A Note on Topological Properties Of Dyck-56 Networks

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Abstract

Recently, Dickson Selvan [21] has obtained analytical closed results for the general Randić index $R_\alpha(G)$ (for diffrent values of α), first Zagreb, ABC and GA indices for the $Dyck-56_{n\times n}(A)$, $Dyck-56_{n\times n}(B)$ and $Dyck-56_{n\times n}(C)$ chemical networks for the first time. Vertex set, edge set, partition of edges based on degrees and results for above said indices of $Dyck-56_{n\times n}(B)$ chemical network prone to error. In this note we correct these errors. In addition, we propose reverse Sombor index, reverse modified Sombor index, first reverse $(a,b)-KA$ index, reverse reduced Sombor index, reverse reduced modified Sombor index and first and second reverse reduced $(a,b)-KA$ indices of graph G. Also, we derive analytical closed results for first reverse $(a,b)-KA$ index, reverse Sombor index, reverse modified Sombor index, first reverse reduced $(a,b)-KA$ index, reverse reduced Sombor index, second reverse reduced $(a,b)-KA$ index, sum connectivity reverse index, product connectivity reverse index, atom bond connectivity reverse index, geometric-arithmetic reverse index and some multiplicative topological indices for the $Dyck-56_{n\times n}(A)$, $Dyck-56_{n\times n}(B)$ and $Dyck-56_{n\times n}(C)$ chemical networks.

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1 Introduction

Chemical graph theory is a branch of mathematical chemistry, which has an major effect on the development of the chemical sciences. In molecular graph, graph is used to represent a molecule by considering the atoms as the vertices and molecular bonds as the edges. A topological index is a molecular descriptor that is calculated based on the molecular graph of a chemical compound. These topological indices are useful in development of quantitative structure activity relationships (QSAR) and quantitative structure property relationship (QSPR) in which the biological activity or other properties of molecules are correlated with their chemical structure. Let $G = (V, E)$ be a simple, undirected graph. Let $V(G)$ be the vertex set and $E(G)$ be the edge set of the graph G. The degree $d_G(v)$ of a vertex $v \in V(G)$ is the number of edges incident to it in G. Let $\Delta(G)$ denote the maximum degree among the vertices of a graph G. The reverse vertex degree of a vertex v in a graph G is defined as $c_v = \Delta(G) - d_G(v) + 1$[11]. The reverse edge connecting the reverse vertices u and v will be denoted by uv. We refer [4] for undefined terms and notation.
One of the oldest degree based topological index is Randić index [20] and is defined as,

$$R_{-\frac{1}{2}} = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)\times d_G(v)}}.$$

The general Randć index $R_\alpha(G)$ [2] is defined as,

$$R_\alpha = \sum_{uv \in E(G)} (d_G(u) \times d_G(v))^\alpha.$$

The atom bond connectivity ABC index [3] is defined as,

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u)d_G(v)}}.$$

The geometric-arithmetic (GA) index [22] is defined as,

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u) + d_G(v)}.$$

In [5], I. Gutman introduced the Sombor index of a graph G and it is defined as,

$$SO(G) = \sum_{un \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2}.$$

We now define the reverse Sombor index of a graph G

$$CSO(G) = \sum_{un \in E(G)} \sqrt{c_u^2 + c_v^2}.$$

In [17], V. R. Kulli introduced modified Sombor index of a graph G and it is defined as,

$$^mSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2}}.$$

We now define the reverse modified Sombor index of a graph G

$$^mCSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{c_u^2 + c_v^2}}.$$

In [18], the first $(a, b) - KA$ index of a graph was introduced and it is defined as,

$$KA^1_{a,b}(G) = \sum_{uv \in E(G)} [d_G(u)^a + d_G(v)^a]^b.$$

We now define the first reverse $(a, b) - KA$ index of a graph G

$$CKA^1_{a,b}(G) = \sum_{uv \in E(G)} [c_u^a + c_v^a]^b. \tag{1.1}$$

Where a and b are suitably chosen real-number parameters. The reduced Sombor index [5] was defined as,

$$RSO(G) = \sum_{uv \in E(G)} \sqrt{(d_G(u) - 1)^2 + (d_G(v) - 1)^2}.$$

We now define the reverse reduced Sombor index of a graph G

$$CRSO(G) = \sum_{uv \in E(G)} \sqrt{(c_u - 1)^2 + (c_v - 1)^2}.$$

In [17], V. R. Kulli introduced reduced modified Sombor index of a graph G and it is defined as,

$$^{m}RSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_G(u) - 1)^2 + (d_G(v) - 1)^2}}.$$

We now define the reverse reduced modified Sombor index of a graph G

$$^{m}CRSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(c_u - 1)^2 + (c_v - 1)^2}}.$$

In [17], V. R. Kulli introduced the first and second reduced $(a, b) - KA$ indices of a graph G and it is defined as,

$$RKA^1_{a,b}(G) = \sum_{uv \in E(G)} \left[(d_G(u) - 1)^a + (d_G(v) - 1)^a\right]^b,$$

$$RKA^2_{a,b}(G) = \sum_{uv \in E(G)} \left[(d_G(u) - 1)^a (d_G(v) - 1)^a\right]^b.$$

We now define the first and second reverse reduced $(a, b) - KA$ indices of a graph G

$$CRKA^1_{a,b}(G) = \sum_{uv \in E(G)} \left[(c_u - 1)^a + (c_v - 1)^a\right]^b, \tag{1.2}$$

$$CRKA^2_{a,b}(G) = \sum_{uv \in E(G)} \left[(c_u - 1)^a (c_v - 1)^a\right]^b. \tag{1.3}$$

The sum connectivity reverse index was introduced by V. R. Kulli in [2] and it is defined as,

$$SC(G) = \sum_{uv \in E(G)} \left[\frac{1}{\sqrt{c_u + c_v}}\right]. \tag{1.4}$$

The product connectivity reverse index was proposed in [11] and it is defined as,

$$PC(G) = \sum_{uv \in E(G)} \left[\frac{1}{\sqrt{c_u \cdot c_v}}\right]. \tag{1.5}$$

In [4], V. R. Kulli introduced atom bond connectivity reverse index of a graph G and it is defined as,

$$ABCC(G) = \sum_{uv \in E(G)} \sqrt{\frac{c_u + c_v - 2}{c_u \cdot c_v}}. \tag{1.6}$$

The geometric-arithmetic reverse index was introduced in [5] and it is defined as,

$$GAC(G) = \sum_{uv \in E(G)} \frac{2\sqrt{c_u \cdot c_v}}{c_u + c_v}. \tag{1.7}$$

V. R. Kulli introduced the first multiplicative inverse sum indeg index [8] of a graph G. The multiplicative inverse sum indeg index of a graph G is defined as,

$$ISIII(G) = \prod_{uv\in E(G)} \frac{d_G(u)\cdot d_G(u)}{d_G(u)+d_G(u)}. \tag{1.8}$$

V. R. Kulli in [14] introduced the multiplicative sum connectivity index, multiplicative product connectivity index, multiplicative atom bond connectivity index and multiplicative geomertic arithmetic index of a graph as follows:
The multiplicative sum connectivity index of a graph G is defined as,

$$XII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(u)}}. \tag{1.9}$$

The multiplicative product connectivity index of a graph G is defined as,

$$\chi^{II(G)} = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)\cdot d_G(u)}}. \tag{1.10}$$

The multiplicative atom bond connectivity index of a graph G is defined as,

$$ABCII(G) = \prod_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(u)-2}{d_G(u)\cdot d_G(u)}}. \tag{1.11}$$

The multiplicative geomertic-arithmetic index of a graph G is defined as,

$$GAII(G) = \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)\cdot d_G(u)}}{d_G(u)+d_G(u)}. \tag{1.12}$$

The multiplicative arithmetic-geometric index [9] of a graph G is defined as,

$$AGII(G) = \prod_{uv\in E(G)} \frac{d_G(u)+d_G(u)}{2\sqrt{d_G(u)\cdot d_G(u)}}. \tag{1.13}$$

The topological indices of some networks were studied in [11, 12, 13, 14, 19, 21].

2 Results for $Dyck-56_{n\times n}(B)$ Network

The vertex set and edge set in Theorem 2.5 in [21] fails for the $Dyck-56_{3\times 3}(B)$ network as shown in Figure 1. It holds for only $Dyck-56_{2\times 2}(B)$ network not for $Dyck-56_{n\times n}(B)$ network, because $V(Dyck-56_{3\times 3}(B)) = 144$ and $E(Dyck-56_{3\times 3}(B)) = 216$. From Theorem 2.5 in [21] it is $V(Dyck-56_{3\times 3}(B)) = 153$ and $E(Dyck-56_{3\times 3}(B)) = 212$.
The correct version of Theorems 2.5, 2.6, 2.7 and 2.8 and Table 2 in [21] is as follows.
Let G be the $Dyck-56_{n\times n}(B)$ network. It has $15n^2+3n$ vertices and $24n^2$ edges. Edge partition of $Dyck-56_{3\times 3}(B)$ network as shown in Table 3. Clearly, $\Delta(G) = 4$. Therefore $c_u = \Delta(G)-d_G(u)+1 = 5-d_G(u)$. Thus there are three types of reverse edges as given in Table 9.

Theorem 2.1. *Let G be the $Dyck-56_{n\times n}(B)$ network, then its general Randić index is equal to*

$$R_\alpha(G) = \begin{cases} 252n^2-72n & \text{if } \alpha=1;\\ 12n\sqrt{6}+36n^2+24n(n-1)\sqrt{3} & \text{if } \alpha=\frac{1}{2};\\ \frac{3n+7n^2}{3} & \text{if } \alpha=-1;\\ 2n(\sqrt{6}+2n+(n-1)\sqrt{3}) & \text{if } \alpha=-\frac{1}{2}; \end{cases}$$

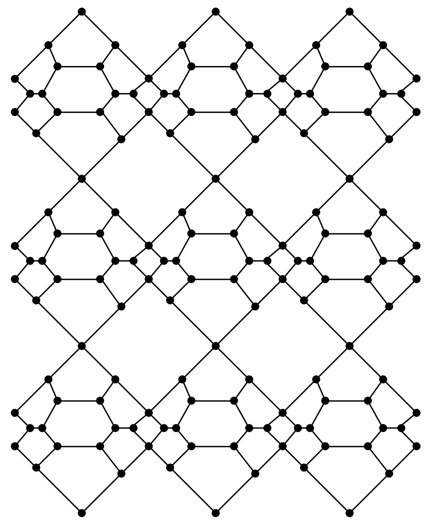

Fig. 1: $Dyck - 56_{3\times3}(B)$ network

Tab. 8: Edge partition of $Dyck - 56_{n\times n}(B)$ Network

$(d_G(u), d_G(v)); uv \in E(G)$	Number of edges
$(2,3)$	$12n$
$(3,3)$	$12n^2$
$(3,4)$	$12n(n-1)$

Tab. 9: Reverse edge partition of $Dyck - 56_{n\times n}(B)$ Network

$(c_u, c_v); uv \in E(G)$	Number of edges
$(2,3)$	$12n$
$(2,2)$	$12n^2$
$(1,2)$	$12n(n-1)$

Proof. The number of vertices and edegs in G are $15n^2 + 3n$ and $24n^2$ respectively. There are three types of edges in G, based on degrees of end vertices of each edge. Table 3 shows the edge partition of G.
By using edge partition given in Table 3, we get

$$R_\alpha(G) = \sum_{uv\in E(G)} (d_G(u) \times d_G(v))^\alpha$$

For $\alpha = 1$,

$$\begin{aligned} R_1(G) &= \sum_{uv\in E(G)} (d_G(u) \times d_G(v)) \\ &= 12n(2\times 3) + 12n^2(3\times 3) + 12n(n-1)(3\times 4) \\ &= 252n^2 - 72n. \end{aligned}$$

For $\alpha = \frac{1}{2}$,

$$\begin{aligned} R_{\frac{1}{2}}(G) &= \sum_{uv \in E(G)} \sqrt{(d_G(u) \times d_G(v))} \\ &= 12n\sqrt{6} + 12n^2\sqrt{9} + 12n(n-1)\sqrt{12} \\ &= 12n\sqrt{6} + 36n^2 + 24n(n-1)\sqrt{3}. \end{aligned}$$

For $\alpha = -1$,

$$\begin{aligned} R_{-1}(G) &= \sum_{uv \in E(G)} \frac{1}{(d_G(u) \times d_G(v))} \\ &= \frac{12n}{6} + \frac{12n^2}{9} + \frac{12n(n-1)}{12} \\ &= \frac{3n + 7n^2}{3}. \end{aligned}$$

For $\alpha = -\frac{1}{2}$,

$$\begin{aligned} R_{-\frac{1}{2}}(G) &= \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_G(u) \times d_G(v))}} \\ &= \frac{12n}{\sqrt{6}} + \frac{12n^2}{\sqrt{9}} + \frac{12n(n-1)}{\sqrt{12}} \\ &= 2n(\sqrt{6} + 2n + (n-1)\sqrt{3}). \end{aligned}$$

□

Theorem 2.2. *Let G be the Dyck* $-$ $56_{n\times n}(B)$ *network, then its first zagreb index is equal to*

$$M_1(G) = n(156n - 24).$$

Proof. By using edge partition from Table 3, the result follows.

$$\begin{aligned} M_1(G) &= \sum_{uv \in E(G)} (d_G(u) + d_G(v)) \\ &= 12n(2+3) + 12n^2(3+3) + 12n(n-1)(3+4) \\ &= n(156n - 24). \end{aligned}$$

□

Theorem 2.3. *Let G be the Dyck* $-$ $56_{n\times n}(B)$ *network, then its ABC index is equal*

$$ABC(G) = 6n\sqrt{2} + 8n^2 + 2n(n-1)\sqrt{15}.$$

Proof. By using edge partition from Table 3, the result follows.

$$\begin{aligned} ABC(G) &= \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u)d_G(v)}} \\ &= 12n\sqrt{\frac{2+3-2}{6}} + 12n^2\sqrt{\frac{3+3-2}{9}} + 12n(n-1)\sqrt{\frac{3+4-2}{12}} \\ &= 6n\sqrt{2} + 8n^2 + 2n(n-1)\sqrt{15}. \end{aligned}$$

□

Theorem 2.4. *Let G be the Dyck* $-\ 56_{n\times n}(B)$ *network, then its GA index is equal*

$$GA(G) = 12n^2 + \frac{24n\sqrt{6}}{5} + \frac{48n(n-1)\sqrt{3}}{7}.$$

Proof. By using edge partition from Table 3, the result follows.

$$\begin{aligned} GA(G) &= \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)} \\ &= 12n\left(\frac{2\sqrt{6}}{5}\right) + 12n^2\left(\frac{2\sqrt{9}}{6}\right) + 12n(n-1)\left(\frac{2\sqrt{12}}{7}\right) \\ &= 12n^2 + \frac{24n\sqrt{6}}{5} + \frac{48n(n-1)\sqrt{3}}{7}. \end{aligned}$$

□

Theorem 2.5. *Let G be the Dyck* $-\ 56_{n\times n}(B)$ *network, then its first reverse* $(a,b)-KA$ *index is equal to*

$$CKA^1_{a,b}(G) = 12n(3^a+2^a)^b + 3n^2 2^{b(a+1)+2} + 12n(n-1)(2^a+1^a)^b.$$

Proof. By Eq. (1.1) and Table 9, we deduce

$$\begin{aligned} CKA^1_{a,b}(G) &= \sum_{uv\in E(G)} [c_u^a + c_v^a]^b \\ &= 12n(3^a+2^a)^b + 12n^2(2^a+2^a)^b + 12n(n-1)(2^a+1^a)^b \\ &= 12n(3^a+2^a)^b + 12n^2(2\times 2^a)^b + 12n(n-1)(2^a+1^a)^b \\ &= 12n(3^a+2^a)^b + 3n^2 2^{b(a+1)+2} + 12n(n-1)(2^a+1^a)^b. \end{aligned}$$

□

Corollary 2.1. *Let G be the Dyck* $-\ 56_{n\times n}(B)$ *network, then its reverse Sombor index is equal to*

$$CSO(G) = 12n\sqrt{13} + 3n^2 2^{\frac{7}{2}} + 12n(n-1)\sqrt{5}.$$

Corollary 2.2. *Let G be the Dyck* $-\ 56_{n\times n}(B)$ *network, then its reverse modified Sombor index is equal to*

$$^{m}CSO(G) = \frac{12n}{\sqrt{13}} + 3n^2\sqrt{2} + \frac{12n(n-1)}{\sqrt{5}}.$$

Theorem 2.6. *Let G be the Dyck* $-\ 56_{n\times n}(B)$ *network, then its first reverse reduced* $(a,b)-KA$ *index is equal to*

$$CRKA^1_{a,b}(G) = 12n(2^a+1^a)^b + 3n^2 2^{b+2} 1^{ab} + 12n(n-1)1^{ab}.$$

Proof. By Eq. (1.2) and Table 9, we deduce

$$\begin{aligned} CRKA^1_{a,b}(G) &= \sum_{uv\in E(G)} [(c_u-1)^a + (c_v-1)^a]^b \\ &= 12n(2^a+1^a)^b + 12n^2(1^a+1^a)^b + 12n(n-1)(1^a)^b \\ &= 12n(2^a+1^a)^b + 12n^2(2\times 1^a)^b + 12n(n-1)(1^a)^b \\ &= 12n(2^a+1^a)^b + 3n^2 2^{b+2} 1^{ab} + 12n(n-1)1^{ab}. \end{aligned}$$

□

Corollary 2.3. *Let G be the Dyck* $-$ $56_{n\times n}(B)$ *network, then its reverse reduced Sombor index is equal to*

$$CRSO(G) = 12n\sqrt{5} + 3n^2 2^{\frac{5}{2}} + 12n(n-1).$$

Corollary 2.4. *Let G be the Dyck* $-$ $56_{n\times n}(B)$ *network, then its reverse reduced modified Sombor index of a graph G*

$$^{m}CRSO(G) = \frac{12n}{\sqrt{5}} + 6n^2\sqrt{2} + 12n(n-1).$$

Theorem 2.7. *Let G be the Dyck* $-$ $56_{n\times n}(B)$ *network, then its second reverse reduced* $(a, b) - KA$ *index is equal to*

$$CRKA^2_{a,b}(G) = 3n2^{ab+2}1^{ab} + 12n^2 1^{2ab}.$$

Proof. By Eq. (1.3) and Table 9, we deduce

$$\begin{aligned} CRKA^2_{a,b}(G) &= \sum_{uv\in E(G)} [(c_u-1)^a \cdot (c_v-1)^a]^b \\ &= 12n(2^a\cdot 1^a)^b + 12n^2(1^a\cdot 1^a)^b \\ &= 12n(2^a\cdot 1^a)^b + 12n^2(1^{2a})^b \\ &= 3n2^{ab+2}1^{ab} + 12n^2 1^{2ab}. \end{aligned}$$

□

Theorem 2.8. *Let G be the Dyck* $-$ $56_{n\times n}(B)$ *network, then its sum connectivity reverse index is equal to*

$$SC(G) = 6n\left(\frac{2}{\sqrt{5}} + \frac{2(n-1)}{\sqrt{3}} + n\right).$$

Proof. By Eq. (1.4) and Table 9, we deduce

$$\begin{aligned} SC(G) &= \frac{1}{\sqrt{3+2}}12n + \frac{1}{\sqrt{2+2}}(12n^2) + \frac{1}{\sqrt{2+1}}(12n(n-1)) \\ &= 6n\left(\frac{2}{\sqrt{5}} + \frac{2(n-1)}{\sqrt{3}} + n\right). \end{aligned}$$

□

Theorem 2.9. *Let G be the Dyck* $-$ $56_{n\times n}(B)$ *network, then its product connectivity reverse index is equal to*

$$PC(G) = 2n(\sqrt{6} + 3n + 2(n-1)\sqrt{3}).$$

Proof. By Eq. (1.5) and Table 9, we deduce

$$\begin{aligned} PC(G) &= \frac{1}{\sqrt{3\cdot 2}}(12n) + \frac{1}{\sqrt{2\cdot 2}}(12n^2) + \frac{1}{\sqrt{2\cdot 1}}(12n(n-1)) \\ &= 2n(\sqrt{6} + 3n + 3(n-1)\sqrt{2}). \end{aligned}$$

□

Theorem 2.10. *Let G be the Dyck* $-$ $56_{n\times n}(B)$ *network, then its atom bond connectivity reverse index is equal to*

$$ABCC(G) = 12n^2\sqrt{2}.$$

Proof. By Eq. (1.6) and Table 9, we deduce

$$\begin{aligned} ABCC(G) &= \left(\sqrt{\frac{3+2-2}{3\cdot 2}}\right)12n + \left(\sqrt{\frac{2+2-2}{2\cdot 2}}\right)12n^2 + \left(\sqrt{\frac{2+1-2}{2\cdot 1}}\right)12n(n-1) \\ &= \frac{12n}{\sqrt{2}} + \frac{12n^2}{\sqrt{2}} + \frac{12n(n-1)}{\sqrt{2}} \\ &= 12n^2\sqrt{2}. \end{aligned}$$

□

Theorem 2.11. *Let G be the* $Dyck-56_{n\times n}(B)$ *network, then its geometric arithmetic connectivity reverse index is equal to*

$$GAC(G) = 4n\left(3n^2 + 2^{\frac{3}{2}}(n-1) + \frac{6^{\frac{3}{2}}}{\sqrt{5}}\right).$$

Proof. By Eq. (1.7) and Table 9, we deduce

$$\begin{aligned} GAC(G) &= \left(\frac{2\sqrt{3\cdot 2}}{3+2}\right)12n + \left(\frac{2\sqrt{2\cdot 2}}{2+2}\right)12n^2 + \left(\frac{2\sqrt{2\cdot 1}}{2+1}\right)12n(n-1) \\ &= 4n\left(3n^2 + 2^{\frac{3}{2}}(n-1) + \frac{6^{\frac{3}{2}}}{\sqrt{5}}\right). \end{aligned}$$

□

Theorem 2.12. *For the* $Dyck-56_{n\times n}(B)$ *network, then we have*

1. $ISIII(G) = 2^{36n^2-12n} \times 3^{24n^2} \times 7^{12n(n-1)}$
2. $XII(G) = 5^{-6n} \times 6^{-6n^2} \times 7^{6n(1-n)}$
3. $\chi^{II(G)} = 6^{-6n} \times 9^{-6n^2} \times 12^{6n(1-n)}$
4. $ABCII(G) = 3^{6n-18n^2} \times 2^{6n} \times 5^{6n(n-1)}$
5. $GAII(G) = 2^{24n^2-6n} \times 3^{6n^2} \times 5^{-12n} \times 7^{12n(1-n)}$
6. $AGII(G) = 2^{6n-24n^2} \times 3^{-6n^2} \times 5^{12n} \times 7^{12n(n-1)}$.

Proof. Let G be the $Dyck-56_{n\times n}(B)$ network. By Eqs. (1.8), (1.9), (1.10), (1.11), (1.12) and

(1.13) and Table 3. We compute

$$\begin{aligned}
ISIII(G) &= \prod_{uv\in E(G)} \frac{d_G(u)\cdot d_G(u)}{d_G(u)+d_G(u)}\\
&= \left[\frac{2\cdot 3}{2+3}\right]^{12n} \times \left[\frac{3\cdot 3}{3+3}\right]^{12n^2} \times \left[\frac{3\cdot 4}{3+4}\right]^{12n(n-1)}\\
&= 2^{36n^2-12n} \times 3^{24n^2} \times 7^{12n(n-1)}\\
XII(G) &= \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(u)}}\\
&= \left(\frac{1}{\sqrt{2+3}}\right)^{12n} \times \left(\frac{1}{\sqrt{3+3}}\right)^{12n^2} \times \left(\frac{1}{\sqrt{3+4}}\right)^{12n(n-1)}\\
&= 5^{-6n} \times 6^{-6n^2} \times 7^{6n(1-n)}\\
\chi^{II(G)} &= \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)\cdot d_G(u)}}\\
&= \left(\frac{1}{\sqrt{2\cdot 3}}\right)^{12n} \times \left(\frac{1}{\sqrt{3\cdot 3}}\right)^{12n^2} \times \left(\frac{1}{\sqrt{3\cdot 4}}\right)^{12n(n-1)}\\
&= 6^{-6n} \times 9^{-6n^2} \times 12^{6n(1-n)}\\
ABCII(G) &= \prod_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(u)-2}{d_G(u)\cdot d_G(u)}}\\
&= \left[\sqrt{\frac{2+3-2}{2\cdot 3}}\right]^{12n} \times \left[\sqrt{\frac{3+3-2}{3\cdot 3}}\right]^{12n^2} \times \left[\sqrt{\frac{3+4-2}{3\cdot 4}}\right]^{12n(n-1)}\\
&= 3^{6n-18n^2} \times 2^{6n} \times 5^{6n(n-1)}\\
GAII(G) &= \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)\cdot d_G(u)}}{d_G(u)+d_G(u)}\\
&= \left[\frac{2\sqrt{2\cdot 3}}{2+3}\right]^{12n} \times \left[\frac{2\sqrt{3\cdot 3}}{3+3}\right]^{12n^2} \times \left[\frac{2\sqrt{3\cdot 4}}{3+4}\right]^{12n(n-1)}\\
&= 2^{24n^2-6n} \times 3^{6n^2} \times 5^{-12n} \times 7^{12n(1-n)}\\
AGII(G) &= \prod_{uv\in E(G)} \frac{d_G(u)+d_G(u)}{2\sqrt{d_G(u)\cdot d_G(u)}}\\
&= \left[\frac{2+3}{2\sqrt{2\cdot 3}}\right]^{12n} \times \left[\frac{3+3}{2\sqrt{3\cdot 3}}\right]^{12n^2} \times \left[\frac{3+4}{2\sqrt{3\cdot 4}}\right]^{12n(n-1)}\\
&= 2^{6n-24n^2} \times 3^{-6n^2} \times 5^{12n} \times 7^{12n(n-1)}.
\end{aligned}$$

□

3 Results for $Dyck - 56_{n\times n}(A)$ network

Let G be the $Dyck - 56_{n\times n}(A)$ [21] network. It has $12n^2 + 4n$ vertices and $18n^2 + 2n$ edges. Edge partition of $Dyck - 56_{3\times 3}(A)$ network as shown in Table 3. Clearly, $\Delta(G) = 3$. Therefore

$c_u = \Delta(G) - d_G(u) + 1 = 4 - d_G(u)$. Thus there are three types of reverse edges as given in Table 2.1.

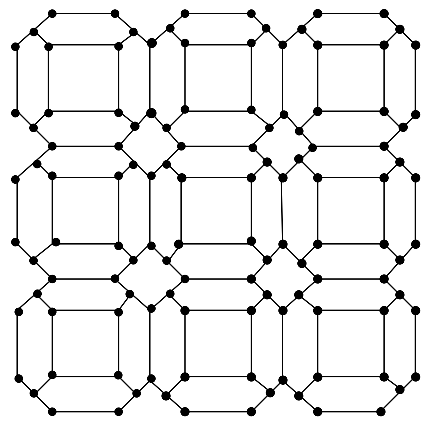

Fig. 2: $Dyck - 56_{3\times3}(A)$ network

Tab. 10: Edge partition of $Dyck - 56_{n\times n}(A)$ Network

$(d_G(u), d_G(v)); uv \in E(G)$	Number of edges
$(2,2)$	$4n$
$(2,3)$	8n
$(3,3)$	$18n^2 - 10n$

Tab. 11: Reverse edge partition of $Dyck - 56_{n\times n}(A)$ Network

$(c_u, c_v); uv \in E(G)$	Number of edges
$(2,2)$	$4n$
$(2,1)$	8n
(1,1)	$18n^2 - 10n$

Theorem 3.1. *Let G be the* $Dyck - 56_{n\times n}(A)$ *network, then its first reverse* $(a,b) - KA$ *index is equal to*

$$CKA^1_{a,b}(G) = 2^{b(a+1)+2}n + 8n(2^a + 1^a)^b + (18n^2 - 10n)2^b1^{ab}.$$

Proof. By Eq. (1.1) and Table 2.1, we get the desired result. □

Corollary 3.1. *Let G be the* $Dyck - 56_{n\times n}(A)$ *network, then its reverse Sombor index is equal to*

$$CSO(G) = n2^{\frac{7}{2}} + 8n\sqrt{5} + (18n^2 - 10n)\sqrt{2}.$$

Corollary 3.2. *Let G be the* $Dyck - 56_{n\times n}(A)$ *network, then its reverse modified Sombor index is equal to*

$$^{m}CSO(G) = n\sqrt{2} + \frac{8n}{\sqrt{5}} + \frac{18n^2 - 10n}{\sqrt{2}}.$$

Theorem 3.2. *Let G be the $Dyck - 56_{n\times n}(A)$ network, then its first reverse reduced $(a,b) - KA$ index is equal to*

$$CRKA^1_{a,b}(G) = 4n2^b1^{ab} + 8n1^{ab}.$$

Proof. By Eq. (1.2) and Table 2.1, we get the desired result. □

Corollary 3.3. *Let G be the $Dyck - 56_{n\times n}(A)$ network, then its reverse reduced Sombor index is equal to*

$$CRSO(G) = 4n(\sqrt{2} + 2).$$

Theorem 3.3. *Let G be the $Dyck - 56_{n\times n}(A)$ network, then its second reverse reduced $(a,b) - KA$ index is equal to*

$$CRKA^2_{a,b}(G) = 4n(1)^{2ab}.$$

Proof. By Eq. (1.3) and Table 2.1, we get the desired result. □

Theorem 3.4. *Let G be the $Dyck - 56_{n\times n}(A)$ network, then its sum connectivity reverse index is equal to*

$$SC(G) = \left(\frac{8\sqrt{2} - 2(9n-5)}{\sqrt{6}} + 2\right)n.$$

Proof. By Eq. (1.4) and Table 2.1, we get the desired result. □

Theorem 3.5. *Let G be the $Dyck - 56_{n\times n}(A)$ network, then its product connectivity reverse index is equal to*

$$PC(G) = \frac{8n}{\sqrt{2}} + 18n^2 - 8n.$$

Proof. By Eq. (1.5) and Table 2.1, we get the desired result. □

Theorem 3.6. *Let G be the $Dyck - 56_{n\times n}(A)$ network, then its atom bond connectivity reverse index is equal to*

$$ABCC(G) = \frac{12n}{\sqrt{2}}.$$

Proof. By Eq. (1.6) and Table 2.1, we get the desired result. □

Theorem 3.7. *Let G be the $Dyck - 56_{n\times n}(A)$ network, then its geometric arithmetic connectivity reverse index is equal to*

$$GAC(G) = \frac{16\sqrt{2}n}{3} + 18n^2 - 6n.$$

Proof. By Eq. (1.7) and Table 2.1, we get the desired result. □

Theorem 3.8. *For the $Dyck - 56_{n\times n}(A)$ network, then we have*

1. $ISIII(G) = 2^{18n(1-n)} \times 3^{2n(9n-1)} \times 5^{-8n}$
2. $XII(G) = 2^{n(1-9n)} \times 5^{-4n} \times 3^{5n-9n^2}$
3. $\chi^{II(G)} = 2^{-8n} \times 3^{3n(2-6n)}$
4. $ABCII(G) = 2^{2n(9n-8)} \times 3^{2n(5-9n)}$
5. $GAII(G) = 2^{12n} \times 3^{4n} \times 5^{-8n}$
6. $AGII(G) = 2^{-12n} \times 3^{-4n} \times 5^{8n}$.

Proof. Let G be the $Dyck - 56_{n\times n}(A)$ network. By Eqs. (1.8), (1.9), (1.10), (1.11), (1.12) and (1.13) and Table 3. We get the desired results. □

4 Results for $Dyck - 56_{n\times n}(C)$ network

Let G be the $Dyck - 56_{n\times n}(C)$ [21] network. It has $32n^2 - 8n$ vertices and $42n^2 - 14n$ edges. Edge partition of $Dyck - 56_{3\times 3}(C)$ network as shown in Table 3. Clearly, $\Delta(G) = 3$. Therefore $c_u = \Delta(G) - d_G(u) + 1 = 4 - d_G(u)$. Thus there are three types of reverse edges as given in Table 13.

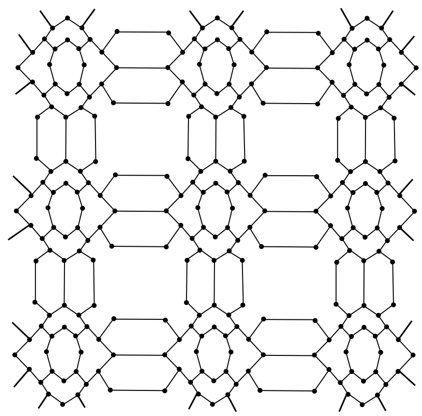

Fig. 3: $Dyck - 56_{3\times 3}(C)$ network

Tab. 12: Edge partition of $Dyck - 56_{n\times n}(C)$ Network

$(d_G(u), d_G(v)); uv \in E(G)$	Number of edges
$(2, 3)$	$16n^2$
$(3, 3)$	$22n^2 - 10n$
$(2, 2)$	$4n(n-1)$

Tab. 13: Reverse edge partition of $Dyck - 56_{n\times n}(C)$ Network

$(c_u, c_v); uv \in E(G)$	Number of edges
$(2, 1)$	$16n^2$
$(1, 1)$	$22n^2 - 10n$
$(2, 2)$	$4n(n-1)$

Theorem 4.1. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its first reverse $(a, b) - KA$ index is equal to*

$$CKA^1_{a,b}(G) = 16n^2(2^a + 1^a)^b + (22n^2 - 10n)(2^b 1^{ab}) + n(n-1)2^{b(a+1)+2}.$$

Proof. By Eq. (1.1) and Table 13, we get the desired result. □

Corollary 4.1. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its reverse Sombor index is equal to*

$$CSO(G) = 16n^2\sqrt{5} + (22n^2 - 10n)\sqrt{2} + n(n-1)2^{\frac{7}{2}}.$$

Corollary 4.2. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its reverse modified Sombor index is equal to*

$$^{m}CSO(G) = \frac{16n^2}{\sqrt{5}} + \frac{(22n^2 - 10n)}{\sqrt{2}} + n\sqrt{2(n-1)}.$$

Theorem 4.2. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its first reverse reduced $(a, b) - KA$ index is equal to*

$$CRKA^{1}_{a,b}(G) = 16n^2 1^{ab} + n(n-1)2^{b+2}1^{ab}.$$

Proof. By Eq. (1.2) and Table 13, we get the desired result. □

Corollary 4.3. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its reverse reduced Sombor index is equal to*

$$CRSO(G) = 16n^2 + n(n-1)2^{\frac{5}{2}}.$$

Theorem 4.3. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its second reverse reduced $(a, b) - KA$ index is equal to*

$$CRKA^{2}_{a,b}(G) = 4n(n-1)1^{2ab}.$$

Proof. By Eq. (1.3) and Table 13, we get the desired result. □

Theorem 4.4. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its sum connectivity reverse index is equal to*

$$SC(G) = \frac{16\sqrt{2}n^2 + \sqrt{3}(22n^2 - 10n)}{\sqrt{6}} + 2n(n-1).$$

Proof. By Eq. (1.4) and Table 13, we get the desired result. □

Theorem 4.5. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its product connectivity reverse index is equal to*

$$PC(G) = \frac{8n^2(2 + 3\sqrt{2})}{\sqrt{2}} - 12n.$$

Proof. By Eq. (1.5) and Table 13, we get the desired result. □

Theorem 4.6. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its atom bond connectivity reverse index is equal to*

$$ABCC(G) = \frac{20n^2 - 4n}{\sqrt{2}}.$$

Proof. By Eq. (1.6) and Table 13, we get the desired result. □

Theorem 4.7. *Let G be the $Dyck - 56_{n\times n}(C)$ network, then its geometric arithmetic connectivity reverse index is equal to*

$$GAC(G) = \frac{n^2(32\sqrt{2} + 78)}{3} - 14n.$$

Proof. By Eq. (1.7) and Table 13, we get the desired result. □

Theorem 4.8. *For the $Dyck - 56_{n\times n}(C)$ network, then we have*

1. $ISIII(G) = 2^{10n-6n^2} \times 3^{38n^2-10n} \times 5^{-16n^2}$
2. $XII(G) = 5^{-8n^2} \times 2^{9n-15n^2} \times 3^{5n-11n^2}$
3. $\chi^{II(G)} = 2^{4n(1-3n)} \times 3^{10n(1-3n)}$

4. $ABCII(G) = 2^{4n(3n-2)} \times 3^{2n(5-11n)}$

5. $GAII(G) = 2^{24n^2} \times 3^{8n^2} \times 5^{-16n^2}$

6. $AGII(G) = 2^{-24n^2} \times 3^{-8n^2} \times 5^{16n^2}$.

Proof. Let G be the $Dyck - 56_{n\times n}(C)$ network. By Eqs. (1.8), (1.9), (1.10), (1.11), (1.12) and (1.13) and Table 3. We get the desired results. □

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