On *ps-ro* fuzzy semi α-irresolute function

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Abstract

Here a class of function between two fuzzy topological spaces termed as *ps-ro* fuzzy semi α-irresolute function is introduced and is found to be independent of existing notion of fuzzy semi α-irresolute function and this motivates to study it in detail and also to use it as a tool to explore fuzzy topological spaces. Further, the interrelation of *ps-ro* fuzzy semi α-irresolute with already existing functions like *ps-ro* fuzzy semicontinuous, *ps-ro* fuzzy irresolute, *ps-ro* fuzzy α-irresolute functions etc. are obtained. It is also shown that the introduced function and *ps-ro* fuzzy continuity do not imply each other. Several characterizations as well as conditions for their existence are studied.

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1 Introduction

L.A Zadeh came up with the extension of the classical set known as fuzzy set [15], which was further used by C.L Chang to initiate and explored the notion of fuzzy topological space (in short, fts) [2]. Later, various concepts of fuzzy topology have been studied by different researchers. The idea of fuzzy semi α-irresolute was introduced by V. Seenivasan, Neyveli, G. Balasubramanian in 2007 [12].

A fts (X,τ) induces a family of topological spaces $(X, i_\alpha(\tau))$, $\forall\alpha \in I_1 = [0,1)$ called strong α-level topology on X, where $i_\alpha(\tau) = \{\mu^{-1}(\alpha,1] : \mu \in \tau\}$ [11]. In [8], it is observed that a strong α-level set μ^α need not be regular open if μ is fuzzy regular open. On the other hand, even the regular openness of μ^α, for all $\alpha \in I_1$ may fail to imply the fuzzy regularity of μ. This observation leads to the introduction of *ps-ro* fuzzy topology in [8] and opened a new direction to study fts. In *ps-ro* fuzzy topology, different type of functions between two fuzzy topological spaces such as *ps-ro* fuzzy continuous [9, 10], *ps-ro* fuzzy semicontinuous [3], *ps-ro* fuzzy irresolute [5], *ps-ro* fuzzy strongly α-irresolute function [6] and *ps-ro* fuzzy α-irresolute [7] etc. were introduced and explored.

Here, introducing *ps-ro* fuzzy semi α-irresolute function, we have shown that it is independent of the familiar notions of both fuzzy semi α-irresolute and *ps-ro* fuzzy continuity. Also, their interrelations with other parallel existing functions on *ps-ro* fuzzy topology are studied. Several characterizations of these functions are studied.

2 Preliminaries

On a nonempty set P, a fuzzy set A is a set of ordered pair $\{(a, \mu_A(a)) : a \in P\}$ where $\mu_A(a)$ is called membership function of A which maps elements of P to $[0,1] = I$. For our convenience, we denote the fuzzy set $\{(a, \mu_A(a)) : a \in P\}$ by $\{(a, A(a)) : a \in P\}$ or simply by A. If g is a mapping between two sets P and Q and U, V are fuzzy sets on P and Q respectively, then $1 - U$, $g^{-1}(V)$ and $g(U)$ are fuzzy sets respectively on P, P and Q and are given by $(1-U)(a) = 1 - U(a)$, $g^{-1}(V)(a) = V(g(a))\ \forall\ t \in P$ and $g(U)(t) = \begin{cases} \sup_{r \in g^{-1}(t)} U(r), when g^{-1}(t) \neq \emptyset \\ 0,\ otherwise \end{cases}$. Any fuzzy sets U, V on A, U is subset of V if $U(t) \leq V(t)\ \forall\ t \in A$ and is written as $U \leq V$. A fuzzy set x_r is called fuzzy point whose value is $r \in (0,1]$ at x, otherwise the value is 0, also it is q-coincident to a fuzzy set U if $r + U(x) > 1$ and is denoted by $x_r q U$ [15].
A fts (U, σ) is a pair where σ is a collection of some fuzzy sets on a set U such that $0, 1 \in \sigma$ and finite intersection and arbitrary union of members of σ belongs to σ [2]. A set B is called regular open if $int(clB) = B$, where B is a subset of a topological space [14]. A fuzzy set P defined on fts (U, σ) is fuzzy regular open for $P = int(cl(P))$ [1]. For a fts (U, σ), the collection $i_\alpha(\sigma) = \{A^\alpha : A \in \sigma$ and $\alpha \in [0, 1) = I_1$, where $A^\alpha = \{s \in U; A(s) > \alpha\}$, is a topology on U named as strong α-level topology. A^α for being regular open in $(U, i_\alpha(\sigma))$, $\forall\ \alpha \in I_1$, the fuzzy open set A on fts (U, σ) is called pseudo regular open fuzzy set, the collection of which generates a fuzzy topology on U, named as *ps-ro* fuzzy topology on U, the elements of which are termed as *ps-ro* open fuzzy sets and as usual complements as *ps-ro* closed fuzzy sets [8, 9, 10]. Fuzzy *ps*-interior of a fuzzy set P in the fts (U, σ), written as *ps-int*(P) is the biggest *ps-ro* open fuzzy set on U that is subset of P also its fuzzy *ps*-closure written as *ps-cl*(P) is the tiniest *ps-ro* closed fuzzy set that contains P [9, 10]. A fuzzy set P on a fts (U, σ) is known as fuzzy α-open [13](resp. fuzzy α-closed [13], *ps-ro* semiopen [3], *ps-ro* semiclosed [3], *ps-ro* α-open [4], *ps-ro* α-closed [4]) on U if $P \leq int(cl(int(P)))$ (resp. $P \geq cl(int(cl(P)))$, $P \leq ps\text{-}cl(ps\text{-}int(P))$, $ps\text{-}int(ps\text{-}cl(P)) \leq P$, $P \leq ps\text{-}int(ps\text{-}cl(ps\text{-}int(P)))$, $P \geq ps\text{-}cl(ps\text{-}int(cl(P))))$. Also, P is *ps-ro* fuzzy dense, nowhere *ps-ro* fuzzy dense respectively for $ps\text{-}cl(P) = 1$ and $ps\text{-}int(ps\text{-}cl(P)) = 0$ [7] and P is called *ps-ro* fuzzy semi-nbd of x_t, if we get Q, a *ps-ro* semiopen fuzzy set satisfying $x_t \in Q \leq P$ [3]. Let us denote by $ps\text{-}(r, U_\sigma)$, $ps\text{-}(o, U_\sigma)$, $ps\text{-}(s, U_\sigma)$ and $ps\text{-}(\alpha, U_\sigma)$ respectively for the set of all pseudo regular open, *ps-ro* open, *ps-ro* semiopen, *ps-ro* α-open fuzzy sets on the fts (U, σ) and also by $ps\text{-}(r^c, U_\sigma)$, $ps\text{-}(o^c, U_\sigma)$, $ps\text{-}(s^c, U_\sigma)$ and $ps\text{-}(\alpha^c, U_\sigma)$ respectively for the set of all pseudo regular closed, *ps-ro* closed, *ps-ro* semiclosed, *ps-ro* α-closed fuzzy sets on the fts (U, σ). In the line of *ps-int* and *ps-cl*, similar concepts of *ps*-semi closure(*ps-scl*), *ps*-semi interior(*ps-sint*) and *ps*-α closure(*ps*-α*cl*), *ps*-α interior(*ps*-α*int*) operators are defined [3, 4].

A function g between two fts U and V is termed as:
(i) fuzzy semi α-irresolute [12](resp. *ps-ro* fuzzy continuous [10], *ps-ro* fuzzy semicontinuous [3], *ps-ro* fuzzy irresolute [5], *ps-ro* fuzzy α-irresolute [7]) if $g^{-1}(Q)$ is semiopen fuzzy set (resp. *ps-ro* open, *ps-ro* semiopen, *ps-ro* semiopen, *ps-ro* α-open fuzzy set) on U for any α-open fuzzy set (resp. *ps-ro* open, *ps-ro* open, *ps-ro* semiopen, *ps-ro* α-open fuzzy set) Q on V.

3 *ps-ro* fuzzy semi α-irresolute function

Definition 3.1. A mapping g from a fts (U, σ_1) to fts (V, σ_2) is termed as *ps-ro* fuzzy semi α-irresolute if inverse image of every *ps-ro* α-open V under g is *ps-ro* semiopen on U.

We shall establish that *ps-ro* fuzzy semi α-irresolute function is independent of the existing notion of fuzzy semi α-irresolute function and this motivates us to investigate this new concept further.

Let us consider two sets $P = \{e, b, s, d\}$ and $Q = \{w, i, m, z\}$. Let us take fuzzy sets K, L, M and N on P given by $K(e) = 0.2, K(b) = 0.3, K(s) = 0.3, K(d) = 0.2$; $L(t) = 0.3\ \forall\ r \in P$; $M(r) = 0.1\ \forall\ r \in P$ and $N(r) = 0.6\ \forall\ r \in P$. Let U, V, W and G be the fuzzy sets on Q given by $U(r) = 0.4\ \forall\ r \in Q$; $V(w) = 0.6, V(i) = 0.6, V(m) = 0.5, V(z) = 0.5$; $W(r) = 0.1\ \forall\ r \in Q$ and $G(w) = 0.7, G(i) = 0.8, G(m) = 0.8, G(z) = 0.7$. Then $\sigma_1 = \{0, 1, K, L, M, N\}$ and $\sigma_2 = \{0, 1, U, V, W, G\}$ are fuzzy topologies on P and Q respectively. In the correspondent strong α-level topological space $(P, i_\alpha(\sigma_1))$, $\forall\ \alpha \in I_1 = [0, 1)$, the open sets are $\phi, P, K^\alpha, L^\alpha, M^\alpha$ and

N^α where $K^\alpha = \begin{cases} P, & \text{if } \alpha < 0.2 \\ \{b, s\}, & \text{if } 0.2 \leq \alpha < 0.3 \\ \phi, & \text{if } \alpha \geq 0.3 \end{cases}$, $L^\alpha = \begin{cases} P, & \text{if } \alpha < 0.3 \\ \phi, & \text{if } \alpha \geq 0.3 \end{cases}$, $M^\alpha = \begin{cases} P, & \text{if } \alpha < 0.1 \\ \phi, & \text{if } \alpha \geq 0.1 \end{cases}$ *and*
$N^\alpha = \begin{cases} P, & \text{if } \alpha < 0.6 \\ \phi, & \text{if } \alpha \geq 0.6 \end{cases}$
For $0.2 \leq \alpha < 0.3$, $int(cl(K^\alpha)) = P$ which implies that K^α fails to be regular open on $(P, i_\alpha(\sigma_1))$ and so K fails to be pseudo regular open fuzzy set on (P, σ_1) i.e. $K \notin ps\text{-}(r, P_{\sigma_1})$. Again, $int(cl(L^\alpha)) = L^\alpha$, $int(cl(M^\alpha)) = M^\alpha$ and $int(cl(N^\alpha)) = N^\alpha$, $\forall \alpha \in I_1$. This implies that L^α, M^α and N^α are regular open on $(P, i_\alpha(\sigma_1))$, $\forall \alpha \in I_1$. Hence, $0, 1, L, M$ and N are pseudo regular open on (P, σ_1) i.e. $0, 1, L, M, N \in ps\text{-}(r, P_{\sigma_1})$ and ps-ro fuzzy topology on P is $\{L, M, N, 1, 0\}$. Also, in the corresponding strong α-level topological space $(Q, i_\alpha(\sigma_2))$, $\forall \alpha \in I_1$, the open sets are $\phi, Q, U^\alpha, V^\alpha, W^\alpha$ and G^α where $U^\alpha = \begin{cases} Q, & \text{if } \alpha < 0.4 \\ \phi, & \text{if } \alpha \geq 0.4 \end{cases}$, $V^\alpha = \begin{cases} Q, & \text{if } \alpha < 0.5 \\ \{w, i\}, & \text{if } 0.5 \leq \alpha < 0.6 \\ \phi, & \text{if } \alpha \geq 0.6 \end{cases}$,
$W^\alpha = \begin{cases} Q, & \text{if } \alpha < 0.1 \\ \phi, & \text{if } \alpha \geq 0.1 \end{cases}$ *and* $G^\alpha = \begin{cases} Q, & \text{if } \alpha < 0.7 \\ \{i, m\}, & \text{if } 0.7 \leq \alpha < 0.8 \\ \phi, & \text{if } \alpha \geq 0.8 \end{cases}$. *$V^\alpha$ and G^α are not regular open in $(Q, i_\alpha(\sigma_2))$, since $int(cl(V^\alpha)) = Q$ for $0.5 \leq \alpha < 0.6$ and $int(cl(G^\alpha)) = Q$ for $0.7 \leq \alpha < 0.8$. Thus $V, G \notin ps\text{-}(r, Q_{\sigma_2})$. Again, $int(cl(U^\alpha)) = U^\alpha$ and $int(cl(W^\alpha)) = W^\alpha$, for all $\alpha \in I_1$. Thus U^α and W^α are regular open on $(Q, i_\alpha(\sigma_2))$ for all $\alpha \in I_1$. Hence $\{0, U, W, 1\}$ is ps-ro fuzzy topology on Q.*
Let h be a function from P to Q defined by $h(e) = w, h(b) = i, h(c) = m, h(d) = z$. As U, $W \in ps\text{-}(o, Q_{\sigma_2})$ we have U, $W \in ps\text{-}(\alpha, Q_{\sigma_2})$. Here, $h^{-1}(U)(r) = 0.4$, $\forall r \in P$ and $h^{-1}(U) \leq ps\text{-}cl(ps\text{-}int(h^{-1}(U)))$, thus $h^{-1}(U) \in ps\text{-}(s, P_{\sigma_1})$. Again, $h^{-1}(W)(r) = 0.1$, $\forall r \in P$ and $h^{-1}(W) \leq ps\text{-}cl(ps\text{-}int(h^{-1}(W)))$, thus $h^{-1}(W) \in ps\text{-}(s, P_{\sigma_1})$ and $ps\text{-}(\alpha, Q_{\sigma_2}) = \{0, U, W, V, 1\}$, where $W \leq V \leq U$. Here, for any $A \in ps\text{-}(\alpha, Q_{\sigma_2})$, $h^{-1}(A) \in ps\text{-}(s, P_{\sigma_1})$ but G is fuzzy α-open on Q and $h^{-1}(G)$ is not fuzzy semiopen on P. Hence h is ps-ro fuzzy semi α-irresolute but not fuzzy semi α-irresolute.

Consider two sets $P = \{e, b, c, d\}$ and $Q = \{w, i, j, z\}$. Let K, L, M, N and S be the fuzzy sets on P given by $K(e) = 0.1, K(b) = 0.1, K(c) = 0.2, K(d) = 0.2$; $L(e) = 0.2, L(b) = 0.2, L(c) = 0.3, L(d) = 0.3$; $M(r) = 0.1$ $\forall r \in P$; $N(r) = 0.4$ $\forall r \in P$ and $S(r) = 0.8$ $\forall r \in P$. Consider the fuzzy sets U, V, W, I and R on Q given by $U(w) = 0.1, U(i) = 0.1, U(j) = 0.2, U(z) = 0.2$; $V(w) = 0.2, V(i) = 0.2, V(j) = 0.3, V(z) = 0.3$; $W(r) = 0.2$ $\forall r \in Q$; $I(r) = 0.4$ $\forall r \in Q$ and $R(r) = 0.6$ $\forall r \in Q$.
Clearly, (P, σ_1) and (Q, σ_2) are fts where $\sigma_1 = \{0, K, L, M, N, S, 1\}$ and $\sigma_2 = \{0, U, V, W, I, R, 1\}$. K and L fails to be pseudo regular open on P respectively for $\alpha \in [.1, .2)$ and $\alpha \in [.2, .3)$. Hence, ps-ro fuzzy topology on P is $\{1, M, N, S, 0\}$. Now, U and V fails to be pseudo regular open on Q for $\alpha \in [0.1, .2)$ and $\alpha \in [0.2, .3)$ respectively. Thus, on Q, $\{1, W, I, R, 0\}$ is the ps-ro fuzzy topology.
Let h be a function between (P, σ_1) to (Q, σ_2) defined as $h(e) = w, h(b) = i, h(c) = j$ and $h(d) = z$. It is easy to verify that $h^{-1}(B)$ is fuzzy semiopen on P for any fuzzy α-open set B on Q. Therefore, h is fuzzy semi α-irresolute but $\forall A \in ps\text{-}(\alpha, Q_{\sigma_2})$ satisfying $W < A < I$, $h^{-1}(A) \notin ps\text{-}(s, P_{\sigma_1})$, proving h is not ps-ro fuzzy semi α-irresolute.

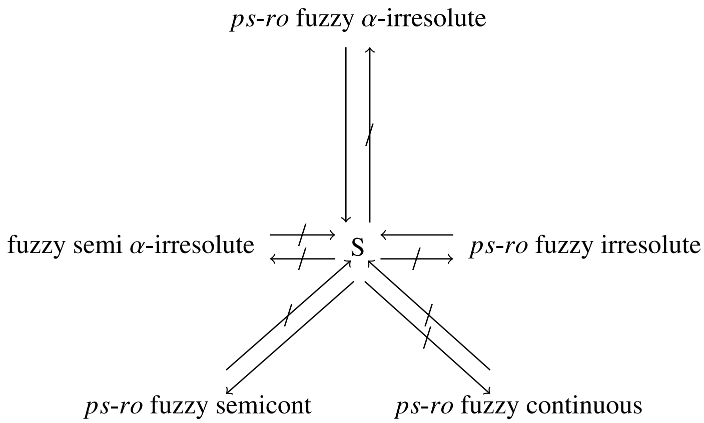
Remark 3.1. From Example (3) and Example (3), we can conclude that fuzzy semi α-irresolute and *ps-ro* fuzzy semi α-irresolute donot imply each other.

Remark 3.2. Evidently, every *ps-ro* fuzzy α-irresoluteness as well as *ps-ro* fuzzy irresoluteness imply *ps-ro* fuzzy semi α-irresoluteness. However, converses are not true are shown below:
In Example (3), $S \in ps\text{-}(\alpha, Q_{\sigma_2})$ whereas $h^{-1}(S) \notin ps\text{-}(\alpha, Q_{\sigma_2})$ and hence h fails to be *ps-ro* fuzzy α-irresolute but h happens to be *ps-ro* fuzzy semi α-irresolute.
In Example (3), $U \in ps\text{-}(s, Q_{\sigma_2})$ but $h^{-1}(U) \notin ps\text{-}(s, P_{\sigma_1})$, h fails to be *ps-ro* fuzzy irresolute whereas h is *ps-ro* fuzzy semi α-irresolute.

Remark 3.3. From the definition it is clear that every *ps-ro* fuzzy semi α-irresoluteness is *ps-ro* fuzzy semicontinuity but not the converse which is given below:
In Example (3), $\forall\, T \in ps\text{-}(o, Q_{\sigma_2})$, $h^{-1}(T) \in ps\text{-}(s, P_{\sigma_1})$. Thus, h is *ps-ro* fuzzy semicontinuous but fails to be *ps-ro* fuzzy semi α-irresolute, which concludes that *ps-ro* fuzzy semicontinuity do not imply *ps-ro* fuzzy semi α-irresolute.

Remark 3.4. Again, *ps-ro* fuzzy semicontinuous function does not imply *ps-ro* fuzzy continuous function[3], hence *ps-ro* fuzzy semi α-irresoluteness and *ps-ro* fuzzy continuity does not imply each other.

All above relations can be put in the following diagram:



Here, S denots *ps-ro* fuzzy semi α-irresolute.

Further, *ps-ro* fuzzy continuity and fuzzy continuity being independent of each other, *ps-ro* fuzzy semi α-irresolute is also independent of fuzzy continuity.
Now we shall discuss several properties of *ps-ro* fuzzy semi α-irresolute function.

Theorem 3.1. For any function h between fts (U, σ_1) and (V, σ_2), the following are equivalent.
(a) h is *ps-ro* fuzzy semi α-irresolute.
(b) $\forall\, Q \in ps\text{-}(\alpha^c, V_{\sigma_2})$, $h^{-1}(Q) \in ps\text{-}(s^c, U_{\sigma_1})$.
(c) $\forall x_t \in U$ and $Q \in ps\text{-}(\alpha, V_{\sigma_2})$ and $h(x_t) \in Q$, $\exists P \in ps\text{-}(s, U_{\sigma_1})$ satisfying $x_t \in P$ and $h(P) \leq Q$.
(d) $ps\text{-}int(ps\text{-}cl(h^{-1}(Q))) \leq h^{-1}(ps\text{-}\alpha cl(Q))$, for any fuzzy set Q on V.
(e) $h(ps\text{-}int(ps\text{-}cl(P))) \leq ps\text{-}\alpha cl(h(P))$, $\forall$ fuzzy set P on U.

Proof. $(a) \Rightarrow (b)$ $\forall Q \in ps\text{-}(\alpha^c, V_{\sigma_2})$, $h^{-1}(1-Q) \in ps\text{-}(s, U_{\sigma_1})$. Since $1 - h^{-1}(Q) = h^{-1}(1-Q)$, $h^{-1}(Q) \in ps\text{-}(s^c, U_{\sigma_1})$.
$(b) \Rightarrow (a)$ $\forall Q \in ps\text{-}(\alpha, V_{\sigma_2})$, $h^{-1}(1-Q) = 1 - h^{-1}(Q) \in ps\text{-}(s^c, U_{\sigma_1})$, which implies that $h^{-1}(Q) \in ps\text{-}(s, U_{\sigma_1})$, proving h is *ps-ro* fuzzy semi α-irresolute.
$(a) \Rightarrow (c)$ $\forall x_t \in U$ and $Q \in ps\text{-}(\alpha, V_{\sigma_2})$ with $h(x_t) \in Q$, $h^{-1}(Q) \in ps\text{-}(s, U_{\sigma_1})$ and containing x_t. The result holds taking $h^{-1}(Q) = P$.
$(c) \Rightarrow (a)$ $\forall Q \in ps\text{-}(\alpha, V_{\sigma_2})$, for if $h^{-1}(Q) = 0$, then proved. Now, for $h^{-1}(Q) \neq 0$, $\exists\, x_t$ on $h^{-1}(Q)$ and $U_{x_t} \in ps\text{-}(s, U_{\sigma_1})$ containing x_t and satisfying $x_t \in U_{x_t} \leq h^{-1}(Q)$. x_t being arbitrary, taking union we get $h^{-1}(Q) = \vee\{x_t : x_t \in h^{-1}(Q)\} \leq \vee\{U_{x_t} : x_t \in h^{-1}(Q)\} \leq h^{-1}(Q)$. This shows that $h^{-1}(Q) = \vee\{U_{x_t} : x_t \in h^{-1}(Q)\}$, so $h^{-1}(Q) \in ps\text{-}(s, U_{\sigma_1})$. Hence, h is *ps-ro* fuzzy semi α-irresolute.
$(b) \Rightarrow (d)$ $ps\text{-}\alpha cl(Q) \in ps\text{-}(\alpha^c, V_{\sigma_2})$ for every fuzzy set Q on V and $h^{-1}(ps\text{-}\alpha cl(Q)) \in ps\text{-}(s^c, U_{\sigma_1})$. Thus, $ps\text{-}int(ps\text{-}cl(h^{-1}(ps\text{-}\alpha cl(Q)))) \leq h^{-1}(ps\text{-}\alpha cl(Q))$. So, $ps\text{-}int(ps\text{-}cl(h^{-1}(Q))) \leq h^{-1}(ps\text{-}\alpha cl(Q))$, as $P \leq ps\text{-}\alpha cl(P)$, $\forall$ fuzzy set P on U.
$(d) \Rightarrow (e)$ Forall fuzzy set P on U set $h(P) = Q$. Now $ps\text{-}int(ps\text{-}cl(h^{-1}(Q))) \leq h^{-1}(ps\text{-}\alpha cl(Q))$.

So ps-$int(ps$-$cl(P))) \leq h^{-1}(ps$-$\alpha cl(Q)) = h^{-1}(ps$-$\alpha cl(h(P)))$. Now $h(ps$-$int(ps$-$cl(P))) \leq h(h^{-1}(ps$-$\alpha cl(h(P)))) \leq ps$-$\alpha clh(P)$. Hence, $h(ps$-$int(ps$-$cl(P)) \leq ps$-$\alpha cl(h(P))$, $\forall$ fuzzy set P on U.
$(e) \Rightarrow (b)$ For any $Q \in ps$-(α^c, V_{σ_2}), taking $P = h^{-1}(Q)$, $h(ps$-$int(ps$-$cl(P))) \leq ps$-$\alpha cl(h(P)) \leq ps$-$\alpha cl(Q) = Q$.
Now $h^{-1}(h(ps$-$int(ps$-$cl(P)))) \leq h^{-1}(Q)$. ps-$int(ps$-$cl(P)) \leq h^{-1}(Q)$. So ps-$int(ps$-$cl(h^{-1}(Q))) \leq h^{-1}(Q)$. Hence, ps-$int(ps$-$cl(h^{-1}(Q))) \leq h^{-1}(Q)$, showing $h^{-1}(Q) \in ps$-(s^c, U_{σ_1}). □

Theorem 3.2. Let (U, σ_1), (V, σ_2) and (W, σ_3) be three fts and $h : U \to V$ and $i : V \to W$ be two functions then the following holds:
(a) $i \circ h$ is *ps-ro* fuzzy semi α-irresolute if i is *ps-ro* fuzzy semi α-irresolute and h is *ps-ro* fuzzy irresolute.
(b) $i \circ h$ is *ps-ro* fuzzy semi α-irresolute if h is *ps-ro* fuzzy semicontinuous and i is *ps-ro* fuzzy strongly α-irresolute.

Proof. (a) For any $P \in ps$-(α, W_{σ_3}), $i^{-1}(P) \in ps$-(s, V_{σ_2}). Under the given conditions, $h^{-1}(i^{-1}(P)) \in ps$-(s, U_{σ_1}). Hence, $i \circ h$ is *ps-ro* fuzzy semi α-irresolute.
(b) For any $P \in ps$-(α, W_{σ_3}), $i^{-1}(P) \in ps$-(o, V_{σ_2}). Now $(i \circ h)^{-1}(P) = h^{-1}(i^{-1}(P))$. As $i^{-1}(P) \in ps$-(o, V_{σ_2}) and h a *ps-ro* fuzzy semicontinuous, we can conclude that $h^{-1}(i^{-1}(P)) \in ps$-(s, U_{σ_1}). Hence $i \circ h$ is *ps-ro* fuzzy semi α-irresolute. □

Theorem 3.3. Let (U, σ_1) and (V, σ_2) be two fts and g be a *ps-ro* fuzzy semi α-irresolute function between them. Then $g^{-1}(P) \in ps$-(s^c, U_{σ_1}) where P is a nowhere *ps-ro* fuzzy dense set on V.

Proof. For every nowhere *ps-ro* fuzzy dense set P on V, ps-$int(ps$-$cl(P)) = 0$. So, ps-$cl(1 - (ps$-$cl(P))) = 1$. Now ps-$cl(ps$-$int(1-P)) = 1$ and ps-$int(ps$-$cl(ps$-$int(1-P)) = 1$. Hence $(1-P) \leq ps$-$int(ps$-$cl(ps$-$int(1 - P)))$ showing $(1 - P) \in ps$-(α, V_{σ_2}). Thus, $g^{-1}(1 - P) \in ps$-(s, U_{σ_1}). Since $g^{-1}(1 - P) = 1 - g^{-1}(P)$, $g^{-1}(P) \in ps$-(s^c, U_{σ_1}). □

Theorem 3.4. ps-$int(ps$-$cl(g^{-1}(Q))) \leq g^{-1}(ps$-$cl(Q))$ and ps-$scl(g^{-1}(Q)) \leq g^{-1}(ps$-$cl(Q))$, $\forall$ fuzzy set Q on V, where g *ps-ro* fuzzy semi α-irresolute between two fts (U, σ_1) and (V, σ_2).

Proof. Let g be a *ps-ro* fuzzy semi α-irresolute. For every fuzzy set Q on V, ps-$cl(Q) \in ps$-(o^c, V_{σ_2}) and so ps-$cl(Q) \in ps$-(α^c, V_{σ_2}). Hence, $g^{-1}(ps$-$cl(Q)) \in ps$-$cl(Q) \in ps$-(s^c, U_{σ_1}).
By the definition, ps-$int(ps$-$cl(g^{-1}(ps$-$cl(Q)))) \leq g^{-1}(ps$-$cl(Q))$ which implies that ps-$int(ps$-$cl(g^{-1}(Q)) \leq g^{-1}(ps$-$cl(Q))$, since $Q \leq ps$-$cl(Q)$ for any fuzzy set Q on U. Again, $Q \leq ps$-$cl(Q)$. So $g^{-1}(Q) \leq g^{-1}(ps$-$cl(Q))$. Now ps-$scl(g^{-1}(Q)) \leq ps$-$scl(g^{-1}(ps$-$cl(Q))) = g^{-1}(ps$-$cl(Q))$. Hence, ps-$scl(g^{-1}(Q)) \leq g^{-1}(ps$-$cl(Q))$. □

Theorem 3.5. A mapping g between fts (U, σ_1) and (V, σ_2) is *ps-ro* fuzzy semi α-irresolute iff $g^{-1}(ps$-$\alpha int(Q)) \leq ps$-$sint(g^{-1}(Q))$ $\forall$ fuzzy set Q of V.

Proof. Let Q be fuzzy set on V and g be *ps-ro* fuzzy semi α-irresolute. ps-$\alpha int(Q) \in ps$-(α, V_{σ_2}), $g^{-1}(ps$-$\alpha int(Q)) \in ps$-(s, U_{σ_1}). $g^{-1}(ps$-$\alpha int(Q)) = ps$-$sint(g^{-1}(ps$-$\alpha int(Q))) \leq ps$-$sint(g^{-1}(Q))$. This proves the result.
Conversely, $Q = ps$-$\alpha int(Q)$ and $g^{-1}(Q) = g^{-1}(ps$-$\alpha int(Q)) \leq ps$-$sint(g^{-1}(Q)) \leq g^{-1}(Q)$, for every $Q \in ps$-(α, V_{σ_2}). So $g^{-1}(Q) = ps$-$sint(g^{-1}(Q))$ and thus $g^{-1}(Q) \in ps$-(s, U_{σ_1}), which proves that g is *ps-ro* fuzzy semi α-irresolute function. □

Theorem 3.6. A bijective mapping g between two fts (U, σ_1) and (V, σ_2) is *ps-ro* fuzzy semi α-irresolute function iff ps-$\alpha int(g(P)) \leq g(ps$-$sint(P))$ for any fuzzy set P of U.

Proof. Let g be bijective and *ps-ro* fuzzy semi α-irresolute function. For every fuzzy set P on U, $g^{-1}(ps$-$\alpha int(g(P))) \in ps$-(s, U_{σ_1}). Therefore, $g^{-1}(ps$-$\alpha int(g(P))) \leq ps$-$sint(g^{-1}(g(P)))$ [By Theorem (3.5), taking $g(P) = Q$]. So $g^{-1}(ps$-$\alpha int(g(P))) \leq ps$-$sint(P)$. Now $g(g^{-1}(ps$-$\alpha int(g(P)))) \leq g(ps$-$sint(P))$. So ps-$\alpha int(g(P)) \leq g(ps$-$sint(P))$. Conversely, for $Q \in ps$-

(α, V_{σ_2}), $Q = ps\text{-}\alpha int(Q) = ps\text{-}\alpha int(g(g^{-1}(Q))) \leq g(ps\text{-}sint(g^{-1}(Q)))$. Thus $g^{-1}(Q) \leq g^{-1}(g(ps\text{-}sint(g^{-1}(Q))))$. $g^{-1}(Q) \leq ps\text{-}sint(g^{-1}(Q)) \leq g^{-1}(Q)$. So $g^{-1}(Q) = ps\text{-}sint(g^{-1}(Q))$. Hence, $g^{-1}(Q) \in ps\text{-}(s, U_{\sigma_1})$. □

Theorem 3.7. For a functions g between two fts (U, σ_1) and (V, σ_2), we have the following equivalent statements:
(a) g is *ps-ro* fuzzy semi α-irresolute.
(b) $\forall$ x_t of U and any *ps-ro* fuzzy α-nbd B of $g(x_t)$ on V, its inverse under g is *ps-ro* fuzzy semi-nbd of x_t on U.
(c) $\forall$ x_t of U and *ps-ro* fuzzy α-nbd B of $g(x_t)$ on V, $\exists$ a *ps-ro* fuzzy semi-nbd A of x_t on U with $g(A) \leq B$.
(d) $\forall$ x_t on U and $B \in ps\text{-}(\alpha, V_{\sigma_2})$ with $g(x_t) \leq B$, $\exists$ $A \in ps\text{-}(s, U_{\sigma_1})$ with $x_t \leq A$ and $g(A) \leq B$.

Proof. (a) $\Rightarrow$ (b) Take g to be *ps-ro* fuzzy semi α-irresolute. For any x_t and *ps-ro* fuzzy α-nbd B of $g(x_t)$ on V, there is a $P \in ps\text{-}(\alpha, V_{\sigma_2})$ with $g(x_t) \leq P \leq B$. Now, $g^{-1}(P) \in ps\text{-}(s, U_{\sigma_1})$, so $x_t \leq g^{-1}(P) \leq g^{-1}(B)$ showing $g^{-1}(B)$ is *ps-ro* fuzzy semi-nbd of x_t on U.
(b) $\Rightarrow$ (c) For every x_t on U and every *ps-ro* fuzzy α-nbd B of $g(x_t)$ on V, $g^{-1}(B)$ is a *ps-ro* fuzzy semi-nbd of x_t on U. Let $g^{-1}(B) = A$. Then $g(g^{-1}(B)) = g(A)$. But $g(g^{-1}(B)) \leq B$. Hence, $g(A) \leq B$.
(c) $\Rightarrow$ (d) For any x_t on U and $B \in ps\text{-}(\alpha, V_{\sigma_2})$ with $g(x_t) \leq B$, $\exists$ a *ps-ro* fuzzy semi-nbd P of x_t on U, such that $g(P) \leq B$. So, $\exists$ $A \in ps\text{-}(s, U_{\sigma_1})$ with $x_t \leq A \leq P$ which gives $g(x_t) \leq g(A) \leq g(P) \leq B$. Hence, $g(A) \leq B$.
(d) $\Rightarrow$ (a) Let $B \in ps\text{-}(\alpha, V_{\sigma_2})$ and x_t be a fuzzy point on $g^{-1}(B)$. Then $x_t \leq g^{-1}(B) \Rightarrow g(x_t) \leq g(g^{-1}(B)) \leq B$. Thus, $\exists$ $A \in ps\text{-}(s, U_{\sigma_1})$ with $x_t \leq A$ and $g(A) \leq B$. Then, $A \leq g^{-1}(B)$. So, $x_t \leq A \leq ps\text{-}cl(ps\text{-}int(A)) \leq ps\text{-}cl(ps\text{-}int(g^{-1}(B)))$. x_t being arbitrary and $g^{-1}(B)$ is the union of all fuzzy points in $g^{-1}(B)$, we have $g^{-1}(B) = \vee\{x_t : x_t \in g^{-1}(B)\} \leq \vee\{A : x_t \in g^{-1}(B)\} \leq g^{-1}(B)$. So, $\vee\{A : x_t \in g^{-1}(B)\} = g^{-1}(B)$, showing $g^{-1}(B) \in ps\text{-}(s, U_{\sigma_1})$. Thus, g is *ps-ro* fuzzy semi α-irresolute. □

Lemma 3.1. *[1]:* Let $h : U \to (U \times V)$ be the graph of a function $r : (U, \sigma_1) \to (V, \sigma_2)$ given by $h(x) = (x, r(x))$. If P and Q are fuzzy sets on U and V respectively, then $h^{-1}(P \times Q) = P \wedge r^{-1}(Q)$.

Theorem 3.8. Let $r : (U, \sigma_1) \to (V, \sigma_2)$ be a function and $h : U \to (U \times V)$ be the graph of r. If h is *ps-ro* fuzzy semi α-irresolute, then r is also so.

Proof. Let $Q \in ps\text{-}(\alpha, V_{\sigma_2})$. By Lemma (3.1), $r^{-1}(Q) = 1 \wedge r^{-1}(Q) = h^{-1}(1 \times Q)$. As, $(1 \times Q)$ is *ps-ro* α-open on $(U \times V)$, $h^{-1}(1 \times Q) \in ps\text{-}(s, U_{\sigma_1})$. So, $r^{-1}(Q) \in ps\text{-}(s, U_{\sigma_1})$. Therefore, r is *ps-ro* fuzzy semi α-irresolute. □

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