))(-W(me^m)(3+W(me^m))(24+W(me^m)(13+3W(me^m)))+ \\ & 72(1+mP)W(me^{m(1-P)}) + 9(7+6mP)W(me^{m(1-P)})^2 + 2(11+6mP)W(me^{m(1-P)})^3 \\ & + 3W(me^{m(1-P)})^4 + 3m^4P^3(-8+3P+12A_1))+ \\ & \frac{1}{\beta}\left(2\sqrt{k}\pi(-\sqrt{k}P(-1+\sigma^2) - \beta I_1\left(\frac{1}{\sqrt{k}}\right)C_1 + \beta\sigma I_1\left(\frac{\sigma}{\sqrt{k}}\right)C_1 + \beta\left(K_1\left(\frac{1}{\sqrt{k}}\right) - \sigma K_1\left(\frac{\sigma}{\sqrt{k}}\right)\right)C_2\right). \end{aligned} \tag{6.3}$$

Expression for wall shear stress

We will evaluate the wall shearing stress using the formula

$$\tau_w^* = \left(\mu_2{}^* \frac{dv_2^*}{dr^*} \right)_{r=r_2^*}. \tag{6.4}$$

Expression for non-dimensional wall shear stress τ_w is obtained by using equation (3.1)

$$\tau_w = \frac{\tau_w^*}{\tau_y^*} = \left(\beta \frac{dv_2}{dr} \right)_{r=\sigma}. \tag{6.5}$$

By substituting the value of v_2 from the equation (5.6) in the above dimensionless expression of wall shear stress, we get

$$\tau_w = \frac{\beta}{\sqrt{k}} \left(C_1 I_1\left(\frac{\sigma}{\sqrt{k}}\right) - C_2 K_1\left(\frac{\sigma}{\sqrt{k}}\right) \right). \tag{6.6}$$

The values of C_1 and C_2 are given by equations (5.8) and (5.9), respectively. It is seen that wall shear stress is depends on the pressure gradient P and permeability k of porous medium.

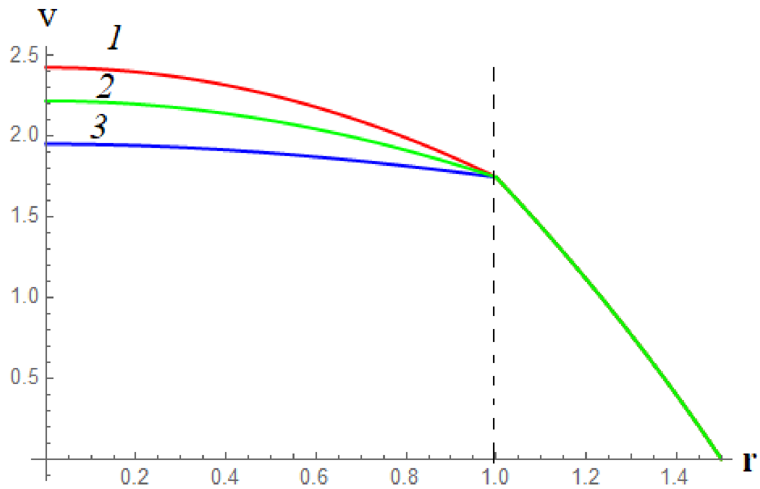

Fig. 2: Variation in velocity with material parameter m when (1) $m = 0.1$, (2) $m = 0.5$, (3) $m = 2$.

7 Discussion of the results

In this section, we have described the graphical representaion of velocity profiles, volumetric flow rate and wall shear stress on different flow parameters.

(a) Influence of various parameters on velocity profiles

Here, we will investigate the effect of different parameters on flow velocities of modified Bingham fluid and Newtonian liquid. Figure 2 sketches the effect of material parameter ($m > 0$) on velocity of the Bingham fluid in region I ($0 \leq r \leq 1$) and velocity of liquid in region II ($1 \leq r \leq \sigma$) when $P = -1.5, \beta = 0.5, k = 1$ and $\sigma = 1.5$ for different values of material parameter $m = 0.1, 0.5, 2$. It is seen clearly from the figure that material parameter m does not affect the velocity of porous region liquid (Newtonian liquid) and velocity of modified Bingham fluid is decreases with increasing the value of material parameter. Thus, it is concluded that material parameter is the controlling property of modified Bingham fluid, so it is not affect velocity of Newtonian liquid. Figure 3 depicts the influence of viscosity ratio parameter β on velocity profiles in both regions I and II when $P = -2.5, m = 0.5, k = 1, \sigma = 1.5$ for different values of viscosity ratio $\beta = 0.3, 0.5, 0.9$. It is observed from the figure that when the value of viscosity ratio increases, then velocities of modified Bingham fluid and Newtonian liquid decreases in both regions.

Figure 4 describes the effect of permeability parameter k on the velocities of modified Bingham fluid and Newtonian liquid when $P = -1.5, m = 2, \beta = 0.5, \sigma = 1.5$ for different values of permeability parameters $k = 15, 1, 0.5.$. The figure shows that when the value of permeability parameter increases, then velocity of Bingham fluid and Newtonian liquid increases in both regions. Effects of pressure gradient on the velocity profiles when $m = 3, k = 1, \sigma = 1.5, \beta = 0.9$ for different values of pressure gradient $P = -2.5, -1.5, -1$ have been described in the figure 5. It is seen that when negative pressure gradient increases, then velocity of Bingham fluid and Newtonian liquid in both regions increases.

(b) Influence of different parameters on volumetric rate flow

Here, we will discussed the effect of different parameters on the volumetric flow rate Q.

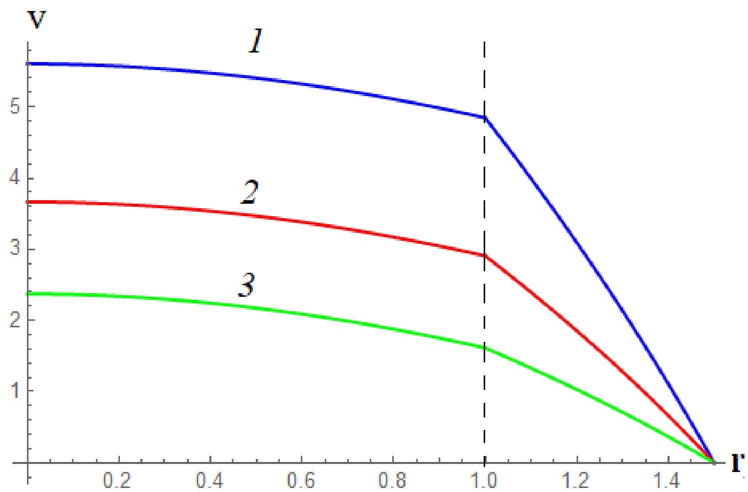

Fig. 3: Variation in velocity with viscosity ratio β when (1) $\beta = 0.3$, (2) $\beta = 0.5$, (3) $\beta = 0.9$.

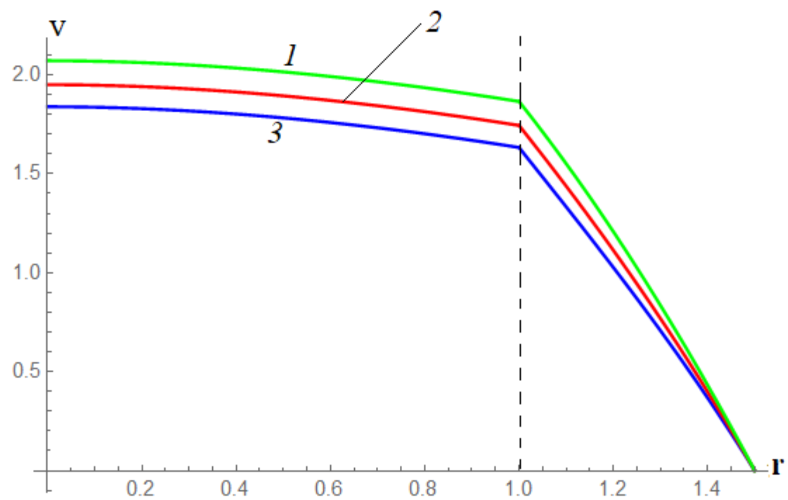

Fig. 4: Variation in velocity with permeability k when(1) $k = 15$, (2) $k = 1$, (3) $k = 0.5$.

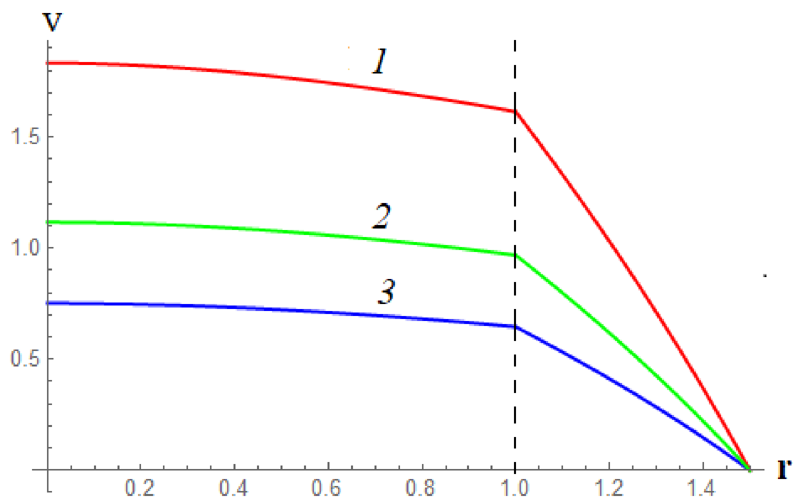

Fig. 5: Variation in velocity with pressure gradient P when(1) $P = -2.5$, (2) $P = -1.5$, (3) $P = -1$.

In figure 6, variation of volumetric flow rate on pressure gradient P when $\beta = 0.5, k = 1$ and $\sigma = 1.5$ for different values of material parameters $m = 0.1, 0.5, 2.$ is plotted. It is found that volumetric flow rate decreases with respect to pressure gradient P, when values of material parameter increases. It can be also observed that volumetric flow rate depends linearly on the pressure gradient P. The variation of volumetric flow rate with respect to permeability parameter k when $\beta = 0.9, P = -1.5, \sigma = 1.5$ for different values of material parameter $m = 2, 5, 30$ is shown in figure 6. It is seen that volumetric flow rate increases abruptly when permeability $k \leq 1$ and smoothly when permeability $k \geq 1$, for all values of m. It is also observed that volumetric flow rate decreases with respect to permeability k, when the values of material parameter m increases. In the next figure 7, the graphical behaviors of volumetric flow rate with respect to material parameter m when $P = -1.5, \sigma = 1.5$ and $\beta = 0.3$ for different values of permeability parameter $k = 10, 2$ and 1 are shown. From the figure, it is observed that flow rate decreases with increasing values of material parameter for small values of m and become almost constant for large values of material parameter m. It is also seen that flow rate increases with respect to material parameter m, when the values of permeability k increases. Figure 8 depicts the graphical behaviors of volumetric flow rate with respect to viscosity ratio β when $k = 2, \sigma = 1.5$ and $m = 2$ for different value of pressure gradient $P = -1.5, -2.5$ and -5.5. It is seen that volumetric flow rate Q decreases exponentially with respect to viscosity ratio β, for all values of the pressure gradient P.

(c)Influence of permeability on wall shear stress τ_w

It is found from the expression of wall shear stress that it depends only on dimensionless permeability parameter k, ratio of radii σ and constant pressure gradient P. Figure 9 shows the variation of wall shear stress with respect to permeability k. It is seen from the figure that wall shear stress decreases with increasing the values of permeability.

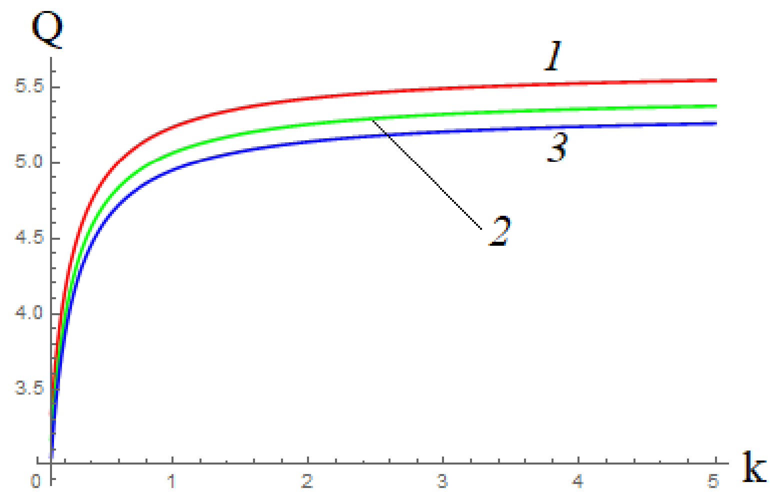

Fig. 6: Variation in flow rate with permeability k when (1) $m = 2$, (2) $m = 5$, (3) $m = 30$.

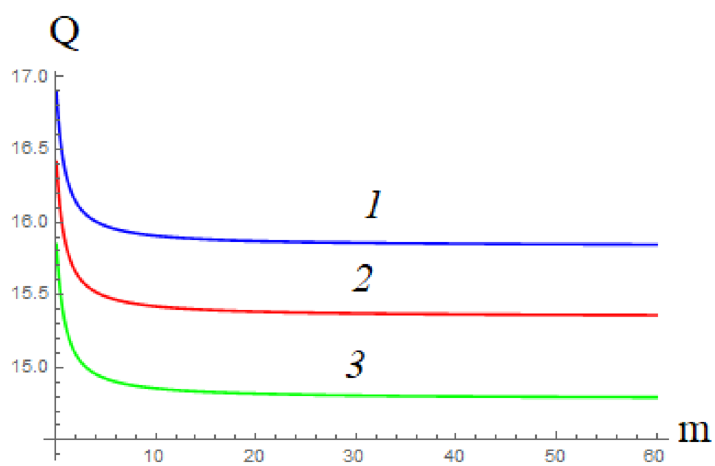

Fig. 7: Variation in flow rate with material parameter m when $(1)k = 10, (2)k = 2, (3)k = 1$.

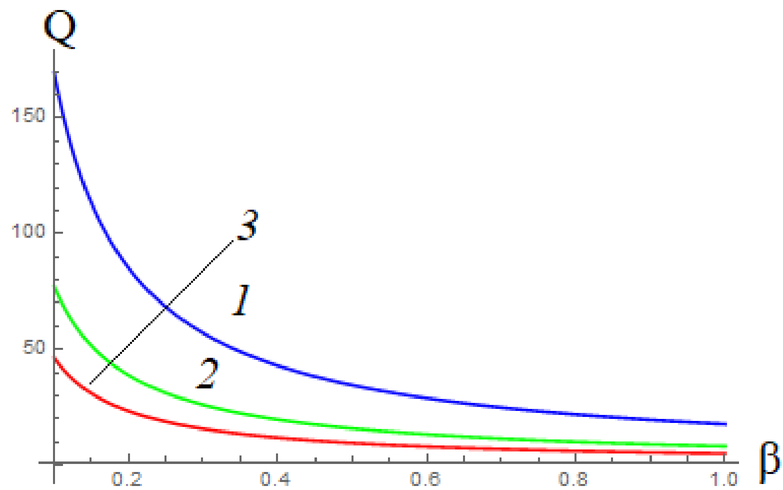

Fig. 8: Variation in flow rate with respect to viscosity ratio β when (1) $P = -1.5$, (2) $P = -2.5$, (3) $P = -5.5$.

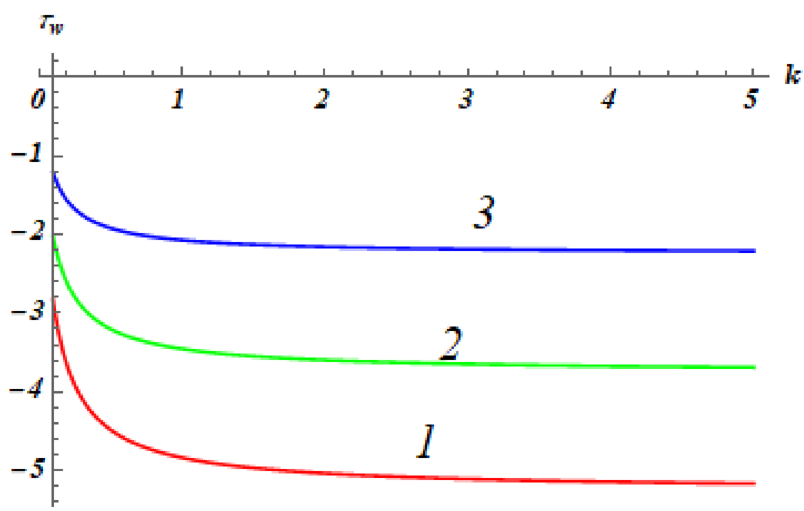

Fig. 9: Variation of wall shearing stress with respect to permeability k when $\sigma = 1.5$,(1)$P = -1.5$, (2)$P = -2.5$, (3)$P = -3.5$.

8 Conclusions

In this work, we have solved the fully developed laminar flow problem of modified Bingham fluid through a cylindrical cavity and Newtonian liquid through porous shell. Analytical expressions for velocity profiles, volumetric flow rate and wall shearing stress have been obtained with the help of appropriate boundary conditions (No-slip condition, Regularity condition and Interface conditions).
Following conclusions are drawn from the above analysis:

- The material parameter m has affected flow velocity of modified Bingham fluid but not affected velocity of Newtonian liquid. Bingham fluid velocity decreases with increasing material parameter m.
- When the values of viscosity ratios β increases, then velocity decreases, in both regions I and II.
- The volumetric flow rate Q increases when permeability k of porous medium increases but for large value of permeability k, volumetric flow rate will become almost constant.
- The volumetric flow rate Q decreases with increasing value of material parameter m and become approximately constant for large value of material parameter m.
- The volumetric flow rate decreases exponentially with respect to viscosity ratios (β).
- The wall shearing stress depends only on pressure gradient P and permeability k. In fact, wall shear stress depends linearly on the pressure gradient and decreases with increasing values of permeability.

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