Analytical Solution of Price Adjustment Equation Involving Constant Proportional Caputo Derivative by Sumudu Transform

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Abstract

An attempt is made to obtain analytical solution of price adjustment equation involving constant proportional Caputo fractional derivative. Price adjustment equation with and without prospects of agents is considered in the study under equilibrium condition. Sumudu transform method is applied to obtain analytical solution of price adjustment model involving constant proportional Caputo derivative. Analytical solution of price adjustment equation with suitable parameters is obtained and are simulated using MATLAB.

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1 Introduction

Mathematical models are being used by the economists which enable them to predict optimal profit, to view the link between demand and supply.The concept of price adjustment is determined by inefficient competitive company's lower demand and different price. A market structure in which price adjustment is free from individual behavior and satisfaction consists of a great number of economic agents competing with each other can be referred for more information about price adjustment.Economics provides the interactions between price, supply and demand,the dependence of supply and demand on price and also how equilibrium point is reached on supply and demand curves.A competitive market is directly linked to competitive equilibrium which means the demand of a quantity of goods demanded by customers is met with supply by the providers [13]. A competitive market is directly related to competitive equilibrium that is quantity of goods demanded by buyers equal to the quantity of goods supplied by vendors. We consider the following economic models as follows:

The object of an economic mathematical model is to formulate economic process in mathematical form so as to achieve equilibrium in price adjustment model [8, 13].

$$q_d(t) = d_0 - d_1 w(t); \qquad q_v(t) = -v_0 + v_1 w(t),$$

where $w(t)$ is the price of goods and d_0, v_0, d_1, v_1 are positive constants.
When $q_d(t) = q_v(t)$, that is demand and supply of goods are equal, the equilibrium price is

$$w(t) = \frac{d_0 + v_0}{d_1 + v_1}.$$

That is, there is no shortage and surplus of goods.
Now consider the price adjustment equation without prospects of agents as follows:

$$w'(t) = p(q_d - q_v)$$

where $p > 0$, is the speed of adjustment constant. That is,

$$w'(t) + p(d_1 + v_1)w(t) = p(d_0 + v_0)$$

The solution of above equation is given by

$$w(t) = \frac{(d_0 + v_0)}{(d_1 + v_1)} + \left[w(0) - \frac{(d_0 + v_0)}{(d_1 + v_1)}\right]e^{-p(d_1+v_1)t},$$

where $w(0)$ is the price at time $t = 0$.
If we consider the prospects of agents, the demand and supply functions involving d_2 and v_2 is

$$q_d(t) = d_0 - d_1 w(t) + d_2 w'(t); \qquad q_v(t) = v_0 - v_1 w(t) - v_2 w'(t).$$

Equating $q_d(t)$ and $q_v(t)$, we obtain

$$w'(t) - \frac{(d_1 + v_1)}{(d_2 + v_2)} w(t) = \frac{(d_0 + v_0)}{(d_2 + v_2)}.$$

The solution of above linear differential equation is

$$w(t) = \frac{(d_0 + v_0)}{(d_2 + v_2)} + \left[w(0) - \frac{(d_0 + v_0)}{(d_1 + v_1)}\right]e^{\frac{(d_1+v_1)}{(d_2+v_2)}t}.$$

If the price of goods raise, buyers may purchase more before price rises to break and accordingly vendors disposed to offer less in order to make a earn higher prices in later times. When $w'(t) = 0$ for all $t > 0$, market is in dynamic equilibrium.

It is well known that any economic process around us can be modeled by ordinary or partial differential equations but there are some dynamical process which are modeled via fractional differential equations [4, 6, 7, 8, 15]. Analytical and approximate solution methods for fractional differential equations have been recently given in [1, 2, 9, 10]. Akgul et.al.[3] studied the solutions of new type of fractional differential equations in the study of electro-hydrodynamic flow, analysis and dynamical behaviour and the solutions of fractional order telegraph equations by Crank-Nicholson finite difference method. Acay et.al.[2] investigated the solutions of economic models with three different derivatives Caputo,Caputo-Fabrizio and Atangana-Baleanu using Laplace transform methods. These models have been studied in [11] using Sumudu transforms. The economic model with constant proportional Caputo derivative by Laplace transforms was considered and obtained analytical solutions for these models [3]. This motivates to study economic models price adjustment equation with and without prospects of agents. In this paper an attempt is made to obtain analytical solution of price adjustment equation involving constant proportional Caputo drivative by Sumudu transform.
The manuscript is arranged as follows:

Section 2 involves the basic definitions and results that plays vital role in the further study. In section 4, result for Sumudu transform of constant proportional Caputo derivative and the price adjustment equation with constant proportional Caputo derivative with and without prospects of agents is considered. The analytical solution of these economic models is obtained by Sumudu transform. An example is given to ensure the obtained results. Conclusion section is given at the end.

2 Main Results

We first prove the following:

Theorem 2.1. *The Sumudu transform of constant proportional Caputo (CPC) derivative of $w(t)$ is*

$$S\left[{}_0^{CPC}D_t^{\alpha}w(t)\right] = [k_1(\alpha)u + k_0(\alpha)]u^{-\alpha}W(u) - k_0(\alpha)u^{-\alpha}w(0).$$

Proof. Using

$$S\left[{}_0^{RL}I_t^{\alpha}w(t)\right] = u^{\alpha}S[w(t)]$$
$$S\left[{}_0^{c}D^{\alpha}w(t)\right] = u^{-\alpha}S[w(t)] - u^{-\alpha}[w(0)] \quad for \quad 0 < \alpha \leq 1,$$

we obtain

$$\begin{aligned} S\left[{}_0^{CPC}D_t^{\alpha}w(t)\right] &= S\left[k_1(\alpha){}_0^{RL}D^{1-\alpha}w(t) + k_0(\alpha){}_0^{c}D^{\alpha}w(t)\right] \\ &= k_1(\alpha)u^{1-\alpha}W(u) + k_0(\alpha)[u^{-\alpha}W(u) - u^{-\alpha}w(0)] \\ &= [k_1(\alpha)u + k_0(\alpha)]u^{-\alpha}W(u) - k_0(\alpha)u^{-\alpha}w(0). \end{aligned}$$

□

We consider the price adjustment equation with constant proportional Caputo derivative without prospects of agent as:

$$\left[{}_0^{CPC}D_t^{\alpha}w(t)\right] + k(d_1 + v_1)w(t) = k(d_0 + v_0).$$

Applying the Sumudu transform of both sides, we obtain

$$S\left[{}_0^{CPC}D_t^{\alpha}w(t)\right] + k(d_1 + v_1)S[w(t)] = S[k(d_0 + v_0)].$$

By using Theorem 2.1,we obtain

$$[uk_1(\alpha) + k_0(\alpha)]u^{-\alpha}S[w(t)] - u^{-\alpha}k_0(\alpha)w(0) + k(d_1 + v_1)S[w(t)] = k(d_0 + v_0)$$
$$S[w(t)][u^{1-\alpha}k_1(\alpha) + u^{-\alpha}k_0(\alpha) + k(d_1 + v_1)] = k(d_0 + v_0) + u^{-\alpha}k_0(\alpha)w(0)$$

$$\begin{aligned} S[w(t)] &= \frac{k(d_0 + v_0)}{k_1(\alpha)u^{1-\alpha} + k_0(\alpha)u^{-\alpha} + k(d_1 + v_1)} + \frac{u^{-\alpha}k_0(\alpha)w(0)}{k_1(\alpha)u^{1-\alpha} + k_0(\alpha)u^{-\alpha} + k(d_1 + v_1)} \\ &= \frac{(d_0 + v_0)}{(d_1 + v_1)}\left[\frac{1}{1 + \frac{k_1(\alpha)u^{1-\alpha} + k_0(\alpha)u^{-\alpha}}{k(d_1+v_1)}}\right] + w(0)\left[\frac{1}{1 + \frac{k_1(\alpha)u + k(d_1+v_1)u^{\alpha}}{k_0(\alpha)}}\right] \end{aligned}$$

$$= \frac{(d_0 + v_0)}{(d_1 + v_1)}\left[1 - \frac{-k_1(\alpha)u^{1-\alpha} - k_0(\alpha)u^{-\alpha}}{k(d_1 + v_1)}\right]^{-1} + w(0)\left[1 - \frac{-k_1(\alpha)u - k(d_1 + v_1)u^{\alpha}}{k_0(\alpha)}\right]^{-1}$$

$$= \frac{(d_0 + v_0)}{(d_1 + v_1)}\sum_{m=0}^{\infty}\left[\frac{-k_1(\alpha)u^{1-\alpha} - k_0(\alpha)u^{-\alpha}}{k(d_1 + v_1)}\right]^m + w(0)\sum_{m=0}^{\infty}\left[\frac{-k_1(\alpha)u - k(d_1 + v_1)u^{\alpha}}{k_0(\alpha)}\right]^m$$

$$\begin{aligned} &= \frac{(d_0 + v_0)}{(d_1 + v_1)}\sum_{m=0}^{\infty}\frac{1}{k^m(d_1 + v_1)^m}\sum_{n=0}^{m}\binom{m}{n}[-k_1(\alpha)u^{1-\alpha}]^{m-n}[-k_0(\alpha)u^{-\alpha}]^n \\ &\quad + w(0)\sum_{m=0}^{\infty}\frac{1}{k_0(\alpha)^m}\sum_{n=0}^{m}\binom{m}{n}[-k_1(\alpha)u]^{m-n}[-k(d_1 + v_1)u^{\alpha}]^n \\ &= \frac{(d_0 + v_0)}{(d_1 + v_1)}\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(-1)^m k_1(\alpha)^{m-n}k_0(\alpha)^n}{k^m(d_1 + v_1)^m}\binom{m}{n}\frac{u^{(1-\alpha)(m-n)-\alpha n}}{(1-\alpha)m - n} \\ &\quad + w(0)\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(-1)^m(\alpha)^{m-n}k^n(d_1 + v_1)^n}{k_0(\alpha)^m}\binom{m}{n}\frac{u^{(m-n+\alpha n)}}{u^{(\alpha-1)n+m}}. \end{aligned}$$

Taking the inverse Sumudu transform of both sides, we obtain

$$w(t) = \frac{(d_0+v_0)}{(d_1+v_1)}\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(-1)^m k_1(\alpha)^{m-n}k_0(\alpha)^n}{k^m(d_1+v_1)^m}\binom{m}{n}\frac{t^{(1-\alpha)m-n}}{\Gamma((1-\alpha)m-n+1)}$$
$$+\, w(0)\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(-1)^m k_1(\alpha)^{m-n}k^n(d_1+v_1)^n}{k_0(\alpha)^m}\binom{m}{n}\frac{t^{m+(\alpha-1)n}}{\Gamma((\alpha-1)n+m+1)}.$$

Let $r = m - n$, we get

$$w(t) = \frac{(d_0+v_0)}{(d_1+v_1)}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(n+r)!}{n!r!}.\frac{(-k_1(\alpha))^r(-k_0(\alpha))^n}{k^{r+n}(d_0+v_0)^{r+n}}.\frac{t^{(1-\alpha)r-\alpha n}}{\Gamma((1-\alpha)r-\alpha n+1)}$$
$$+\, w(0)\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(n+r)!}{n!r!}.\frac{(-k_1(\alpha))^r(-k(d_1+v_1))^n}{k_0(\alpha)^{r+n}}.\frac{t^{p+\alpha n}}{\Gamma(r+\alpha n+1)}$$
$$= \frac{(d_0+v_0)}{(d_1+v_1)}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(n+r)!}{n!r!}\Big[\frac{-k_0(\alpha)}{k(d_0+v_0)}t^{-\alpha}\Big]^n\Big[\frac{-k_1(\alpha)}{k(d_0+v_0)}t^{1-\alpha}\Big]^r\frac{1}{\Gamma((1-\alpha)r-\alpha n+1)}$$
$$+\, w(0)\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(n+r)!}{n!r!}.\Big[\frac{-k(d_1+v_1)}{k_0(\alpha)}t^{\alpha}\Big]^n\Big[\frac{-k_1(\alpha)}{k_0(\alpha)}t\Big]^r\frac{1}{\Gamma(r+\alpha n+1)}.$$

The solution in geometric series [10] can be written as

$$w(t) = \frac{(d_0+v_0)}{(d_1+v_1)}E^1_{1-\alpha,-\alpha,1}\Big[\frac{-k_1(\alpha)}{k(d_0+v_0)}t^{1-\alpha}, \frac{-k_0(\alpha)}{k(d_0+v_0)}t^{-\alpha}\Big]$$
$$+\, w(0)E^1_{1,\alpha,1}\Big[\frac{-k_1(\alpha)}{k_0(\alpha)}t, \frac{-k(d_1+v_1)}{k_0(\alpha)}t^{\alpha}\Big].$$

Next, we consider the price adjustment equation involving constant proportional Caputo derivative with prospects of agents as:

$${}_0^{CPC}D_t^{\alpha}w(t) - \frac{d_1+v_1}{d_2+v_2}w(t) = -\frac{d_0+v_0}{d_2+v_2}.$$

Applying the Sumudu transform of both sides, we obtain

$$S\Big[{}_0^{CPC}D_t^{\alpha}w(t)\Big] - \frac{d_1+v_1}{d_2+v_2}S[w(t)] = S\Big[-\frac{d_0+v_0}{d_2+v_2}\Big].$$

By using

$$[uk_1(\alpha)+k_0(\alpha)]u^{-\alpha}S[w(t)] - u^{-\alpha}k_0(\alpha)w(0) - \frac{d_1+v_1}{d_2+v_2}S[w(t)] = -\frac{d_0+v_0}{d_2+v_2}$$

$$S[w(t)]\Big[u^{1-\alpha}k_1(\alpha)(d_2+v_2) + k_0(\alpha)u^{-\alpha}(d_2+v_2)\Big] - u^{\alpha}k_0(\alpha)w(0)(d_2+v_2) -$$
$$(d_1+v_1)S[w(t)] = -(d_0+v_0).$$

$$
\begin{aligned}
S[w(t)] &= \frac{-(d_0+v_0)}{(d_2+v_2)k_1(\alpha)u^{1-\alpha}+(d_2+v_2)k_0(\alpha)u^{-\alpha}-(d_1+v_1)} \\
&+\frac{w(0)u^{-\alpha}k_0(\alpha)(d_2+v_2)}{(d_2+v_2)k_1(\alpha)u^{1-\alpha}+(d_2+v_2)k_0(\alpha)u^{-\alpha}-(d_1+v_1)} \\
&= \frac{d_0+v_0}{d_1+v_1}\Big[\frac{1}{1-\frac{(d_2+v_2)k_1(\alpha)u^{1-\alpha}+(d_2+v_2)k_0(\alpha)u^{-\alpha}}{d_1+v_1}}\Big] \\
&+w(0)\Big[\frac{1}{1+\frac{(d_2+v_2)k_1(\alpha)u-(d_1+v_1)u^{\alpha}}{k_0(\alpha)(d_2+v_2)}}\Big] \\
&= \frac{d_0+v_0}{d_1+v_1}\Big[1-\frac{(d_2+v_2)k_1(\alpha)u^{1-\alpha}+(d_2+v_2)k_0(\alpha)u^{-\alpha}}{d_1+v_1}\Big]^{-1} \\
&+w(0)\Big[1-\frac{-(d_2+v_2)k_1(\alpha)u+(d_1+v_1)u^{\alpha}}{k_0(\alpha)(d_2+v_2)}\Big]^{-1} \\
&= \frac{d_0+v_0}{d_1+v_1}\sum_{m=0}^{\infty}\Big[\frac{(d_2+v_2)u^{1-\alpha}+k_0(\alpha)(d_2+v_2)u^{-\alpha}}{d_1+v_1}\Big]^{m} \\
&+w(0)\sum_{m=0}^{\infty}\Big[\frac{-k_1(\alpha)(d_2+v_2)u+(d_1+v_1)u^{\alpha}}{k_0(\alpha)(d_2+v_2)}\Big]^{m} \\
&= \frac{d_0+v_0}{d_1+v_1}\sum_{m=0}^{\infty}\frac{1}{(d_1+v_1)^m}\sum_{n=0}^{m}\binom{m}{n}[(d_2+v_2)^{1-\alpha}]^{m-n}[k_0(\alpha)(d_2+v_2)u^{-\alpha}]^{n} \\
&+w(0)\sum_{m=0}^{\infty}\frac{1}{k_0(\alpha)^m(d_2+v_2)^m}\sum_{n=0}^{m}\binom{m}{n}[-k_1(\alpha)(d_2+v_2)u]^{m-n}[(d_1+v_1)u^{\alpha}]^{n} \\
&= \frac{d_0+v_0}{d_1+v_1}\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(d_2+v_2)^{m-n}k_0(\alpha)^n(d_2+v_2)^n}{(d_1+v_1)^m}\binom{m}{n}u^{(1-\alpha)(m-n)-\alpha n} \\
&+w(0)\frac{d_0+v_0}{d_1+v_1}\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(-k_1(\alpha))^{m-n}(d_2+v_2)^{m-n}(d_1+v_1)^n}{k_0(\alpha)^m(d_2+v_2)^m}\binom{m}{n}u^{m-n+\alpha n}.
\end{aligned}
$$

Taking the inverse Sumudu transform of both sides, we obtain

$$
\begin{aligned}
w(t) &= \frac{d_0+v_0}{d_1+v_1}\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(d_2+v_2)^{m-n}k_0(\alpha)^n(d_2+v_2)^n}{(d_1+v_1)^m}\binom{m}{n}\frac{t^{(1-\alpha)(m-n)-\alpha n}}{\Gamma((1-\alpha)(m-n)-\alpha n+1)} \\
&+w(0)\frac{d_0+v_0}{d_1+v_1}\sum_{m=0}^{\infty}\sum_{n=0}^{m}\frac{(-k_1(\alpha))^{m-n}(d_2+v_2)^{m-n}(d_1+v_1)^n}{k_0(\alpha)^m(d_2+v_2)^m}\binom{m}{n}\frac{t^{m-n+\alpha n}}{\Gamma(j-r+\alpha n+1)}.
\end{aligned}
$$

Let $r = m - n$, we get

$$\begin{aligned} w(t) &= \frac{d_0+v_0}{d_1+v_1}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(n+r)!}{n!r!}\frac{(d_2+v_2)^{r+n}(k_0(\alpha))^n}{(d_1+v_1)^{r+n}}.\frac{t^{(1-\alpha)r-\alpha n}}{\Gamma((1-\alpha)r-\alpha n+1)} \\ &+ w(0)\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(n+r)!}{n!r!}\frac{(-k_1(\alpha))^r(d_1+v_1)^n}{k_0(\alpha)^{r+n}}(d_2+v_2)^n\frac{t^{r+\alpha n}}{\Gamma(r+\alpha n+1)} \\ &= \frac{d_0+v_0}{d_1+v_1}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(n+r)!}{n!r!}\Big[\frac{(d_2+v_2)k_0(\alpha)}{d_1+v_1}t^{-\alpha}\Big]^n\Big[\frac{d_2+v_2}{d_1+v_1}t^{1-\alpha}\Big]^r\frac{1}{\Gamma((1-\alpha)r-\alpha n+1)} \\ &+ w(0)\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{(n+r)!}{n!r!}\Big[\frac{d_1+v_1}{(d_2+v_2)k_0(\alpha)}t^{\alpha}\Big]^n\Big[\frac{-k_1(\alpha)}{k_0(\alpha)}t\Big]^r\frac{1}{\Gamma(r+\alpha n+1)}. \end{aligned}$$

The solution in geometric series [10] is written as

$$\begin{aligned} w(t) &= \frac{d_0+v_0}{d_1+v_1}E^1_{1-\alpha,-\alpha,1}\Big[\frac{d_2+v_2}{d_1+v_1}t^{1-\alpha}, \frac{(d_2+v_2)k_0(\alpha)}{d_1+v_1}t^{-\alpha}\Big] \\ &+ w(0)E^1_{1,\alpha,1}\Big[\frac{-k_1(\alpha)}{k_0(\alpha)}t, \frac{d_1+v_1}{k_0(\alpha)(d_2+v_2)}t^{\alpha}\Big]. \end{aligned}$$

3 Example

We assign some values to constants $d_0, v_0, d_1, v_1, d_2, v_2$ influencing the market equilibrium as $d_0 = 10, v_0 = 100, d_1 = 14, v_1 = 97, d_2 = 18$ and $v_2 = 94$. The price adjustment equation involving constant proportional Caputo derivative without prospects of agents for above parameter is

$$[{}^{CPC}_{0}D^{\alpha}_{t}w(t)] + 111kw(t) = 110k. \tag{3.1}$$

The solution of above price adjustment equation is

$$w(t) = \frac{110}{111}E^1_{1-\alpha,-\alpha,1}\Big[\frac{-k_1(\alpha)}{(110)k}t^{1-\alpha}, \frac{-k_0(\alpha)}{(110)k}t^{-\alpha}\Big] + w(0)E^1_{1,\alpha,1}\Big[\frac{-k_1(\alpha)}{k_0(\alpha)}t, \frac{-(111)k}{k_0(\alpha)}t^{\alpha}\Big].$$

Also, the price adjustment equation involving constant proportional Caputo derivative with prospects of agents for above parameter is

$${}^{CPC}_{0}D^{\alpha}_{t}w(t) - \frac{111}{112}w(t) = -\frac{110}{112}. \tag{3.2}$$

The solution of above price adjustment equation is

$$w(t) = \frac{110}{111}E^1_{1-\alpha,-\alpha,1}\Big[\frac{112}{111}t^{1-\alpha}, \frac{112k_0(\alpha)}{111}t^{-\alpha}\Big] + w(0)E^1_{1,\alpha,1}\Big[\frac{-k_1(\alpha)}{k_0(\alpha)}t, \frac{111}{112k_0(\alpha)}t^{\alpha}\Big].$$

Following figure 1 to figure 3 and figure 4 to figure 6 are the graphs of solutions of fractional differential equations (3.1) and (3.2) respectively involving constant proportional Caputo derivative of order $\alpha = 0.5, 0.2, 0.7$.

4 Conclusion

The price adjustment equations involving constant proportional Caputo derivative without and with prospects of agents are studied using Sumudu transform. Analytical solutions are obtained using

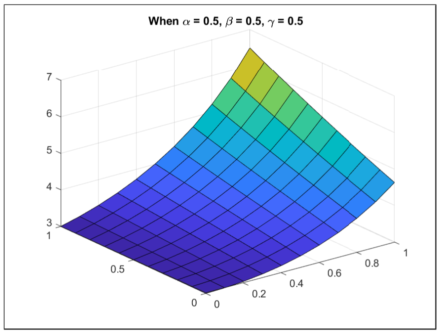

Fig. 1: Solution of (3.1) for $\alpha = 0.5$

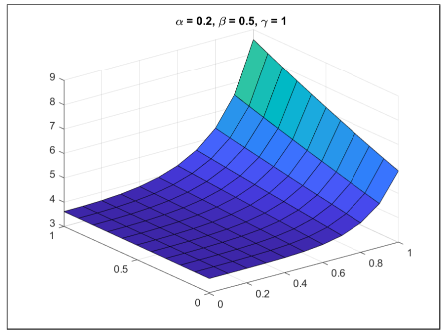

Fig. 2: Solution of (3.1) for $\alpha = 0.2$

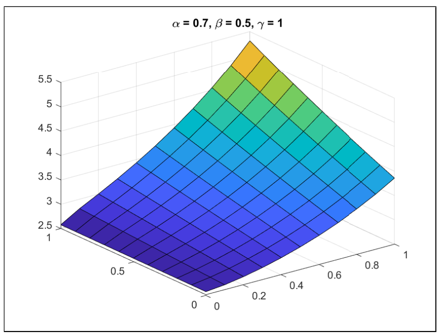

Fig. 3: Solution of (3.1) for $\alpha = 0.7$

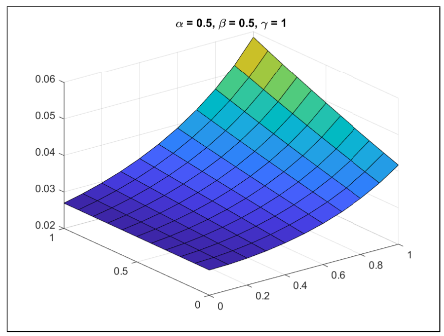

Fig. 4: Solution of (3.2) for $\alpha = 0.5$

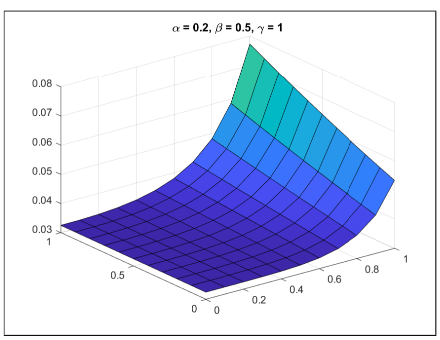

Fig. 5: Solution of (3.2) for $\alpha = 0.2$

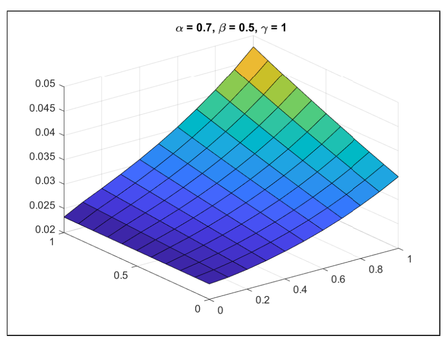

Fig. 6: Solution of (3.2) for $\alpha = 0.7$

Sumudu transform method. The method is illustrated with suitable example by considering values to parameters with equilibrium of market. Solutions are simulated using MATLAB

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References

[1] *T. Abdeljawad*, Fractional Operators with Generalized Mittag-Leffler Kernels and their Iterated Differintegrals, Chaos: Interdiscip J Nonlinear Sci. **29**(2019), no. 2,023102.

[2] *B. Acay, E. Bas and T. Abdeljawad* , Fractional Economic Models Based on Market Equilibrium in the Frame of Different Type Kernels, Chaos, Solitons Fractals.**130**(2020), 109438.

[3] *E. K. Akgul, A. Akgul and D. Baleanu*, Laplace Transform Method for Economic Models with Constant Proportional Caputo Derivative, Fractal and Fractional. **4**(2020), 30.

[4] *D. R. Anderson and D.J. Ulness*, Newly Defined Conformable Derivatives, Adv.Dyn.Syst.Appl. **10** (2015),109–137.

[5] *D. S. Bodkhe and S. K. Panchal*, On Sumudu Transform of Fractional Dertivatives and Its Applications to Fractional Differential Equations, Asian Journal Mathematics and Computer Research. **11** (2016), no. 1, 69–77.

[6] *E. Bas and R. Ozarslan*, Real World Applications of Fractional Models by Atangana-Baleanu Fractional Derivative, Chaos, Solitons Fract. (2018),116–121.

[7] *M. Caputo and M. Fabrizio*, A New Definition of Fractional Derivative without Singular Kernel, Prog. Fract Differ Appl. **1**(2015),no. 2,1-–13.

[8] *D. Cohen-Vernik and A. Pazgal*, Price Adjustment Policy with Partial Refunds, J. Retailing. **93**(2017), no. 4, 507-–526.

[9] *D. Baleanu, H. Mohammadi and S. Rezapour*, A Fractional Differential Equation Model for the COVID-19 Transmission by Using the Caputo-Fabrizio Derivative, Advances in Difference Equations.(2020), 1–27.

[10] *A. Fernandez, C. Kürt and M. A. Özarslan*, A Naturally Emerging Bivariate Mittag-Leffler Function and Associated Fractional-Calculus Operators, **2020**, arXiv:12171(2002).

[11] *J. A. Nanware, N. G. Patil and G. A. Birajdar*, Applications of Sumudu Transform to Economic Models, Palestine Journal of Mathematics. **11** (2022), no. 2,636–649.

[12] *I. Podlubny*, Fractional Differential Equations, Academic Press. San Diego, CA, USA, 1999.

[13] *A. Takayama*, Mathematical Economics, Cambridge University Press. 1985.

[14] *G. K. Watugala*,Sumudu transform- A New Integral Transform to Solve Differential Equations and Control Engineering Problems, Int.J.Math.Educ.Sci.Technol. **24**(1993), no. 1, 35–43 (1993). https://doi.org/10.1080/0020739930240105.

[15] *M. Yavuz and N. Ozdemir*, A Different Approach to the European Option Pricing Model with New Fractional Operator, Math. Model. Nat. Phenom. **13**(2018), no. 1,:12.