Screen Bi-Slant Lightlike Submersions

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Abstract

In this article, we introduce the notion of screen bi-slant lightlike submersions from an indefinite K*ä*hler manifold onto a lightlike manifold with screen semi-slant and screen pseudo-slant lightlike submersions as its particular cases. We study some properties of proper screen bi-slant lightlike submersions giving non-trivial examples. Moreover, we give a characterization theorem and obtain integrability conditions of distributions involved in the definition of such submersions.
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1 Introduction

A C^∞ map $f : (M, g) \to (B, g')$ between Riemannian manifolds M and B is said to be Riemannian submersion if the derivative map f_* is onto and preserves the length of horizontal vectors. O'Neill [3] and Gray [2] initiated the theory of Riemannian submersions. Later, Sahin [16] gave the notion of screen lightlike submersions from lightlike manifolds onto semi Riemannian manifolds. Sahin and G*ü*nd*ü*zalp [8], defined lightlike submersions from semi-Riemannian manifolds onto lightlike manifolds. As a generalization of semi-slant and hemi-slant submersions, C. Sayar et al. [11] studied the geometry of bi-slant submersions from almost Hermitian manifolds onto Riemannian manifolds. In the present article we study screen bi-slant lightlike submersions from an indefinite K*ä*hler manifold onto a lightlike manifold.

The paper is organized as follows. In section 2, we give the basic notion and definitions needed for the present study. In section 3, we introduce the notion of screen bi-slant lightlike submersions from an indefinite K*ä*hler manifold onto a lightlike manifold with non-trivial examples. We study some properties of proper screen bi-slant lightlike submersions, giving a characterization theorem and obtain integrability conditions of distributions Δ, D^{θ_1}, D^{θ_2}, D' and D'' involved in the definition of these submersions.

2 Preliminaries

Let (M, g) be a real m-dimensional C^∞ manifold, the radical (or null) space $Rad\ T_pM$ of T_pM is given by $Rad\ T_pM = \{\xi \in T_pM : g(\xi, X) = 0,\ \forall X \in T_pM\}$. If $Rad\ TM : p \in M \to Rad\ T_pM$ defines a smooth distribution of rank $r > 0$ on M such that $0 < r \leq m$, then $Rad\ TM$ is called a radical or null distribution of M and the manifold M is called an r-lightlike manifold.

Let $f : (M, g) \to (B, g')$ be a smooth submersion from a semi-Riemannian manifold M onto an r-lightlike manifold, $Ker\ f_{*p} = \{X \in T_pM : f_{*p}X = 0\}$ and $(Ker\ f_{*p})^\perp = \{Y \in T_pM : g(Y, X) = 0,\ \forall X \in Ker\ f_{*p}\}$. As T_pM is a semi-Riemannian vector space, $(Ker\ f_{*p})^\perp$ may not be a complementary space to $Ker f_{*p}$. Assume that $Ker\ f_{*p}\ \cap\ (Ker\ f_{*p})^\perp = \Delta_p \neq \{0\}$. In this case

$\Delta : p \to \Delta_p$ is said to be a radical distribution on M at $p \in M$. As Δ is a lightlike distribution, we have $Ker\ f_* = \Delta \perp S(Ker\ f_*)$. Similarly $(Ker\ f_*)^\perp = \Delta \perp S(Ker\ f_*)^\perp$. Here $S(Ker\ f_*)^\perp$ is the complementary distribution to Δ in $(Ker\ f_*)^\perp$. Now, let $dim(\Delta) = r > 0$. Since $\Delta \subset (S(ker\ f_*)^\perp)^\perp$ and $(S(kerf_*)^\perp)^\perp$ is non-degenerate, then there exists null vectors $N_1, N_2..., N_r$, such that $g(N_i, N_j) = 0$, $g(\xi_i, N_j) = \delta_{ij}$, where $\{N_i\}$ and $\{\xi_i\}$ are smooth null vector fields of $S(Ker\ f_*)^\perp$ and lightlike basis of Δ, respectively. The distribution spanned by null vector fields $N_1, N_2..., N_r$ is denoted by $ltr(ker\ f_*)$. Then we have $tr(kerf_*) = ltr(kerf_*) \perp S(kerf_*)^\perp$. Moreover, we have

$$TM = (\Delta \oplus ltr(Ker\ f_*)) \perp S(Ker\ f_*) \perp S(Ker\ f_*)^\perp. \tag{2.1}$$

A Riemannian submersion $f : (M, g) \to (B, g')$ is called an r-lightlike submersion if $dim\ \Delta = dim\{(Ker\ f_*) \cap (Ker\ f_*)^\perp\} = r$, $0 < r < min\{dim(ker\ f_*), dim(ker\ f_*)^\perp\}$; isotropic submersion if $dim\ \Delta = dim(Ker\ f_*) < dim(Ker\ f_*)^\perp$; co-isotropic submersion if $dim\ \Delta = dim(Ker\ f_*)^\perp < dim(Ker\ f_*)$ and totally lightlike submersion if $dim\ \Delta = dim(Ker\ f_*)^\perp = dim(Ker\ f_*)$. A lightlike submersion $f : (M, g) \to (B, g')$ determines two (1,2) type tensors fields T and A on M, given as

$$T_X Y = h\nabla_{vX} vY + v\nabla_{vX} hY, \tag{2.2}$$

$$A_X Y = v\nabla_{hX} hY + h\nabla_{hX} vY. \tag{2.3}$$

Here T and A are vertical and horizontal tensors, respectively. For vertical tensor T, we have

$$T_X Y = T_Y X, \quad \forall\ X, Y \in \Gamma(Ker\ f_*). \tag{2.4}$$

Now, we suppose that f is a lightlike submersion from a real (m + n)-dimensional semi-Riemannian manifold (M, g) onto a lightlike manifold (B, g'), with $m, n > 1$. Further, let $Ker\ f_*$ be an m-dimensional lightlike distribution of M and $tr(Ker\ f_*)$ is the complementary distribution of $Ker\ f_*$ in M with respect to the pair of screen distributions $\{S(Ker\ f_*), S(Ker\ f_*)^\perp\}$. Let us denote by $\hat{g}$ the induced metric on $Ker\ f_*$ of g and by ∇ the Levi-Civita connection on M. Then, in view of (2.2), we write

$$\nabla_U V = \hat{\nabla}_U V + T_U V, \tag{2.5}$$

$$\nabla_U X = T_U X + \nabla^\perp_U X, \tag{2.6}$$

$\forall\ U, V \in \Gamma(Ker\ f_*), X \in \Gamma(Ker\ f_*)^\perp$, where $\hat{\nabla}_U V = v\nabla_U V$ and $\nabla^\perp_U X = h\nabla_U X$. Here $\{\hat{\nabla}_U V, T_U X\}$ and $\{T_U V, \nabla^\perp_U X\}$ belong to $\Gamma(Ker\ f_*)$ and $\Gamma(tr(Ker\ f_*))$, respectively. Let $S(Ker\ f_*)^\perp \neq \{0\}$. Now, we denote by L and S the projections of $tr(Ker\ f_*)$ on $ltr(Ker\ f_*)$ and $S(Ker\ f_*)^\perp$, respectively. Then, from (2.7) and (2.8), we have

$$\nabla_U V = \hat{\nabla}_U V + T^l_U V + T^s_U V, \tag{2.7}$$

$$\nabla_U N = T_U N + \nabla^{\perp l}_U N + D^{\perp s}(U, N), \tag{2.8}$$

$$\nabla_U W = T_U W + D^{\perp l}(U, W) + \nabla^{\perp s}_U W, \tag{2.9}$$

$\forall U, V \in \Gamma(Ker\ f_*), N \in \Gamma(ltr(Ker\ f_*))$ and $W \in \Gamma(S(Ker\ f_*)^\perp)$. If f is either r-lightlike or co-isotropic submersion, then we write

$$\hat{\nabla}_U \xi = T^*_U \xi + \nabla^{*\perp}_U \xi, \tag{2.10}$$

$\forall\ U \in \Gamma(Ker\ f_*)$, $\xi \in \Gamma\Delta$. Here $T^*_U \xi \in \Gamma(S(Ker\ f_*))$ and $\nabla^{*\perp}_U \xi \in \Gamma\Delta$.

Let (M, J) be a 2m-dimensional almost complex manifold, where J is an almost complex structure and g is a semi-Riemannian metric with index $0 < r \leq 2m$. Then M is called an indefinite almost Hermitian manifold, if

$$g(JX, JY) = g(X, Y), \quad \forall X, Y \in \Gamma(TM). \tag{2.11}$$

Also, if J is a complex structure on M, then M is said to be an indefinite Hermitian manifold. Now, let (M, J, g) is an indefinite almost Hermitian manifold with Levi-Civita connection ∇. Then, M is called an indefinite K*ä*hler manifold if

$$(\nabla_X J)Y = 0, \quad \forall X, Y \in \Gamma(TM). \tag{2.12}$$

3 Main Results

In this section, we introduce the notion of screen bi-slant lightlike submersions from an indefinite Kähler manifold onto a lightlike manifold. First we prove the following lemma:

Lemma 3.1. *Let $f : (M, g) \to (B, g')$ be a $2r$-lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B and $Ker\ f_*$ is a lightlike distribution of M. Then screen distribution $S(Ker\ f_*)$ and screen transversal distribution $S(Ker\ f_*)^\perp$ are Riemannian.*

Proof. Let M be a real $(m+n)$-dimensional indefinite Kähler manifold and $Ker\ f_*$ be a lightlike distribution of dimension m. Then there exists a local quasi orthonormal field of frames on M along $Ker\ f_*$

$$\{\xi_i, N_i, U_\alpha, Z_a\}, i \in \{1, ..., 2r\}, \alpha \in \{2r+1, ..., m\}, a \in \{2r+1, ..., n\},$$

where $\{\xi_i\}$, $\{N_i\}$ are lightlike basis of Δ, $ltr(Ker\ f_*)$ and U_α, Z_a are orthonormal basis of $S(Ker\ f_*)$, $S(Ker\ f_*)^\perp$, respectively. With the help of null basis $\{\xi_1, ..., \xi_{2r}, N_1, ..., N_{2r}\}$ of $\Delta \oplus ltr(Ker\ _*)$, we construct following orthonormal basis $\{X_1, ..., X_{4r}\}$

$$\begin{aligned} X_1 &= \frac{1}{\sqrt{2}}(\xi_1 + N_1), & X_2 &= \frac{1}{\sqrt{2}}(\xi_1 - N_1), \\ X_3 &= \frac{1}{\sqrt{2}}(\xi_2 + N_2), & X_4 &= \frac{1}{\sqrt{2}}(\xi_2 - N_2), \\ &... & &... \\ &... & &... \\ X_{4r-1} &= \frac{1}{\sqrt{2}}(\xi_{2r} + N_{2r}), & X_{4r} &= \frac{1}{\sqrt{2}}(\xi_{2r} - N_{2r}). \end{aligned}$$

Thus, Span $\{\xi_i, N_i\}$ is a non-degenerate space of index $2r$, which enables us to conclude that $\Delta \oplus ltr(Ker\ _*)$ is non-degenerate with constant index $2r$ on M. Moreover,

$$ind(TM) = ind(\Delta \oplus ltr(Ker\ f_*)) + ind(S(Ker\ f_*) \perp (S(Ker\ f_*))^\perp),$$

implies that $S(Ker\ f_*) \perp S(Ker\ f_*)^\perp$ has a constant index zero. Hence, $S(Ker\ f_*)$ and $S(Ker\ f_*)^\perp$ are Riemannian distributions. □

With the help of above lemma we define:

Definition 3.1. *Let $f : (M, g, J) \to (B, g')$ be a $2r$-lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B, such that $2r < dim(Ker\ f_*)$. Then f is called a screen bi-slant lightlike submersion if the radical distribution Δ is invariant with respect to J and screen distribution $S(Ker\ f_*)$ admits two non-degenerate orthogonal complementary slant distributions D^{θ_1} and D^{θ_2} with slant angles θ_1 and θ_2 respectively, such that*

$$S(Ker\ f_*) = D^{\theta_1} \oplus D^{\theta_2}.$$

Let n_1 and n_2 be the dimensions of the distributions D^{θ_1} and D^{θ_2} respectively. Then we have the following cases:

(i) if $n_1 = 0$ and $\theta_2 = 0$, then f is called a complex (invariant) lightlike submersion,

(ii) if $n_1 = 0$ and $\theta_2 = \frac{\pi}{2}$, then f is called a screen real (anti-invariant) lightlike submersion,

(iii) if $n_1 \neq n_2 \neq 0$, $\theta_1 = 0$ and $\theta_2 = \frac{\pi}{2}$, then f is called a SCR-lightlike submersion,

(iv) if $n_1 = 0$ and $0 < \theta_2 < \frac{\pi}{2}$, then f is called a screen-slant lightlike submersion,

(v) if $n_1 \neq n_2 \neq 0$, $\theta_1 = 0$ and $0 < \theta_2 < \frac{\pi}{2}$, then f is called a screen semi-slant lightlike submersion,

(vi) if $n_1 \neq n_2 \neq 0$, $\theta_1 = \frac{\pi}{2}$ and $0 < \theta_2 < \frac{\pi}{2}$, then f is called a screen pseudo-slant lightlike submersion.

A screen bi-slant lightlike submersion is said to be proper if slant angles θ_1 and θ_2 are different from zero and $\frac{\pi}{2}$.

Now, we construct some examples of screen bi-slant lightlike submersions. Denote by $\mathbb{R}^n_{r,q,p}$ the space $\mathbb{R}^n$ equipped with the semi-Riemannian metric g given by $g(e_i, e_j)_{r,q,p} = (G_{r,q,p})_{ij}$, $i \in \{1, ..., n\}$. Here e_i is the standard basis of $\mathbb{R}^n$ and $G_{r,q,p}$ is the diagonal matrix determined by g, that is,

$$G_{ij} = diagonal(\underbrace{0, ..., 0}_{\text{r-times}}, \underbrace{-1, ..., -1}_{\text{q-times}}, \underbrace{1, ..., 1}_{\text{p-times}}).$$

Example 3.1. *Let $\mathbb{R}^{12}_{0,2,10}$ and $\mathbb{R}^6_{2,0,4}$ endowed with the semi-Riemannian metric*

$$g = -(dx_1)^2 - (dx_2)^2 + (dx_3)^2 + (dx_4)^2 + (dx_5)^2 + (dx_6)^2 + (dx_7)^2 + (dx_8)^2 + (dx_9)^2 + (dx_{10})^2 + (dx_{11})^2 + (dx_{12})^2,$$

and degenerate metric $g' = (dy_3)^2 + (dy_4)^2 + (dy_5)^2 + (dy_6)^2$, where $x_1, \ldots, x_{12}$ and $y_1, \ldots, y_6$ are the canonical coordinates on $\mathbb{R}^{12}$ and $\mathbb{R}^6$, respectively. Define the mapping $f : (\mathbb{R}^{12}, g) \to (\mathbb{R}^6, g')$ as $(x_1, ..., x_{12}) \longmapsto \Big(x_1 - x_3,\ x_2 - x_4,\ \frac{x_5 - x_8}{\sqrt{2}},\ x_6,\ \frac{\sqrt{3}x_9 - x_{12}}{2},\ x_{10}\Big)$. Then f is a 2-lightlike submersion with

$$\Delta = Kerf_* \cap (Kerf_*)^{\perp} = Span\Big\{\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_3}, \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_4}\Big\},$$

which is invariant with respect to J. By easy computation we observe that

$$S(Ker\ f_*) = \mathcal{D}^{\theta_1} \oplus \mathcal{D}^{\theta_2},$$

where $\mathcal{D}^{\theta_1} = Span\Big\{\frac{1}{\sqrt{2}}\Big(\frac{\partial}{\partial x_5} + \frac{\partial}{\partial x_8}\Big), \frac{\partial}{\partial x_7}\Big\}$ and $\mathcal{D}^{\theta_2} = Span\Big\{\frac{1}{2}\Big(\frac{\partial}{\partial x_9} + \sqrt{3}\frac{\partial}{\partial x_{12}}\Big), \frac{\partial}{\partial x_{11}}\Big\}$ are slant distributions with slant angles $\theta_1 = \frac{\pi}{4}$ and $\theta_2 = \frac{\pi}{6}$ respectively. Thus, f is a proper screen bi-slant lightlike submersion.

Example 3.2. *Let $\mathbb{R}^8_{0,2,6}$ and $\mathbb{R}^4_{2,0,2}$ be endowed with the semi-Riemannian metric*

$$g = -(dx_1)^2 - (dx_2)^2 + (dx_3)^2 + (dx_4)^2 + (dx_5)^2 + (dx_6)^2 + (dx_7)^2 + (dx_8)^2,$$

and degenerate metric $g' = (dy_3)^2 + (dy_4)^2$, where $x_1, \ldots, x_8$ and $y_1, \ldots, y_4$ are the canonical coordinates on $\mathbb{R}^8$ and $\mathbb{R}^4$, respectively. Define the map $f : (\mathbb{R}^8, g) \to (\mathbb{R}^4, g')$ as $(x_1, ..., x_8) \longmapsto \Big(\frac{x_1 - x_5}{2}, \frac{x_2 - x_6}{2}, \frac{x_3 + x_8}{\sqrt{2}}, \frac{x_4 - x_7}{\sqrt{2}}\Big)$. Then, the radical distribution $\Delta = Span\Big\{\frac{1}{2}\Big(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_5}\Big), \frac{1}{2}\Big(\frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_6}\Big)\Big\}$ is invariant with respect to J. Moreover $\mathcal{D}^{\theta_1}$ is trivial and $\mathcal{D}^{\theta_2} = Span\Big\{\frac{1}{\sqrt{2}}\Big(\frac{\partial}{\partial x_3} - \frac{\partial}{x_8}\Big), \frac{1}{\sqrt{2}}\Big(\frac{\partial}{\partial x_4} + \frac{\partial}{\partial x_7}\Big)\Big\}$ is invariant. Thus, f is a complex lightlike submersion.

Example 3.3. *Let $\mathbb{R}^8_{0,2,6}$ and $\mathbb{R}^4_{2,0,2}$ be endowed with the semi-Riemannian metric*

$$g = (dx_1)^2 + (dx_2)^2 - (dx_3)^2 - (dx_4)^2 + (dx_5)^2 + (dx_6)^2 + (dx_7)^2 + (dx_8)^2,$$

and degenerate metric $g' = (dy_1)^2 + (dy_4)^2$, where $x_1, \ldots, x_8$ and $y_1, \ldots, y_4$ are the canonical coordinates on $\mathbb{R}^8$ and $\mathbb{R}^4$, respectively. Define the map $f : (\mathbb{R}^8, g) \to (\mathbb{R}^4, g')$ as $(x_1, ..., x_8) \longmapsto \Big(x_1,\ x_3 + x_5,\ x_4 + x_6,\ x_8\Big)$. Then f is a screen real lightlike submersion as $\Delta = Span\Big\{\frac{\partial}{\partial x_3} - \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_4} - \frac{\partial}{\partial x_6}\Big\}$ is invariant, $\mathcal{D}^{\theta_2} = Span\Big\{\frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_7}\Big\}$ is anti-invariant and $\mathcal{D}^{\theta_1}$ is trivial.

Example 3.4. *Let $\mathbb{R}^{12}_{0,4,12}$ and $\mathbb{R}^6_{2,0,4}$ be endowed with the semi-Riemannian metric*

$$\begin{aligned} g = &-(dx_1)^2 - (dx_2)^2 - (dx_3)^2 - (dx_4)^2 + (dx_5)^2 + (dx_6)^2 \\ &+ (dx_7)^2 + (dx_8)^2 + (dx_9)^2 + (dx_{10})^2 + (dx_{11})^2 \\ &+ (dx_{12})^2 + (dx_{13})^2 + (dx_{14})^2 + (dx_{15})^2 + (dx_{16})^2, \end{aligned}$$

and degenerate metric $g' = (dy_3)^2 + (dy_4)^2 + (dy_5)^2 + (dy_6)^2$. Here $x_1, \ldots, x_{16}$ and $y_1, \ldots, y_6$ are the canonical coordinates on $\mathbb{R}^{16}$ and $\mathbb{R}^6$, respectively. Let us define the map $f : (\mathbb{R}^{16}, g) \to (\mathbb{R}^6, g')$ as

$$(x_1, ..., x_{16}) \longmapsto \Big(x_1 + x_3 + x_5 + x_7,\ x_2 + x_4 + x_6 + x_8,\ x_9,\ \frac{x_{11} - x_{13}}{\sqrt{2}},\ \frac{x_{12} - x_{14}}{\sqrt{2}},\ x_{16}\Big).$$

Then, f is a 2-lightlike submersion with

$$\Delta = Span\Big\{\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_5} - \frac{\partial}{\partial x_7}, \frac{\partial}{\partial x_2} - \frac{\partial}{\partial x_4} + \frac{\partial}{\partial x_6} - \frac{\partial}{\partial x_8}\Big\},$$

which is invariant. Also

$$\mathcal{D}^{\theta_1} = Span\Big\{\frac{1}{\sqrt{2}}\Big(\frac{\partial}{\partial x_{11}} + \frac{\partial}{\partial x_{13}}\Big), \frac{1}{\sqrt{2}}\Big(\frac{\partial}{\partial x_{12}} + \frac{\partial}{\partial x_{14}}\Big)\Big\} \text{ and } \mathcal{D}^{\theta_2} = Span\Big\{\frac{\partial}{\partial x_{10}}, \frac{\partial}{\partial x_{15}}\Big\}.$$

Since $\mathcal{D}^{\theta_1}$ is invariant and $\mathcal{D}^{\theta_2}$ is anti-invariant, so f is a SCR-lightlike submersion.

Example 3.5. *Let $\mathbb{R}^{12}_{0,2,10}$ and $\mathbb{R}^6_{2,0,4}$ be endowed with the semi-Riemannian metric*

$$\begin{aligned} g = &-(dx_1)^2 - (dx_2)^2 + (dx_3)^2 + (dx_4)^2 + (dx_5)^2 + (dx_6)^2 + (dx_7)^2 \\ &+ (dx_8)^2 + (dx_9)^2 + (dx_{10})^2 + (dx_{11})^2 + (dx_{12})^2 \end{aligned}$$

and degenerate metric $g' = (dy_3)^2 + (dy_4)^2 + (dy_5)^2 + (dy_6)^2$, where $x_1, \ldots, x_{12}$ and $y_1, \ldots, y_6$ are the canonical coordinates on $\mathbb{R}^{12}$ and $\mathbb{R}^6$, respectively. Define the map $f : (\mathbb{R}^{16}, g) \to (\mathbb{R}^8, g')$ as

$$(x_1, ..., x_{12}) \longmapsto \Big(\frac{x_1 - x_{11}}{\sqrt{3}},\ \frac{x_2 - x_{12}}{\sqrt{3}},\ \frac{-\sqrt{3}x_3 + x_6}{2},\ -x_4,\ x_7,\ x_8\Big).$$

Then, the radical distribution $\Delta = Span\Big\{\frac{1}{\sqrt{3}}\Big(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_{11}}\Big), \frac{1}{\sqrt{3}}\Big(\frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_{12}}\Big)\Big\}$ is clearly seen to be invariant. Also, as $\mathcal{D}^{\theta_1} = Span\Big\{\frac{\partial}{\partial x_9}, \frac{\partial}{\partial x_{10}}\Big\}$ is invariant and $\mathcal{D}^{\theta_2} = Span\Big\{\frac{1}{2}\Big(\frac{\partial}{\partial x_3} + \sqrt{3}\frac{\partial}{\partial x_6}\Big), \frac{\partial}{\partial x_5}\Big\}$ is slant with slant angle $\frac{\pi}{3}$, f is proper screen semi-slant lightlike submersion.

Example 3.6. *Let $\mathbb{R}^{12}_{0,2,10}$ and $\mathbb{R}^{6}_{4,0,4}$ be endowed with the semi-Riemannian metric*

$$g = -(dx_1)^2 - (dx_2)^2 + (dx_3)^2 + (dx_4)^2 + (dx_5)^2 + (dx_6)^2 + (dx_7)^2 + (dx_8)^2 + (dx_9)^2 + (dx_{10})^2 + (dx_{11})^2 + (dx_{12})^2$$

and degenerate metric $g' = (dy_5)^2 + (dy_6)^2 + (dy_7)^2 + (dy_8)^2$, where $x_1, \ldots, x_{12}$ and $y_1, \ldots, y_6$ are the canonical coordinates on $\mathbb{R}^{12}$ and $\mathbb{R}^6$, respectively. Define the map $f : (\mathbb{R}^{12}, g) \to (\mathbb{R}^6, g')$ as $(x_1, ..., x_{12}) \longmapsto \Big(x_1 - x_5,\ x_2 - x_6,\ x_3,\ \frac{x_7 + x_{10}}{\sqrt{2}},\ x_9,\ x_{12}\Big)$. Then $\Delta = Span\{\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_5}, \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_6}\}$, which is invariant. Now, since $\mathcal{D}^{\theta_1} = Span\{\frac{\partial}{\partial x_4}, \frac{\partial}{\partial x_{11}}\}$ is anti-invariant and $\mathcal{D}^{\theta_2} = Span\{\frac{1}{\sqrt{2}}\Big(-\frac{\partial}{\partial x_7} + \frac{\partial}{\partial x_{10}}\Big), \frac{\partial}{\partial x_8}\}$ is slant with slant angle $\frac{\pi}{4}$. Hence f is a proper screen pseudo-slant lightlike submersion.

Now, for any $U \in \Gamma(Ker\ f_*)$, we write

$$JU = \phi U + FU. \tag{3.1}$$

Here $\phi U \in \Gamma(ker\ f_*)$ and $FU \in \Gamma(ker\ f_*)^\perp$. Next, we assume that ϕ_1, ϕ_2 and ϕ_3 denotes the projections of $Ker\ f_*$ on Δ, D^{θ_1} and D^{θ_2}, respectively. Also, let Q_1, Q_2 and Q_3 denote the projections of $tr(Ker\ f_*)$ on $ltr(Ker\ f_*)$, D' and D'', respectively. Here D' and D'' are non-null orthogonal complementary distributions of JD^{θ_1} and JD^{θ_2}, respectively in $S(Ker\ f_*)^\perp$. So, for any vector field U tangent to $Ker\ f_*$, we put

$$U = \phi_1 U + \phi_2 U + \phi_3 U, \tag{3.2}$$

which gives

$$JU = J\phi_1 U + J\phi_2 U + J\phi_3 U.$$

Then

$$JU = J\phi_1 U + \psi_1\phi_2 U + \psi_2\phi_2 U + \eta_1\phi_3 U + \eta_2\phi_3 U, \tag{3.3}$$

where $\psi_1\phi_2 U$ (resp. $\psi_2\phi_2 U$) and $\eta_1\phi_3 U$ (resp. $\eta_2\phi_3 U$) denotes the tangential (resp. transversal) components of $J\phi_2 U$ and $J\phi_3 U$, respectively. Thus, $J\phi_1 U \in \Gamma(\Delta)$, $\psi_1\phi_2 U \in \Gamma(D^{\theta_1})$, $\psi_2\phi_2 U \in \Gamma(D')$, $\eta_1\phi_3 U \in \Gamma(D^{\theta_2})$ and $\eta_2\phi_3 U \in \Gamma(D'')$. Further, for any vector field $W \in \Gamma(Ker\ f_*)^\perp$, we put

$$W = Q_1 W + Q_2 W + Q_3 W \tag{3.4}$$

It follows that

$$JW = JQ_1 W + JQ_2 W + JQ_3 W.$$

Then, we have

$$JW = JQ_1 W + B_1 Q_2 W + B_2 Q_2 W + C_1 Q_3 W + C_2 Q_3 W, \tag{3.5}$$

where $B_1 Q_2 W$ (resp. $B_2 Q_2 W$) and $C_1 Q_3 W$ (rep. $C_2 Q_3 W$) denotes the tangential (resp. transversal) components of $JQ_2 W$ and $JQ_3 W$ respectively, where $JQ_1 W \in \Gamma(ltr(Ker\ f_*))$, $B_1 Q_2 W \in \Gamma(D^{\theta_1})$, $B_2 Q_2 W \in \Gamma(D')$, $C_1 Q_3 W \in \Gamma(D^{\theta_2})$ and $C_2 Q_3 W \in \Gamma(D'')$. Now, in view of (2.7)-(2.9), (2.10), (2.12) and (3.2)-(3.5) and identifying the components of Δ, D^{θ_1}, D^{θ_2}, $ltr(Ker\ f_*)$, D' and D'', we have

$$\nabla^{*\perp}_U J\phi_1 V + \phi_1(\hat{\nabla}_U \psi_1\phi_2 V) + \phi_1(T_U \psi_2\phi_2 V) + \phi_1(\hat{\nabla}_U \eta_1\phi_3 V) + \phi_1(T_U \eta_2\phi_3 V) = J\phi_1(\hat{\nabla}_U V), \tag{3.6}$$

(3.7) $$\phi_2(T^*_U J\phi_1 V) + \phi_2(\hat{\nabla}_U \psi_1\phi_2 V) + \phi_2(T_U\psi_2\phi_2 V) + \phi_2(\hat{\nabla}_U \eta_1\phi_3 V) + \phi_2(T_U\eta_2\phi_3 V) = B_1 Q_2 T^s_U V + \psi_1\phi_2\hat{\nabla}_U V,$$

(3.8) $$\phi_3(T^*_U J\phi_1 V) + \phi_3(\hat{\nabla}_U \psi_1\phi_2 V) + \phi_3(T_U\psi_2\phi_2 V) + \phi_3(\hat{\nabla}_U \eta_1\phi_3 V) + \phi_3(T_U\eta_2\phi_3 V) = C_1 Q_3 T^s_U V + \eta_1\phi_3\hat{\nabla}_U V,$$

(3.9) $$T^l_U J\phi_1 V + T^l_U \psi_1\phi_2 V + D^{\perp l}(U, \psi_2\phi_2 V) + T^l_U \eta_1\phi_3 V + D^{\perp l}(U, \eta_2\phi_3 V) = JT^l_U V$$

(3.10) $$Q_2(T^s_U J\phi_1 V) + Q_2(T^s_U \psi_1\phi_2 V) + Q_2(\nabla^{\perp s}_U \psi_2\phi_2 V) + Q_2(T^s_U \eta_1\phi_3 V) + Q_2(\nabla^{\perp s}_U \eta_2\phi_3 V) = \psi_2\phi_2(\hat{\nabla}_U V)$$

(3.11) $$Q_3(T^s_U J\phi_1 V) + Q_3(T^s_U \psi_1\phi_2 V) + Q_3(\nabla^{\perp s}_U \psi_2\phi_2 V) + Q_3(T^s_U \eta_1\phi_3 V) + Q_3(\nabla^{\perp s}_U \eta_2\phi_3 V) = \eta_2\phi_3(\hat{\nabla}_U V) + C_2 Q_3 T^s_U V$$

Now, we give a characterization theorem for screen bi-slant lightlike submersions:

Theorem 3.1. *Let f be a 2r-lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B. Then, f is a screen bi-slant lightlike submersion if and only if*

(i) $ltr(Ker\ f_*)$ is invariant with respect to J,

(ii) there exists a constant $\lambda_1 \in [0,1]$ such that $\phi^2 U = -\lambda_1 U,\ \forall U \in \Gamma(D^{\theta_1}),\ \lambda_1 = cos^2\theta_1,\ \theta_1$ is a slant angle of D^{θ_1},

(iii) there exists a constant $\lambda_2 \in (0,1]$ such that $\phi^2 V = -\lambda_2 V,\ \forall V \in \Gamma(D^{\theta_2}),\ \lambda_2 = cos^2\theta_2,\ \theta_2$ is a slant angle of D^{θ_2},

where D^{θ_1} and D^{θ_2} are non-degenerate orthogonal distributions, such that $S(Ker\ f_*) = D^{\theta_1} \oplus D^{\theta_2}$.

Proof. Let f be a screen bi-slant lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B. Then, in view of (2.11), (3.3) and (3.5), we get

$$g(JN, U) = -g(N, JU) = -g(N, J\phi_1 U + J\phi_2 U + \psi\phi_3 U + F\phi_3 U) = 0,$$
$$g(JN, W) = -g(N, JW) = -g(N, JQ_1 W + JQ_2 W + BQ_3 W + CQ_3 W) = 0,$$

for any $U \in \Gamma(S(Ker\ f_*)),\ N \in \Gamma(ltr(Ker\ f_*)), W \in \Gamma(S(Ker\ f*)^\perp)$, which implies that JN does not belongs to $\Gamma(S(Ker\ f_*))$ and $\Gamma(S(Ker\ f_*)^\perp)$. If $JN \in \Gamma(\Delta)$, then $J^2 N = -N \in \Gamma(ltr(Ker\ f_*))$, which contradicts the fact that Δ is invariant with respect to J. Thus we conclude that $ltr(Ker\ f_*)$ is invariant with respect to J. Now, let $U \in \Gamma(D_2)$, then we have

$$cos(\theta)(U) = \frac{g(JU, \phi U)}{|J(U)||\phi U|} = -\frac{g(U, \phi^2 U)}{|JU||\phi U|}.$$

Also, we have

$$cos(\theta)(U) = \frac{|\phi U|}{|JU|}.$$

Thus, we obtain

$$cos^2\theta(U) = -\frac{\hat{g}(U, \phi^2 U)}{|U|^2}.$$

Since $\theta(U)$ is constant, we have $\phi^2 U = -\lambda U, \lambda \in [0,1)$, where $\lambda = cos^2\theta$.

Now, applying J to (3.1) and comparing the tangential parts, we get $-U = \phi^2 U + BFU,\ \forall U \in \Gamma(D_2)$. It gives $BFU = -\mu U$, where $1 - \lambda = \mu \in [0,1)$. The reverse implication can be proved in a similar way. □

Theorem 3.2. *Let $f : M \to B$ be a screen bi-slant lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B. Then, the radical distribution Δ is integrable if and only if*

(i) $Q_2(T^s_U J\phi_1 V) = Q_2(T^s_V J\phi_1 U)$,

(ii) $Q_3(T^s_U J\phi_1 V) = Q_3(T^s_V J\phi_1 U)$,

(iii) $\phi_3(T^*_U J\phi_1 V) = \phi_3(T^*_V J\phi_1 U)$,

(iv) $\phi_2(T^*_U J\phi_1 V) = \phi_2(T^*_V J\phi_1 U)$,

$\forall\ U, V \in \Gamma(\Delta)$.

Proof. Let $U, V \in \Gamma(\Delta)$. In view of (3.10), we get $Q_2(T^s_U J\phi_1 V) = \psi_2\phi_2\hat{\nabla}_U V$. It follows that $Q_2(T^s_U J\phi_1 V) - Q_2(T^s_V J\phi_1 U) = \psi_2\phi_2[U, V]$. From (3.11), we have $Q_3(T^s_U J\phi_1 V) = \eta_2\phi_3\hat{\nabla}_U V + C_2 Q_3(T^s_U V)$, which gives $Q_3(T^s_U J\phi_1 V) - Q_3(T^s_V J\phi_1 U) = \eta_2\phi_3[U, V]$. Now, using (3.8), we get $\phi_3(T^*_U J\phi_1 V) = \eta_1\phi_3(\hat{\nabla}_U V) + C_1 Q_3 T^s_U V$, which implies $\phi_3(T^*_U J\phi_1 V) - \phi_3(T^*_V J\phi_1 U) = \eta_1\phi_3[U, V]$. Finally, using (3.8), we get $\phi_2(T^*_U J\phi_1 V) = \psi_1\phi_2(\hat{\nabla}_U V) + B_1 Q_2 T^s_U V$. It gives $\phi_2(T^*_U J\phi_1 V) - \phi_2(T^*_V J\phi_1 U) = \psi_1\phi_2[U, V]$, which completes the proof. □

Theorem 3.3. *Let $f : M \to B$ be a screen bi-slant lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B. Then, the distribution D^{θ_1} is integrable if and only if*

(i) $\phi_1(\hat{\nabla}_U \psi_1\phi_2 V) = \phi_1(\hat{\nabla}_V \psi_1\phi_2 U)$,

(ii) $Q_2(T^s_U \psi_1\phi_2 V) = Q_2(T^s_V \psi_1\phi_2 U)$,

(iii) $\phi_3(\hat{\nabla}_U \psi_1\phi_2 V) = \phi_3(\hat{\nabla}_V \psi_1\phi_2 U)$,

(iv) $Q_3(T^s_U \psi_1\phi_2 V) = Q_3(T^s_V \psi_1\phi_2 U)$,

$\forall\ U, V \in \Gamma(D^{\theta_1})$.

Proof. Let $U, V \in \Gamma(D^{\theta_1})$. Then from (3.6), we get $\phi_1(\hat{\nabla}_U \psi_1\phi_2 V) = J\phi_1\hat{\nabla}_U V$. It gives $\phi_1(\hat{\nabla}_U \psi_1\phi_2 V) - \phi_1(\hat{\nabla}_V \psi_1\phi_2 U) = J\phi_1[U, V]$. Using (3.10), we have $Q_2(T^s_U \psi_1\phi_2 V) = \psi_2\phi_2\hat{\nabla}_U V$, which gives $Q_2(T^s_U \psi_1\phi_2 V) - Q_2(T^s_V \psi_1\phi_2 U) = \psi_2\phi_2[U, V]$. Now, in view of (3.8), we get $\phi_3(T^s_U \psi_1\phi_2 V) = \eta_1\phi_3(\hat{\nabla}_U V) + C_1 Q_3 T^s_U V$, which implies $\phi_3(T^s_U \psi_1\phi_2 V) - \phi_3(T^s_V \psi_1\phi_2 U) = \eta_1\phi_3[U, V]$. Finally, from (3.11), we have $Q_3(T^s_U \psi_1\phi_2 V) = \eta_2\phi_3(\hat{\nabla}_U V) + C_2 Q_3 T^s_U V$. It gives $Q_3(T^s_U \psi_1\phi_2 V) - Q_3(T^s_V \psi_1\phi_2 U) = \eta_2\phi_3[U, V]$, which completes the proof. □

Theorem 3.4. *Let $f : M \to B$ be a screen bi-slant lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B. Then, the distribution D^{θ_2} is integrable if and only if $\forall\ U, V \in \Gamma(D^{\theta_2})$, we have*

(i) $\phi_1(\hat{\nabla}_U \eta_1\phi_3 V) = \phi_1(\hat{\nabla}_V \eta_1\phi_3 U)$,

(ii) $\phi_2(\hat{\nabla}_U \eta_1\phi_3 V) = \phi_2(\hat{\nabla}_V \eta_1\phi_3 U)$,

(iii) $Q_2(T^s_U \eta_1\phi_3 V) = Q_2(T^s_V \eta_1\phi_3 U)$,

(iv) $Q_3(T^s_U \eta_1\phi_3 V) = Q_3(T^s_V \eta_1\phi_3 U)$.

Proof. Let $U, V \in \Gamma(D^{\theta_2})$. Using (3.6), we obtain $\phi_1(\hat{\nabla}_U \eta_1\phi_3 V) = J\phi_1\hat{\nabla}_U V$, which gives $\phi_1(\hat{\nabla}_U \eta_1\phi_3 V) - \phi_1(\hat{\nabla}_V \eta_1\phi_3 U) = J\phi_1[U, V]$. In view of (3.7), we have $\phi_2(\hat{\nabla}_U \eta_1\phi_3 V) = B_1 Q_2 T^s_U V + \psi_1\phi_2\hat{\nabla}_U V$. From which, it follows that $\phi_2(\hat{\nabla}_U \eta_1\phi_3 V) - \phi_2(\hat{\nabla}_V \eta_1\phi_3 U) = \psi_1\phi_2[U, V]$. Now, from (3.10), we get $Q_2(T^s_U \eta_1\phi_3 V) = \psi_2\phi_2(\hat{\nabla}_U V)$, which implies $Q_2(T^s_U \eta_1\phi_3 V) - Q_2(T^s_V \eta_1\phi_3 U) = \psi_2\phi_2[U, V]$. Finally, using (3.11), we have $Q_3(T^s_U \eta_1\phi_3 V) = \eta_2\phi_3(\hat{\nabla}_U V) + C_2 Q_3 T^s_U V$. It gives $Q_3(T^s_U \eta_1\phi_3 V) - Q_3(T^s_V \eta_1\phi_3 U) = \eta_2\phi_3[U, V]$. Thus the proof is completed. □

Theorem 3.5. *Let $f : M \to B$ be a screen bi-slant lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B. Then, the distribution D' is integrable if and only if $\forall\ U, V \in \Gamma(D')$, we have*

(i) $\phi_1(T_U\psi_2\phi_2 V) = \phi_1(T_V\psi_2\phi_2 U)$,

(ii) $\phi_2(T_U\psi_2\phi_2 V) = \phi_2(T_V\psi_2\phi_2 U)$,

(iii) $\phi_3(T_U\psi_2\phi_2 V) = \phi_3(T_V\psi_2\phi_2 U)$,

(iv) $Q_3(T_U^{\perp s}\psi_2\phi_2 V) = Q_3(T_V^{\perp s}\psi_1\phi_2 U)$.

Proof. Let $U, V \in \Gamma(D')$. From (3.6), we get $\phi_1(T_U\psi_2\phi_2 V) = J\phi_1\hat{\nabla}_U V$. It follows that $\phi_1(T_U\psi_2\phi_2 V) - \phi_1(T_V\psi_2\phi_2 U) = J\phi_1[U, V]$. Using (3.7), we obtain $\phi_2(T_U\psi_2\phi_2 V) = B_1 Q_2 T_U^s V + \psi_1\phi_2\hat{\nabla}_U V$, which gives $\phi_2(T_U\psi_2\phi_2 V) - \phi_2(T_V\psi_2\phi_2 U) = \psi_1\phi_2[U, V]$. In view of (3.8), we have $\phi_3(T_U\psi_2\phi_2 V) = C_1 Q_3 T_U^s V + \eta_1\phi_3(\hat{\nabla}_U V)$, which implies $\phi_3(T_U\psi_2\phi_2 V) - \phi_3(T_V\psi_2\phi_2 U) = \eta_1\phi_3[U, V]$. Moreover, from (3.11), we have $Q_3(T_U^{\perp s}\psi_2\phi_2 V) = \eta_2\phi_3(\hat{\nabla}_U V) + C_2 Q_3 T_U^s V$.
It gives $Q_3(T_U^{\perp s}\psi_2\phi_2 V) - Q_3(T_V^{\perp s}\psi_2\phi_2 U) = \eta_2\phi_3[U, V]$. Thus the proof of the theorem is completed. □

Theorem 3.6. *Let $f : M \to B$ be a screen bi-slant lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B. Then, the distribution D'' is integrable if and only if $\forall\ U, V \in \Gamma(D'')$, we have*

(i) $\phi_1(T_U\eta_2\phi_3 V) = \phi_1(T_V\eta_2\phi_3 U)$,

(ii) $\phi_2(T_U\eta_2\phi_3 V) = \phi_2(T_V\eta_2\phi_3 U)$,

(iii) $\phi_3(T_U\eta_2\phi_3 V) = \phi_3(T_V\eta_2\phi_3 U)$,

(iv) $Q_2(T_U^{\perp s}\eta_2\phi_3 V) = Q_3(T_V^{\perp s}\eta_2\phi_3 U)$.

Proof. Let $U, V \in \Gamma(D'')$. In view of (3.6), we obtain $\phi_1(T_U\eta_2\phi_3 V) = J\phi_1\hat{\nabla}_U V$. It follows that $\phi_1(T_U\eta_2\phi_3 V) - \phi_1(T_V\eta_2\phi_3 U) = J\phi_1[U, V]$. Using (3.7), we have $\phi_2(T_U\eta_2\phi_3 V) = B_1 Q_2 T_U^s V + \psi_1\phi_2\hat{\nabla}_U V$, which implies $\phi_2(T_U\eta_2\phi_3 V) - \phi_2(T_V\eta_2\phi_3 U) = \psi_1\phi_2[U, V]$. Now, from (3.8), we get $\phi_3(T_U\eta_2\phi_3 V) = C_1 Q_3 T_U^s V + \eta_1\phi_3(\hat{\nabla}_U V)$, which implies $\phi_3(T_U\eta_2\phi_3 V) - \phi_3(T_V\eta_2\phi_3 U) = \eta_1\phi_3[U, V]$. Finally, using (3.10), we obtain $Q_2(T_U^{\perp s}\eta_2\phi_3 V) = \psi_2\phi_2(\hat{\nabla}_U V)$, which gives $Q_2(T_U^{\perp s}\eta_2\phi_3 V) - Q_2(T_V^{\perp s}\eta_2\phi_3 U) = \psi_2\phi_2[U, V]$. Thus, the proof follows. □

Theorem 3.7. *Let $f : M \to B$ be a screen bi-slant lightlike submersion from an indefinite Kähler manifold M onto a lightlike manifold B. Then, the induced connection $\hat{\nabla}$ on $S(Ker\ f_*)$ is a metric connection if and only if $\forall\ U \in \Gamma(S(Ker\ f_*))$ and $\xi \in \Gamma(\Delta)$, we have*

(i) $B_1 Q_2 T_U^s\xi = 0$,

(ii) $C_1 Q_3 T_U^s\xi = 0$,

(iii) $T_U^*\xi = 0$ *on* $\Gamma(Ker\ f_*)$.

Proof. In view of Theorem 2.4, page 161 [12], we see that the induced connection $\hat{\nabla}$ on $S(Ker\ f_*)$ is a metric connection if and only if Δ is a parallel distribution with respect to $\hat{\nabla}$. Using (2.7), (2.10) and (2.12), we derive

$$\begin{aligned}\nabla_U J\xi &= J\nabla_U\xi\\ &= J\hat{\nabla}_U\xi + JT_U^l\xi + JT_U^s\xi\\ &= JT_U^*\xi + J\nabla_U^{*\perp}\xi + JT_U^l\xi + B_1 Q_2 T_U^s\xi + B_2 Q_2 T_U^s\xi + C_1 Q_3 T_U^s\xi + C_2 Q_3 T_U^s\xi,\end{aligned}$$

for $U \in \Gamma(S(Ker\ f_*))$ and $\xi \in \Gamma(\Delta)$. Comparing the tangential components of above equation, we get $\hat{\nabla}_U J\xi = JT_U^*\xi + J\nabla_U^{*\perp}\xi + B_1 Q_2 T_U^s\xi + C_1 Q_3 T_U^s\xi$, which completes the proof. □

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