Mathematical Analysis of Rabies Transmission Dynamics in Nepal

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Abstract

In this paper, we have proposed an SEIV deterministic mathematical model to describe the rabies transmission dynamics within household and stray dogs in the context of Nepal. Some properties of the solution to the model including non-negativity, existence and stability of equilibrium points (rabies free and endemic both) are analyzed in this study.The expression for the value of basic reproduction ratio (R_0) is derived by using the next generation matrix approach. Sensitivity analysis of R_0 identifies that the transmission rate and the dog recruitment rate are the most crucial parameters to target the reduction of R_0. Numerical simulations of the model show that only vaccination is not sufficient to eradicate dog rabies. Significant results have been observed when both vaccination and sterilization of dogs are implememted for controlling the rabies among household and stray dogs.

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1 Introduction

Rabies is mostly caused by a bite from infected animals but occasionally by other form of contact. It is a natural disease of dogs, cats, bats, raccoons, foxes skunks, wolves and other warm blooded organisms [15]. After a human being is bitten by an infected animal, the virus enters the body, it either migrates directly to the brain via the peripheral nervous system to replicate there or stay in the muscles to replicate before migrating to the brain via muscular junctions. At this stage, the virus cannot easily be detected within the host, but vaccination may be used to prevent the virus from reaching the brain. Once the virus reaches the brain, it duplicates itself and it produces acute inflammations of the brain, leading to coma and eventually death. Once the symptoms of rabies have developed, its mortality rate is almost sure [20]. All species of mammals are susceptible to rabies virus infection, but dogs remain the main carrier of rabies and are responsible for most of the human rabies death world wise [6]

Various national, international and governmental organizations are involved in rabies control and elimination in all over the world but we shall take it in the context of Nepal. The National Zoonosis and Food Hygiene Research Center (NZFHRC) have contributed to rabies control by vaccinating dogs. The Alliance Group for Rabies Control in Nepal has vaccinated more than 10,000 household and stray dogs [2].

The main intervention strategy in the dog rabies control blueprint is mass dog vaccination, possibly complemented with dog population management measures. However, proper planning and evaluation are equally crucial components of the blueprint. In the planning phase, information should be gathered on the local rabies epidemiology and the extent of the dog population. Also, awareness should be created and support elicited from both the local population and the relevant governmental agencies. Once a programme is in place, the change in epidemiological, economic and social impact of the disease needs to be monitored to evaluate the effectiveness of the programme. Reliable baseline data and effective rabies surveillance are inevitable to accomplish this goal [10].

Mathematical models have long been used to predict and understand the transmission dynamics of the animal rabies [4]. Leung and Davis studied mathematically the rabies vaccination target for stray dogs. They presented a method to estimate vaccination target for stray dogs when the dog population is made up of stray, free-roaming and confined dogs [14]. Shigui Ruan has constructed a SEIR model for the spread of rabies virus among dogs and from dogs to humans and used rabies data in China from 1996 to 2010 for the estimations of parameter values [19]. Zhang et al. proposed a deterministic model to study the transmission dynamics of rabies in China and explore effective control and prevention measure [23]. Recently, Abdulmajid and Hassan formulated and analyzed a delay differential equations model for assessing the effects of controls and time delay as incubation period on the transmission dynamics of rabies in human and dog population[1].

2 Mathematical Model

In our proposed SEIV deterministic model, the total population $N(t)$ of dogs is divided into four mutually exclusive compartments : susceptible class S, exposed class E, infected class I and vaccinated class V. Thus, we have $N(t) = S(t) + E(t) + I(t) + V(t)$. It is assumed that only susceptible dogs may get vaccinated. For simplicity, per-capita birth and death rates are supposed to be constant throughout the study. The model is the modification of the model given by Eze *et al.* [12] by incorporating the vaccine inefficiency rate (σ) for the dog population.

The compartmental diagram of the proposed model is shown in Figure 1.The mathematical model describing the transmission dynamics of rabies within dogs is governed by the following set of differential equations:

$$\left.\begin{aligned} \frac{dS(t)}{dt} &= \Lambda - \beta S(t) I(t) - (\alpha + \mu) S(t) \\ \frac{dE(t)}{dt} &= \beta S(t) I(t) + \sigma\beta V(t) I(t) - (\gamma + \mu) E(t) \\ \frac{dI(t)}{dt} &= \gamma E(t) - (\delta + \mu) I(t) \\ \frac{dV(t)}{dt} &= \alpha S(t) - \sigma\beta V(t) - \mu V(t) \end{aligned}\right\} \tag{2.1}$$

with initial conditions: $S(0) = S_0 > 0, E(0) = E_0 \geq 0, I(0) = I_0 \geq 0, V(0) = V_0 \geq 0$

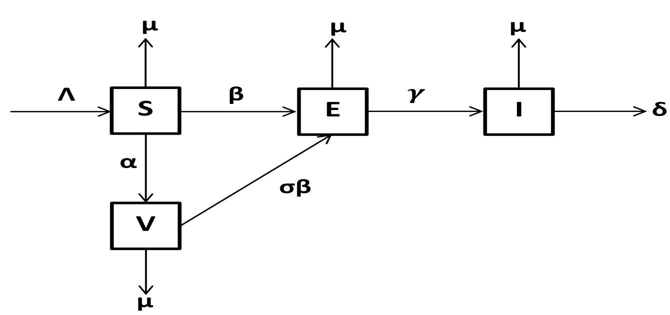

Fig. 1: Compartmental diagram of the proposed model

Tab. 5: Description of parameters and their values used in the model.

Parameter & Value	Description	Source
$\Lambda = 400,000$	Recruitment rate of dog	[13]
$\beta = 2.74 \times 10^{-5}$	Transmission rate from dog to dog	[16]
$\alpha = 0.03$	Dog vaccination rate	[16]
$\gamma = 2$	Dog moving rate from exposed to infected	[21]
$\delta = 36.5$	Dog mortality rate due to rabies	[14]
$\mu = 0.2$	Natural death rate of dog	[14]
$\sigma = 0.08$	Vaccine inefficiency rate	[9]

3 Basic Properties and Existence of Equilibria

To analyze the model, we discuss various properties including the non-negativity and boundedness of the solution set. We compute the equilibrium points and discuss the stability. Again, we find the expression for the basic reproduction ratio of the model (2.1).

3.1 Non-Negativity of the Model

Theorem 3.1. *If $S_0 > 0, E_0 \geq 0, I_0 \geq 0$ and $V_0 \geq 0$, then the solutions of the model* (2.1) *remain non-negative for all $t \geq 0$.*

Proof. Let $t_1 = sup\{t > 0 : S(t) > 0, E(t) \geq 0, I(t) \geq 0, V(t) \geq 0\}$.

From first equation of the system (2.1), we can write

$$\frac{dS(t)}{dt} + [\beta I(t) + (\alpha + \mu)] S(t) = \Lambda$$

Multiplying by integrating factor $exp\left\{(\alpha + \mu)t + \beta \int_0^t I(\tau) d\tau\right\}$, we get

$$\frac{d}{dt}\left[S(t)\, exp\left\{(\alpha + \mu)t + \beta \int_0^t I(\tau) d\tau\right\}\right] = \Lambda exp\left\{(\alpha + \mu)t + \beta \int_0^t I(\tau) d\tau\right\}$$

Integrating it from 0 to t_1, we have

$$S(t_1)\, exp\left\{(\alpha + \mu)t_1 + \beta \int_0^{t_1} I(\tau) d\tau\right\} = \int_0^{t_1} \Lambda exp\left\{(\alpha + \mu)y + \beta \int_0^{y} I(\tau) d\tau\right\} dy + S(0)$$

$$S(t_1) = \left[S(0) + \Lambda \int_0^{t_1} exp\left\{(\alpha + \mu)y + \beta \int_0^{y} I(\tau) d\tau\right\} dy\right] exp\left\{-(\alpha + \mu)t_1 - \beta \int_0^{t_1} I(\tau) d\tau\right\} > 0$$

which implies that $S(t) > 0$

From second equation of system (2.1) , we have

$$\frac{dE(t)}{dt} \geq -(\gamma+\mu)E(t)$$

which implies that $E(t_1) \geq E(0)\, exp\left[-(\gamma+\mu)t_1\right] > 0$

Similarly, from third equation of system (2.1) , we have

$$\frac{dI(t)}{dt} \geq -(\delta+\mu)I(t)$$

which implies that $I(t_1) \geq I(0)\, exp\left[-(\delta+\mu)t_1\right] > 0$

Similarly, from fourth equation of system (2.1) , we have

$$\frac{dV(t)}{dt} \geq -\left[\sigma\beta I(t)+\mu\right]V(t)$$

which implies that

$$V(t_1) \geq V(0)\, exp\left[-\left\{\mu t_1 + \sigma\beta\int_0^{t_1} I(\tau)\, d\tau\right\}\right] > 0$$

Thus, we have $S(t) > 0, E(t) > 0, I(t) > 0$ and $V(t) > 0 \quad \forall t \geq 0$.

□

3.2 Boundedness Analysis

Theorem 3.2. *All the solutions of the proposed model with non-negative initial conditions are bounded for all time. The feasible region* Ω *of the model* (2.1) *is defined by* $\Omega = \left\{(S, E, I, V) \in \mathbb{R}^4_+ : 0 \leq N \leq \frac{\Lambda}{\mu}\right\}$ *with initial conditions* $S(0) > 0, E(0) \geq 0, I(0) \geq 0, V(0) \geq 0$, *is positively invariant.*

Proof. Using $N(t) = S(t) + E(t) + I(t) + V(t)$, we have

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dV}{dt} = \Lambda - \mu N - \delta I$$

Solving this equation with initial condition $N(0) = N_0$ at $t = 0$, we get

$$N(t) \leq \frac{\Lambda}{\mu} + e^{-\mu t}\left(N_0 - \frac{\Lambda}{\mu}\right)$$

Taking limit as $t \to \infty$, we find $N(t) \leq \frac{\Lambda}{\mu}$

$$\therefore \quad S(t) + E(t) + I(t) + V(t) \leq \frac{\Lambda}{\mu}$$

Hence, all the solutions of the model (2.1) are bounded and feasible region for the model is $\Omega = \left\{(S, E, I, V) \in \mathbb{R}^4_+ : 0 \leq N \leq \frac{\Lambda}{\mu}\right\}$, which is positively invariant. □

3.3 Rabies-Free Equilibrium and Basic Reproduction Ratio

3.3.1 Rabies-Free Equilibrium Point

The rabies-free equilibrium point E_0 of the model (2.1) is obtained by equating all the derivatives to zero with $I = 0$.

Thus, we have

$$\frac{dS}{dt} = 0, \frac{dE}{dt} = 0, \frac{dI}{dt} = 0 \text{ and } \frac{dV}{dt} = 0$$

Solving these equations, we get rabies-free equilibrium point as

$$E_0\left(S_0, E_0, I_0, V_0\right) = \left(\frac{\Lambda}{\alpha + \mu}, 0, 0, \frac{\alpha\Lambda}{\mu\left(\alpha + \mu\right)}\right).$$

3.3.2 Basic Reproduction Ratio

We apply next generation matrix approach to derive expression for R_0 [11]. According to this approach, we take two matrices T and W. The elements of the matrix T constitute the new infectious that will arise, while that of matrix W constitute the transfer of infectious from one compartment to another. R_0 is a dominant eigen value of the matrix $K = TW^{-1}$.

The infection compartments are E and I only. So, we take only second and third equations of the model (2.1) for computing the value of R_0. Let $X = \begin{pmatrix} E \\ I \end{pmatrix}$, then the system can be written as

$$\begin{aligned} \frac{dX}{dt} &= \begin{bmatrix} -\mu-\gamma & \beta S + \sigma\beta V \\ \gamma & -\mu-\delta \end{bmatrix} \begin{bmatrix} E \\ I \end{bmatrix} \\ &= \begin{bmatrix} 0 & \beta S + \sigma\beta V \\ 0 & 0 \end{bmatrix} \begin{bmatrix} E \\ I \end{bmatrix} - \begin{bmatrix} \mu+\gamma & 0 \\ -\gamma & \mu+\delta \end{bmatrix} \begin{bmatrix} E \\ I \end{bmatrix} \\ &= T\left(X\right) - W\left(X\right) \end{aligned}$$

Linearizing about the rabies-free equilibrium $E_0\left(\frac{\Lambda}{\alpha+\mu}, 0, 0, \frac{\alpha\Lambda}{\mu(\alpha+\mu)}\right)$ and separating new infectious T from other transitions W, we get

$$T\left(E_0\right) = \begin{bmatrix} 0 & \beta S_0 + \sigma\beta V_0 \\ 0 & 0 \end{bmatrix} \text{ and } W\left(E_0\right) = \begin{bmatrix} \mu+\gamma & 0 \\ -\gamma & \mu+\delta \end{bmatrix}$$

Thus, we have

$$TW^{-1} = \begin{bmatrix} \frac{\beta\gamma(S_0+\sigma V_0)}{(\mu+\gamma)(\mu+\delta)} & \frac{\beta(S_0+\sigma V_0)}{(\mu+\delta)} \\ 0 & 0 \end{bmatrix}$$

According to next generation matrix approach, the basic reproduction ratio is the spectral radius of TW^{-1}, denoted by $\rho\left(TW^{-1}\right)$ and is given by

$$R_0 = \frac{\beta\gamma\left(S_0 + \sigma V_0\right)}{\left(\mu+\gamma\right)\left(\mu+\delta\right)} = \frac{\beta\gamma\Lambda\left(\mu + \alpha\sigma\right)}{\mu\left(\mu+\gamma\right)\left(\mu+\delta\right)\left(\mu+\alpha\right)}$$

The term R_0 is applied to measure the transmission potential of the rabies.

3.4 Endemic Equilibrium

Now, we investigate the existence of endemic equilibrium when $R_0 > 1$ and test its uniqueness.

Let $E^* = (S^*, E^*, I^*, V^*)$ be the endemic equilibrium point of the model (2.1). Then, it is obtained by setting all the derivatives of the model (2.1) to zero and $I^* \neq 0$.

Thus, we have

$$\left.\begin{aligned} \Lambda - \beta S^* I^* - (\alpha+\mu) S^* &= 0 \\ \beta S^* I^* + \sigma\beta V^* I^* - (\gamma+\mu) E^* &= 0 \\ \gamma E^* - (\delta+\mu) I^* &= 0 \\ \alpha S^* - \sigma\beta V^* I^* - \mu V^* &= 0 \end{aligned}\right\} \tag{3.1}$$

Solving first three equations in terms of I^* and then substituting in the fourth equation, we get the endemic- equilibrium point as

$$E^*(S^*, E^*, I^*, V^*) = E^*\left(\frac{\Lambda}{\beta I^* + a_1}, \frac{a_3}{\gamma} I^*, I^*, \frac{\alpha\Lambda}{(\sigma\beta I^* + \mu)(\beta I^* + a_1)}\right).$$

For uniqueness of this endemic equilibrium point, we substitute the values of S^*, E^* and V^* in second equation of (3.1) to get

$$a_2 a_3 \sigma \beta^2 I^{*2} - \beta\{\beta\gamma\sigma\Lambda + a_2 a_3(\sigma a_1 + \mu)\} I^* + \mu a_1 a_2 a_3 (1 - R_0) = 0$$

$$C_2 I^{*2} - C_1 I^* + C_0 = 0 \tag{3.2}$$

where $C_0 = \mu a_1 a_2 a_3 (1 - R_0), C_1 = \beta\{\beta\gamma\sigma\Lambda + a_2 a_3(\sigma a_1 + \mu)\}, C_2 = a_2 a_3 \sigma \beta^2$.

The above quadratic equation (3.2) in I^* yields two roots as

$$I^* = \frac{1}{2C_2}\left\{C_1 \pm \sqrt{C_1^2 - 4C_2 \times C_0}\right\}$$

Here, the values of both C_1 and C_2 are always positive. For $R_0 > 1, C_0 = \mu a_1 a_2 a_3 (1 - R_0)$ has negative value, and in such case, $C_1^2 - 4C_2 \times C_0$ is always positive and greater than C_1^2 . If we denote this value by C_3^2, then we have $C_3^2 = C_1^2 - 4C_2 \times C_0 > C_1^2$ which implies that $C_3 > C_1$.

$$I^* = \frac{1}{2C_2}\left\{C_1 \pm \sqrt{C_1^2 - 4C_2 \times C_0}\right\} = \frac{1}{2C_2}(C_1 \pm C_3)$$
$$\therefore I^* = \frac{1}{2C_2}(C_1 + C_3) > 0 \quad \& \quad I^* = \frac{1}{2C_2}(C_1 - C_3) < 0$$

Among two solutions of I^* of equation (3.2) only one is positive (other negative). Hence, we get a unique positive value of I^* when $R_0 > 1$. Consequently, if $R_0 > 1$, there exists a unique endemic equilibrium point E^*.

3.5 Stability Analysis of Rabies-Free Equilibrium

Theorem 3.3. *The rabies-free equilibrium point E_0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.*

Proof. The Jacobian matrix of the system (2.1) is

$$J = \begin{bmatrix} -\beta I - (\alpha+\mu) & 0 & -\beta S & 0 \\ \beta I & -(\gamma+\mu) & \beta(S + \sigma V) & \sigma\beta I \\ 0 & \gamma & -(\delta+\mu) & 0 \\ \alpha & 0 & -\sigma\beta V & -(\sigma\beta I + \mu) \end{bmatrix}$$

∴ The Jacobian matrix at E_0 is given by

$$J(E_0) = \begin{bmatrix} -(\alpha+\mu) & 0 & -\beta S_0 & 0 \\ 0 & -(\gamma+\mu) & \beta(S_0+\sigma V_0) & 0 \\ 0 & \gamma & -(\delta+\mu) & 0 \\ \alpha & 0 & -\sigma\beta V_0 & -\mu \end{bmatrix} = \begin{bmatrix} -a_1 & 0 & -\beta S_0 & 0 \\ 0 & -a_2 & \beta(S_0+\sigma V_0) & 0 \\ 0 & \gamma & -a_3 & 0 \\ \alpha & 0 & -\sigma\beta V_0 & -\mu \end{bmatrix}$$

where $a_1 = (\alpha+\mu)$, $a_2 = (\gamma+\mu)$ and $a_3 = (\delta+\mu)$.

The characteristic equation of the matrix $J(E_0)$ is given by

$$(\mu+\lambda)(a_1+\lambda)\left\{\lambda^2 + (a_2+a_3)\lambda + a_2a_3(1-R_0)\right\} = 0$$

where λ denotes the eigen values.

Clearly, two eigen values $\lambda_1 = -\mu < 0$ and $\lambda_2 = -a_1 = -(\alpha+\mu\} < 0$ are negatives. The remaining two eigen values are obtained by solving quadratic equation

$$\lambda^2 + (a_2+a_3)\lambda + a_2a_3(1-R_0) = 0$$

This equation yields the two roots (eigen values) as follows:

$$\lambda = \frac{-(a_2+a_3) \pm \sqrt{(a_2+a_3)^2 - 4a_2a_3(1-R_0)}}{2}$$

When $R_0 < 1$, we get $\sqrt{(a_2+a_3)^2 - 4a_2a_3(1-R_0)} = q < (a_2+a_3)$. So, the third and fourth eigen values will be $\lambda_3 = \frac{1}{2}\{-(a_2+a_3)+q\} < 0$ and $\lambda_4 = \frac{1}{2}\{-(a_2+a_3)-q\} < 0$. We find that, when $R_0 < 1$, all the four eigen-values will be negative. Therefore, the rabies-free equilibrium point E_0 will be locally asymptotically stable when $R_0 < 1$. Again, when $R_0 > 1$, we have $\lambda_3 > 0$ and $\lambda_4 < 0$, so the rabies-free equilibrium E_0 is locally asymptotically unstable, when $R_0 > 1$. □

3.6 Stability Analysis of Endemic Equilibrium

Theorem 3.4. *The endemic equilibrium point E^* of system* (2.1) *is locally asymptotically stable provided that $R_0 > 1$ and restrictions* (3.3) *are satisfied.*

Proof. The Jacobian matrix of system (2.1) at E^* is given by

$$J(E^*) = \begin{bmatrix} -\beta I^* - (\alpha+\mu) & 0 & -\beta S^* & 0 \\ \beta I^* & -(\gamma+\mu) & \beta(S^*+\sigma V^*) & \sigma\beta I^* \\ 0 & \gamma & -(\delta+\mu) & 0 \\ \alpha & 0 & -\sigma\beta V^* & -(\sigma\beta I^*+\mu) \end{bmatrix} = \begin{bmatrix} J_{11} & 0 & J_{13} & 0 \\ J_{21} & J_{22} & J_{23} & J_{24} \\ 0 & J_{32} & J_{33} & 0 \\ J_{41} & 0 & J_{43} & J_{44} \end{bmatrix}$$

Using the notation $R_i = \sum_{j=1, j\neq i}^{4} \left|J_{ij}\right|$, for each row $i = 1, 2, 3, 4$, we have

$$
\begin{aligned}
R_1 &= \sum_{j=1, j\neq 1}^{4} \left|J_{ij}\right| = |J_{12}| + |J_{13}| + |J_{14}| = \beta S^* \\
R_2 &= \sum_{j=1, j\neq 2}^{4} \left|J_{ij}\right| = |J_{21}| + |J_{23}| + |J_{24}| = \beta\left\{(1+\sigma) I^* + S^* + \sigma V^*\right\} \\
R_3 &= \sum_{j=1, j\neq 3}^{4} \left|J_{ij}\right| = |J_{31}| + |J_{32}| + |J_{34}| = \gamma \\
R_4 &= \sum_{j=1, j\neq 4}^{4} \left|J_{ij}\right| = |J_{41}| + |J_{42}| + |J_{43}| = \alpha + \sigma\beta V^*
\end{aligned}
$$

For the first hypothesis of the *Gershgarin stability theorem* [5], the system must satisfy the condition $J_{ii} < 0\ \forall i$. It is observed that the condition is satisfied.

Again, the second hypothesis of the theorem, $R_i < |J_{ii}|$ for $i = 1, 2, 3, 4$, is satisfied if the following inequalities hold.

$$
\left.
\begin{aligned}
\beta S^* &< \beta I^* + (\alpha + \mu) \\
\beta\left\{(1+\sigma) I^* + S^* + \sigma V^*\right\} &< (\gamma + \mu) \\
\gamma &< \delta + \mu \\
\alpha + \sigma\beta V^* &< \sigma\beta I^* + \mu
\end{aligned}
\right\} \tag{3.3}
$$

Consequently, the endemic equilibrium point E^* is locally asymptotically stable when inequality (3.3) is satisfied by the system. □

4 Numerical Simulation

For numerical simulations, we use parameter values that are mostly from published literature associated with the rabies in the context of Nepal [17]. We set beginning of the year 2014 as the initial time of our model dynamics. There are approximately 2 million dogs in Nepal [10]. As we do not have dog vaccination data available for the year of 2014, we assume that about $20\% \left(i.e.\ 4 \times 10^5\right)$ of dog population were vaccinated in 2014. In addition, for our base case to be consistent with the actual data, we assume that about $0.85\% \left(i.e.\ 17 \times 10^3\right)$ dogs were exposed to rabies and 0.01% dogs (200) were infected by rabies. According to the study of Leung and Davis, most of the stray dogs in Nepal live for less than 3 years due to poor environmental conditions [14]. So, we take average life expectancy for all dogs in our model as 5 years, implying the natural death rate of a dog as $\mu = 1/5 = 0.5$ dogs per year. We assume the steady state level before the epidemic begins for both dogs so that we have $\Lambda - \mu N(0) = 0$ and so $\Lambda = \mu N(0)$. Thus, recruitment rate for dog population is $\Lambda = 0.2 \times N(0)$ dogs per year. For numerical simulation, we take the initial condition as; $S(0) = 1{,}589{,}800, E(0) = 17{,}000, I(0) = 200$ and $V(0) = 393{,}000$.

4.1 Sensitivity Analysis

Using the baseline parameter values listed in the Table 5, the basic reproduction ratio is calculated as $R_0 = 1.1946$. The normalized forward sensitivity index of R_0 with respect to each of the parameter p involved in R_0, denoted by $C_p^{R_0}$, is defined as follows [7]:

$$
C_p^{R_0} = \frac{p}{R_0} \times \frac{\partial R_0}{\partial p}
$$

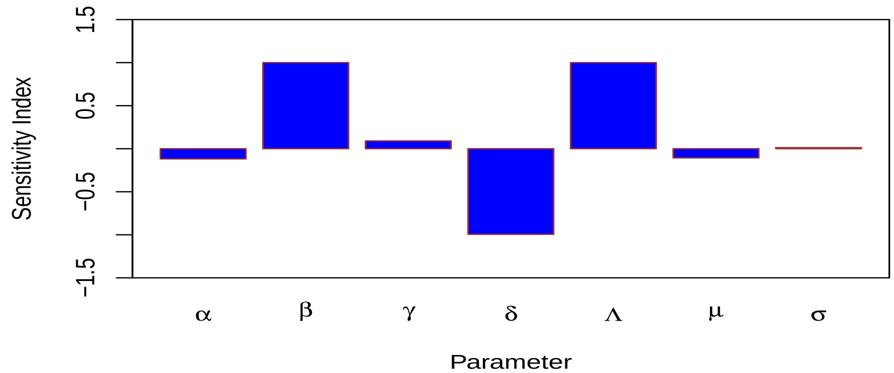

Fig. 2: Sensitivity indices of R_0, using the parameter values from Table 5.

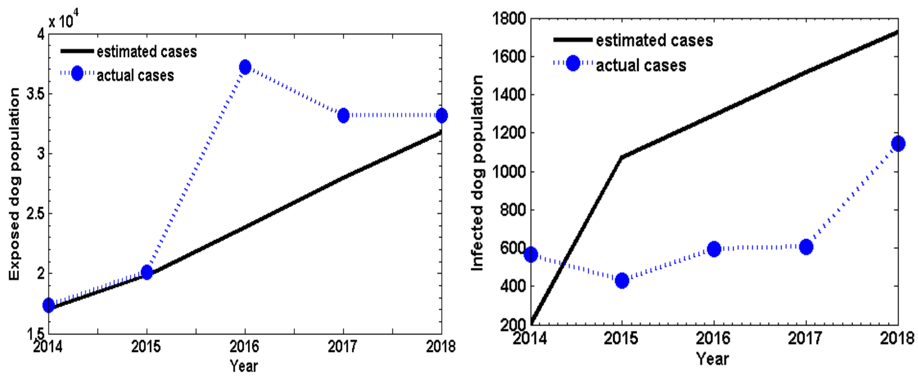

Fig. 3: Comparison between reported data and numerically simulated data of system (2.1) for rabies exposed and infected population of dogs for Nepal from 2014 to 2018, using parameter values from Table 5

Normalized sensitivity indices of R_0 with respect to parameters are in the Table 6. To specify the effect of changing parameter values on R_0, the value of each parameter is increased and decreased by 10 % and consequences on R_0 are displayed in the Table 7.

Tab. 6: Sensitivity indices of basic reproduction ratio R_0.

Parameter	α	β	γ	δ	Λ	μ	σ
Sensitivity Index	-0.1186	1	0.0909	-0.9946	1	-0.1069	0.0119

Bar plot in Figure 2 shows the normalized sensitivity indices of R_0 related to the parameters for the rabies model, evaluated at the baseline parameter values given in Table 5. The most sensitive parameters are the transmission rate (β) and dog recruitment rate (Λ) of the susceptible dogs. The least sensitive parameter is the anti-rabies vaccine inefficiency rate (σ). In general, from the bar plot in Figure 2, parameters that have positive sensitivity indices, namely $\beta, \Lambda, \gamma, \sigma$ have positive impact on R_0 in the condition that the other parameters remain constant. Whereas increase of parameters whose sensitivity indices are negative, δ, α, μ, have negative impact on R_0, minimizing the effect of spread of disease.

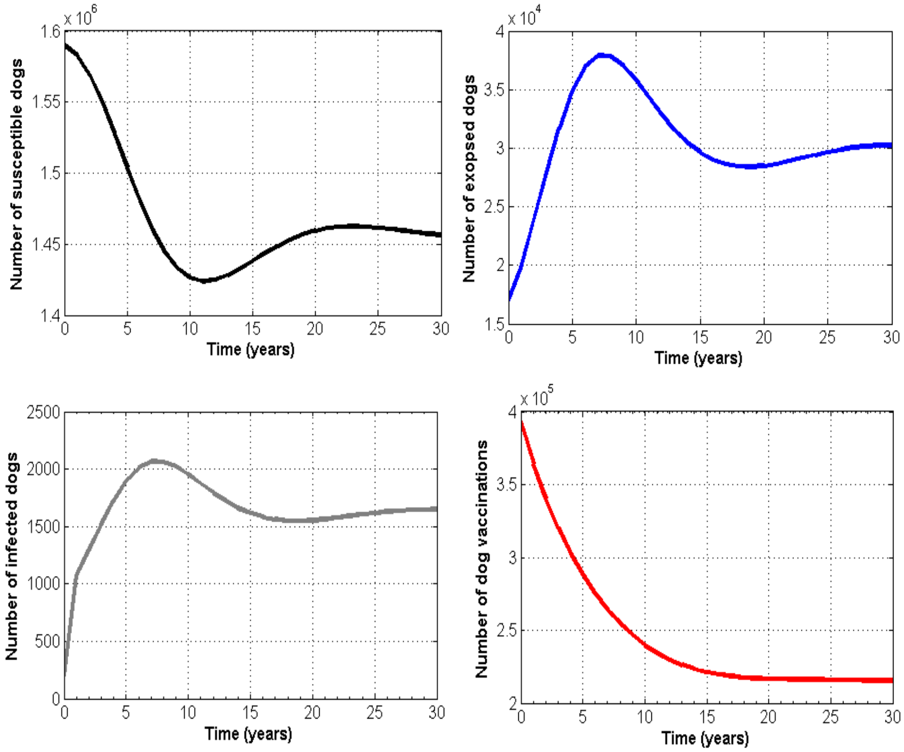

Fig. 4: Numerical simulations displaying the transmission dynamics of rabies in infected dogs for the next 30 years in Nepal, using parameter values from Table 5.

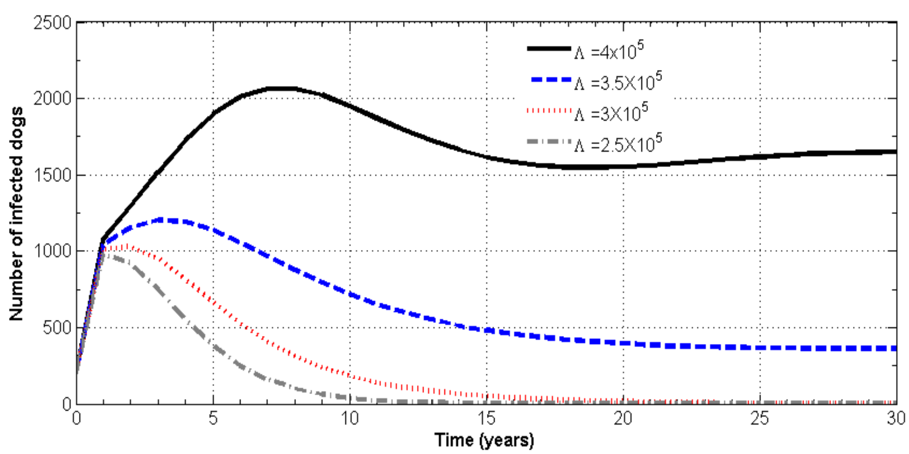

Fig. 5: Numerical simulations displaying the impact of dog sterilization, taking different values of annual new birth of dogs (Λ), $\Lambda = 4 \times 10^5, \Lambda = 3.5 \times 10^5, \Lambda = 3 \times 10^5, \Lambda = 2.5 \times 10^5$ using other parameter values from Table 5.

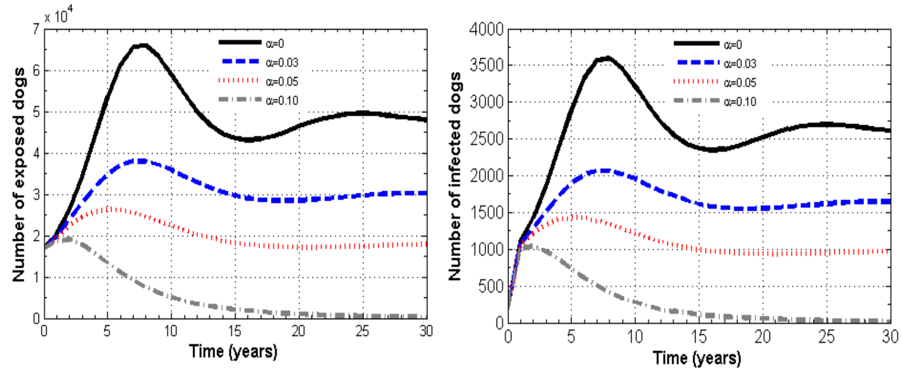

Fig. 6: Numerical simulations displaying the impact of different level of vaccination coverage of dogs in Nepal for next 30 years of period, taking vaccination percentage 0%, 3%, 5% and 10%, using other values of parameters from Table 5.

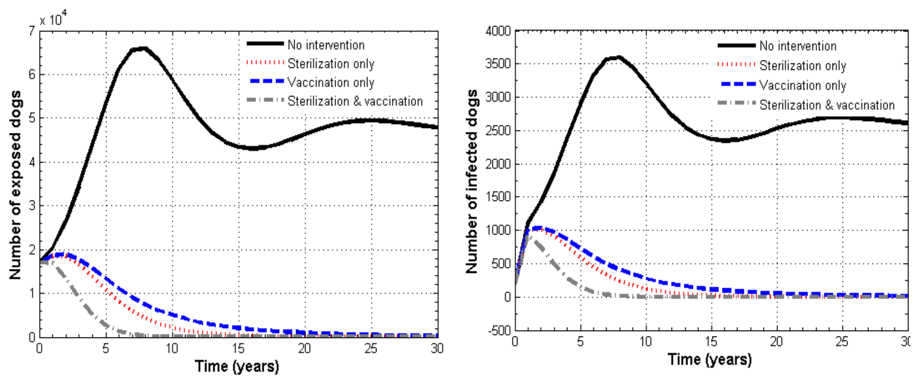

Fig. 7: Numerical simulations displaying the impact of different combination of interventions of rabies in Nepal for next 30 years of period, taking $\alpha = 10\%$, and $\Lambda = 2.5 \times 10^5$, using other parameter values from Table 5 except for Λ and α .

5 Results and Discussion

Figure 3 shows the total estimated from the model (2.1) and observed number of rabies exposed and infected dogs in Nepal from 2014 to 2018. The predicted number of infected cases is all in very good agreement with the observed data. They perfectly match the actual data during initial and terminal period while they tend to slightly underestimate exposed cases and overestimate the infected cases the observed values at the middle of the duration. Overall, our numerical results demonstrate the capability of the proposed model in achieving an excellent prediction skill.

In Figure 4, numerical result of infected dog population shows that the rabies virus will spread very rapidly in the coming years and it will peak in 2023. The bite of rabid dog is the main reason for the transmission of rabies.

5.1 Impact of Dog Sterilization

The annual dog birth rate is one of the sensitive parameters which control the dynamics of the disease. A minor increase in newly born puppies' increases animals' infection and vice versa. The

Tab. 7: Effect of change in parameter (p) values on R_0.

Parameter	α	β	γ	δ	Λ	μ	σ
R_0 for $p + 10\%$	1.1806	1.2948	1.2045	1.0865	1.3140	1.0873	1.1960
R_0 for $p - 10\%$	1.2089	1.0594	1.1826	1.3265	1.0751	1.3228	1.1931

sterilization does not remove directly the current dog population or block the infection; it rather controls fertility of the dogs. Fertility control has long been considered important tools for reducing transmission of some density dependent pathogens, although lethal management techniques are controversial among many stakeholders [18].We have taken the recruitment rate as $\Lambda = \mu \times N$, where N is the total number of dogs and μ represents the per capita birth rate. If k represents the sterilization rate which reduces the recruitment growth rate as $\Lambda = (\mu - k) \times N$. Using this new growth rate, dog population at time t is obtained as $N(t) = N(0)\, e^{(\mu-k)t}$. This implies that the implementation of the dog sterilization program to reduce the dog recruitment by θ % in period of t years, the sterilization rate should be $k = \mu - ln(1-\theta)/t$, we defined θ as the t-year sterilization strength.

Figure 5 shows infectious dog population verses time for different values of the annual birth of the dogs due to the different dog sterilization strength. The trend suggests that applying strategy to control the annul birth of new born puppies with the implementation of the proper sterilization strength; number of rabies infection cases reduces significantly.

5.2 Impact of Dog Vaccination

Figure 6 demonstrates that rabies cases increase rapidly in dog population for coming nine years and will peak in 2023. By the increment of 5% vaccination coverage in the stray dog population, the number of rabies infection cases predicted to decrease about half of number by 2023. With each increase in the 5% of vaccination coverage the total impact in term of reductions of rabies infected cases gets smaller. The prediction shows that even 10% vaccination coverage does not lead to the elimination of the rabies among dogs completely immediately. Stray dogs are typically wary of humans such that they cannot be held, and so vaccination of these of these dogs needs to be achieved either by physically capturing the dogs or by distributing oral baits [22]. In either case, searching a sufficiently high vaccination coverage of the whole dog population becomes a more difficult and costly endeavour. Another contributing factor is success of the vaccination program is the relationship that dogs share with the humans they live with some owned dogs are fully confined and others partially free-roaming [2]. For this reason, we recommend that reducing rabies transmission in the stray dog population is the best method for controlling the transmission in the animals including dogs.

5.3 Impact of combinations of Interventions

Figure 7 shows that when there is no intervention, number of exposed and infected dog population is increasing rapidly for about nine years and then slightly decreases. This is because when infected dogs bite another dogs, the bitten dogs converted into exposed dogs. So, the number of exposed dogs increases and ultimately after sometimes they become infectious dogs. Also, the number of exposed dog is decreasing and that of infected dogs are decreasing. But after some times, infected dogs begin to die as there is no treatment of rabies for dogs. Consequently, the number of infected dogs begins to decrease. When the interventions are implemented, the graph illustrates that the number of exposed and infected dogs decreases rapidly. It is also noticed that implementation of both vaccination and birth control by sterilization of dogs is more effective than using only one mode of intervention either vaccination or sterilization.

6 Conclusion

We have analyzed rabies transmission dynamics model for dog population by incorporating two control strategies; one vaccination and other sterilization of dog. Basic properties of the model are established and it is found that the model is well-posed mathematically and biologically. It is concluded that administrating vaccination at the rate of 10% per annum on the susceptible dogs can eradicate rabies among dogs in about 10 years. Significant results have been observed when both interventions (vaccination and sterilization) are implemented on controlling rabies among dogs.

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References

[1] *S. Abdulmajid and A. S. Hassan*, Analysis of time delayed Rabies model in human and dog populations with controls, Afrika Matematika **32**(2021), no. 5, 1067–1085.

[2] *K. P. Acharya, N. Acharya, S. Phuyal, M. Upadhyaya, and L. Steven*, One-health approach: A best possible way to control rabies, One Health **100161**(2020).

[3] *E.S. Allman and A.R. John*, Mathematical Models in Biology- An Introduction,Cambridge University Press, New York, 2004.

[4] *P. J. Bacon*, Population dynamics of rabies in wildlife, London: Academic Press, 1985.

[5] *D. Bejarano and E. A. Ibarguen-Hernandez*, A stability test for nonlinear systems of ordinary differential equations based on the gershgorin circles, Contemporary engineering sciences **11**(2018), no. 91, 4541–4548.

[6] Centre for Disease Control and Prevention (CDC), (2011), *Rabies*. http://www.cdc.gov/rabies/. Accessed June 1, 2019.

[7] *N. Chitnis, J.M. Hyman and J.M. Cushing*, Determining Important Parameter in the Spread of Malaria. Through the Sensitivity Analysis of Mathematical Model, Bulletin of Mathematical Biology **70**(2008), 1272–1296.

[8] *N. Dandy et al.*, Exploring beliefs behind support for and opposition to wildlife management methods: a qualitative study, European Journal of Wildlife Research **58**(2012), no. 4, 695–706.

[9] *S. Darkaoui et al.*, Immunogenicity and efficacy of Rabivac vaccine for animal rabies control in Morocco, Clinical and experimental vaccine research **5**(2016), no. 1, 60–69.

[10] *B. Devleesschauwer et al.*, Epidemiology, impact and control of rabies in Nepal: a systematic review, PLoS neglected tropical diseases **10**(2017), no. 2, e0004461.

[11] *O. Diekmann, J. A. P. Heesterbeek and M. G. Roberts*, The construction of next-generation matrices for compartmental epidemic models, Journal of the Royal Society Interface **7**(2010), no. 47, 873–885.
DOI: 10.1098/rsif.2009.0386

[12] *O. C. Eze, G. E. Mbah, D. U. Nnaji and N. E. Onyiaji*, Mathematical Modelling of Transmission Dynamics of Rabies Virus, International Journal of Mathematics Trends and Technology **66**(2020), no. 7, 40–64.

[13] *K. Kakati*, Street dog population survey, Kathmandu 2012, Final report to WSPA, WSPA (2012).

[14] *T. Leung and S. A. Davis*, Rabies vaccination targets for stray dog populations, Frontiers in veterinary science **4**(2017), 52.

[15] *T. Nuertey*, Human rabies, Ghana Veterinary Medical Association Newsletter **6**(2007), 16–17.

[16] *B. Pantha, S. Giri, H. R. Joshi and N. K. Vaidya*, Modeling transmission dynamics of rabies in Nepal, Infectious Disease Modelling **6**(2021), 284–301.

[17] *Quarterly Animal Health E-Bulletin (Rabies specific)*, Veterinary Epidemiology Centre,Tripureshwor, Kathmandu, Nepal, 2018, Jan-April.

[18] *R. C. Rosatte, M. J. Pybus and J. R. Gunson*, Population reduction as a factor in the control of skunk rabies in Alberta, Journal of Wildlife Diseases **22**(1986), no. 4, 459–467.

[19] *S. Ruan*, Modeling the transmission dynamics and control of rabies in China, Mathematical biosciences **286**(2017), 65–93.

[20] *C. E. Rupprecht et al.*, Use of a reduced (4-dose) vaccine schedule for postexposure prophylaxis to prevent human rabies: recommendations of the advisory committee on immunization practices, MMWR Recomm Rep **59**(2010), no. RR-2, 1–9.

[21] *VCA Animal Hospital*, Rabies in Dogs, Accessed on 09/2021 https://vcahospitals.com/know-your-pet/rabies-in-dogs

[22] *World Health Organization*, WHO Expert Consultation on Rabies. Second Report, Geneva: World Health Organization (2013).

[23] *J. Zhang, Z. Jin, G. Q. Sun, T. Zhou and S. Ruan*, Analysis of rabies in China: transmission dynamics and control, PLoS one **6**(2011), no. 7, e20891.

[24] *J. Zhang, Z. Jin, G. Q. Sun, X. D. Sun and S. Ruan*, Modeling seasonal rabies epidemics in China, Bulletin of Mathematical Biology, **74**(2012), no. 5, 1226–1251.