Application of I-Function of several Complex Variables in Electric Circuit Theory

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Abstract

In this article, an electric circuit consisting of a source of an electromotive force $E_0P(t)$, inductance, resistance, and capacitance has been considered and the value of charge at any time is obtained, when it is taken in terms of the multivariable $I-$Function. The particular cases of $I-$Function are quite general in nature as they include a number of elementary functions. The results obtained may be applied to a variety of elementary functions which frequently occurs in Mathematical Physics and Engineering.

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1 Introduction and Preliminaries

Special functions and their applications play a significant role in many engineering and science problems. The $H-$function defined by Fox [1], which is well recognized as a generalization of Meijer's $G-$function and Wright's generalized hypergeometric function [2], does attract many researchers, mathematicians, and statisticians.

For the properties and applications of Fox's $H-$fun-ctions, we refer [3-8] to the readers. B. Satyanarayana et. al. [9], established the integrals involving the product of two general classes of polynomials, the H-function of one variable and the H-function of 'r' variables, and derived a large number of integrals involving simpler functions and polynomials as their particular cases. P. Agarwal et al [10], obtained a certain new integral which reduces Fox's $H-$function and generalized wright hypergeometric function, etc.

In 1997, Rathie introduced a new function [11] namely the "I-function", which has wide applications in Mathematics, Physics, and other branches of applied mathematics. This newly defined function contains many important functions such as the Riemann zeta functions, polylogarithms, etc. which were not described by H-functions. It also describes the exact partitions of the Gaussian free energy model from statistical mechanics, Feynmann integrals, and many other useful functions that are being used in testing hypotheses as a special case in Statistics.

In 2012, a very useful application of I-function has been found in wireless communication. Ansari, et. al. [12, 13] developed a Mathematica@ implementation of the I-function to obtain the numerical results of their research. Recently, Mohan and Kumar showed some important applications of I-Functions [14,15].

In 1986, Prasad [6], introduced a new function in the field of special functions, known as the I-function of several variables which is denoted and represented as follows-

(1.1)

$$I[z_1,\cdots,z_r] = I^{0,n_2:0,n_3:\cdots:0,n_r:(m',n');\cdots,(m^{(r)},n^{(r)})}_{p_2,q_2:p_3,q_3:\cdots:p_r,q_r:[p',q'];\cdots;[p^{(r)},q^{(r)}]}\left[z_1,\cdots,z_r\left|\begin{matrix}\left(a_{2j};\alpha'_{2j},\alpha''_{2j}\right)_{1,p_2}: \\ \left(b_{2j};\beta'_{2j},\beta''_{2j}\right)_{1,q_2}:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left(a_{3j};\alpha'_{3j},\alpha''_{3j},\alpha'''_{3j}\right)_{1,p_3}:\cdots:\left(a_{rj};\alpha'_{rj},\alpha^{(r)}_{rj}\right)_{1,p_r}:\left(a'_j,\alpha'_j\right)_{1,p'};\cdots;\left(a^{(r)}_j,\alpha^{(r)}_j\right)_{1,p^r} \\ \left(b_{3j};\beta'_{3j},\beta''_{3j},\beta'''_{3j}\right)_{1,q_3}:\cdots:\left(b_{3j};\beta'_{rj},\beta^{(r)}_{rj}\right)_{1,q_r}:\left(b'_j,\beta'_j\right)_{1,q'};\cdots;\left(b^{(r)}_j,\beta^{(r)}_j\right)_{1,q^r}\end{matrix}\right]$$

$$= \frac{1}{(2\pi i)^r}\int_{\Delta_1}\Phi_1(s_1)\cdots\Phi_1(s_r)\,\Psi(s_1,\cdots,s_r)\,z_1^{s_1},\cdots,z_r^{s_{(r)}}ds_1\cdots ds_r$$

where $i = \sqrt{(-1)}$

$$\Phi_i(s_i) = \frac{\coprod_{j=1}^{m^{(i)}}\left|\left(b^i_j,\beta^i_j\cdot s_i\right)\coprod_{j=1}^{n^{(i)}}\right|\left(1-\alpha^i_j+\alpha^i_j\cdot s_i\right)}{\coprod_{j=1+m^{(i)}}^{q^{(i)}}\left|\left(1-\beta^i_j+\beta^i_j\cdot s_i\right)\coprod_{j=1+n^(i)}^{p^{(i)}}\right|\left(a^i_j,\alpha^i_j\cdot s_i\right)};\forall\, i\in\{1,2,\cdots,r\} \tag{1.2}$$

$$\Psi(s_1,\cdots,s_r) = \frac{\coprod_{j=1}^{n_2}\left|\left(1-a_{2j}+\sum_{i=1}^{2}\alpha^{(i)}_{2j}\cdot s_i\right)\coprod_{j=1}^{n_3}\right|\left(1-a_{3j}+\sum_{i=1}^{3}\alpha^{(i)}_{3j}\cdot s_i\right)\cdots}{\coprod_{j=n_2+1}^{p_2}\left|\left(a_{2j}-\sum_{i=1}^{2}\alpha^{(i)}_{2j}\cdot s_i\right)\coprod_{j=n_3+1}^{p_3}\right|\left(a_{3j}-\sum_{i=1}^{3}\alpha^{(i)}_{3j}\cdot s_i\right)\cdots}$$

$$\frac{\coprod_{j=1}^{n_{(r)}}\left|\left(1-a_{rj}+\sum_{i=1}^{r}\alpha^{(i)}_{rj}\cdot s_i\right)\right.}{\coprod_{j=n_r+1}^{p_r}\left|\left(a_{rj}-\sum_{i=1}^{r}\alpha^{(i)}_{rj}\cdot s_i\right)\coprod_{j=1}^{q_2}\right|\left(1-b_{2j}-\sum_{i=1}^{2}\beta^{(i)}_{2j}\cdot s_i\right)} \tag{1.3}$$

$$\frac{1}{\coprod_{j=1}^{q_s}\left|\left(1-b_{sj}+\sum_{i=1}^{s}\beta^{(i)}_{sj}\cdot s_i\right)\cdots\coprod_{j=1}^{q_r}\right|\left(1-b_{rj}+\sum_{i=1}^{r}\beta^{(i)}_{rj}\cdot s_i\right)}$$

Where the conditions of convergent and restrictions and parameters are defined in [6].

A useful Integral:

$$\int_0^t x^{\rho-1} e^{Rx/2L} \sin\{w(t-x)\}\, I\{z_1 x^{\sigma_1}, \cdots, z_r x^{\sigma_r}\} dx = t^{\rho+1} \sum_{l,k=0}^{\infty} \frac{(-1)^k (R/2L)^l t^{l+2k} w^{2k+1}}{l!} \cdot \tag{1.4}$$

$$I^{0,0:0,0:\cdots:0,1:(m',n');\cdots;(m^{(r)},n^{(r)})}_{p_2,q_2:p_3,q_3:\cdots:p_{r+1},q_{r+1}:[p',q'];\cdots;[p^{(r)},q^{(r)}]} \left[z_1 t^{\sigma_1}, \cdots, z_r t^{\sigma_r} \middle| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2} :\cdots:(1-\rho,\sigma_1,\cdots,\sigma_r) \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2} :\cdots:(-1-\rho-l-2k,\sigma_1,\cdots,\sigma_r) \end{matrix} \right.$$
$$\left. \begin{matrix} (a_{rj},\alpha'_{rj},\cdots,\alpha^{(r)}_{2j})_{1,p_r} :(a'_j,\alpha'_j)_{1,p'} :\cdots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{rj},\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r} :(b'_j,\beta'_j)_{1,q'} :\cdots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right]$$

provided that $\sigma_i > 0,\ Re\{\rho + \sum_{i=1}^r \sigma_i \alpha_i\} > 0, |arg z_i| < \frac{1}{2\pi u_i},\ u_i > 0.$

where α_i and u_i are:

$$\alpha_i = min\ Re\left\{ \left(\frac{b^i_j}{\beta^i_j} \right) : j = 1, \cdots, m^{(i)},\ i = 1, \cdots, r \right\} \tag{1.5}$$

$$\begin{aligned} u_i = & \sum_{j=1}^{n^{(i)}} \alpha^{(i)}_j - \sum_{j=n^{(i)}+1}^{p^{(i)}} \alpha^{(i)}_j + \sum_{j=1}^{m^{(i)}} \beta^{(i)}_j - \sum_{j=m^{(i)}+1}^{q^{(i)}} \beta^{(i)}_j + \left(\sum_{j=1}^{n_2} \alpha^{(i)}_{2j} - \sum_{j=n_2+1}^{p_2} \alpha^{(i)}_{2j} \right) + \left(\sum_{j=1}^{n_3} \alpha^{(i)}_{3j} - \right. \\ & \left. \sum_{j=n_3+1}^{p_3} \alpha^{(i)}_{3j} \right) + \cdots + \left(\sum_{j=1}^{n_r} \alpha^{(i)}_{rj} - \sum_{j=n_r+1}^{p_{(r)}} \alpha^{(i)}_{rj} \right) - \left(\sum_{j=1}^{q_2} \beta^{(i)}_{2j} + \sum_{j=1}^{q_3} \beta^{(i)}_{3j} + \cdots + \sum_{j=1}^{q_r} \beta^{(i)}_{rj} \right) \end{aligned} \tag{1.6}$$

Proof of (1.4):

To prove the result (1.4), we write the expansion of $e^{Rx/2L}$ and $\sin\{w(t-x)\}$ into their series form, write the contour form of $I[z_1 x^{\sigma_1}, \cdots, z_r x^{\sigma_r}]$, interchange the order of integration and summation evaluate the inner integral with help of gamma function and finally interpret the result in the light of definition of I-function, we obtain the result.

2 Main Results

If we consider an electric circuit consisting of resistance R, and inductance L, a condenser of capacity C and a source of electromotive force $E_0 P(t)$, where E_0 is constant and $P(t)$ is known function of time t, the charge $q(t)$ on the plates of condenser at any time t, satisfies the following second order differential equation-

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E_0 P(t) \tag{2.1}$$

The solution of this differential equation subject to initial conditions $q = Q,\ dq/dt = I$ when $t = 0$, is obtained.

The standard result of Sneddon [8; page 98] is

$$q(t) = J(t) + \frac{E_0}{wL} e^{-Rt/2L} \int_0^t p(\eta) e^{R\eta/2L} \sin\{w(t-\eta)\} d\eta \tag{2.2}$$

Where, for convenience

$$J(t) = e^{-Rt/wL}[Q\cos wt + I_1 w \sin wt] \tag{2.3}$$

$$I_1 = I + \frac{RQ}{2L} \quad \text{and} \quad (1/LC) - (R^2/4L^2) = w^2 > 0 \tag{2.4}$$

For the solution of (2.1) when $P(t)$ is taken in terms of the multivariable $I-$function, let

$$P(t) = t^{\rho-1} I[z_1 t^{\sigma_1}, \cdots, z_r t^{\sigma_r}] \tag{2.5}$$

Putting the value of $P(t)$ in (2.2) and evaluating the integral with the help of (1.4), we find the value of the charge $q(t)$ is given by

$$q(t) = J(t) + \tfrac{E_0}{wL} e^{-Rt/2L} t^{\rho+1} \sum_{l,k=0}^{\infty} \frac{(-1)^k (R/2L)^l t^{l+2k}}{l!} I^{0,0:0,0:\cdots:0,1:(m',n');\cdots;(m^{(r)},n^{(r)})}_{p_2,q_2:p_3,q_3:\cdots:p_{r+1},q_{r+1}:[p',q'];\cdots;[p^{(r)},q^{(r)}]} \tag{2.6}$$

$$\left[z_1 t^{\sigma_1}, \cdots, z_r t^{\sigma_r} \middle| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\cdots:(1-\rho,\sigma_1,\cdots,\sigma_r)\ (a_{rj},\alpha'_{rj},\cdots,\alpha^{(r)}_{2j})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'}:\cdots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\cdots:(-1-\rho-l-2k,\sigma_1,\cdots,\sigma_r)\ (b_{rj},\beta'_{rj},\cdots\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'}:\cdots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right]$$

where $J(t)$ stands for the quantity as given by (2.3) and conditions mentioned in (1.4) are satisfied. It is interesting to note that the value of current dq/dt can also be obtained from (2.6) by differentiating the series on its right hand side term by term w.r.t. t. The process of term by term differentiation is assumed to be justified as the multivariable $I-$Function is an analytic function.

A special case of (2.6) which is of practical interest follows easily by putting $R = 0$ (i.e. when circuit has no resistance). The charge $q(t)$ for this case is given by

$$q(t) = [Q\cos wt + I_1 w \sin wt] + \frac{E_0}{wL} t^{\rho+1} \sum_{l,k=0}^{\infty} (-1)^k w^{l+2k} t^{2k}\, I^{0,0:0,0:\cdots:0,1:(m',n');\cdots;(m^{(r)},n^{(r)})}_{p_2,q_2:p_3,q_3:\cdots:p_{r+1},q_{r+1}:[p',q'];\cdots;[p^{(r)},q^{(r)}]} \tag{2.7}$$

$$\left[z_1 t^{\sigma_1}, \cdots, z_r t^{\sigma_r} \middle| \begin{matrix} (a_{2j},\alpha'_{2j},\alpha''_{2j})_{1,p_2}:\cdots:(1-\rho,\sigma_1,\cdots,\sigma_r)\ (a_{rj},\alpha'_{rj},\cdots,\alpha^{(r)}_{2j})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'}:\cdots:(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}} \\ (b_{2j},\beta'_{2j},\beta''_{2j})_{1,q_2}:\cdots:(-1-\rho-l-2k,\sigma_1,\cdots,\sigma_r)\ (b_{rj},\beta'_{rj},\cdots\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'}:\cdots:(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}} \end{matrix} \right]$$

3 Particular Cases

(i) On taking $p_2 = p_3 = 0 = q_2 = q_3$, $p_{r-1} = 0 = q_{r-1}$ in (2.6), then the multivariable $I-$function reduces to multivariable $H-$function, as result, we find

$$P(t) = t^{\rho-1} H[z_1 t^{\sigma_1}, \cdots, z_r t^{\sigma_r}]$$

Hence, the charge at any time t is given by

$$q(t) = J(t) + \frac{E_0}{wL} e^{-Rt/2L} t^{\rho+1} \sum_{l,k=0}^{\infty} \frac{(-1)^k (R/2L)^l t^{l+2k}}{l!} H^{0,1:(m',n');\cdots;(m^{(r)},n^{(r)})}_{p_{r+1},q_{r+1}:[p',q'];\cdots;[p^{(r)},q^{(r)}]}$$

$$\left[z_1 t^{\sigma_1}, \cdots, z_r t^{\sigma_r} \middle| \begin{matrix} (1-\rho;\sigma_1,\cdots,\sigma_r)\ \left(a_{rj},\alpha'_{rj},\cdots,\alpha^{(r)}_{2j}\right)_{1,p_r} : \left(a'_j,\alpha'_j\right)_{1,p'} : \cdots : \left(a^{(r)}_j,\alpha^{(r)}_j\right)_{1,p^{(r)}} \\ (-1-l-\rho-2k;\sigma_1,\cdots,\sigma_r)\ \left(b_{rj},\beta'_{rj},\cdots,\beta^{(r)}_{rj}\right)_{1,q_r} : \left(b'_j,\beta'_j\right)_{1,q'} : \cdots : \left(b^{(r)}_j,\beta^{(r)}_j\right)_{1,q^{(r)}} \end{matrix} \right] \tag{3.1}$$

provided that the conditions appropriate to (1.4) are satisfied. The above result has already been given by Mishra [1, equation (6.10.3), P.211], but in different notations and $J(t)$ stands for the quantity as given by (2.3).

(ii) On taking $r = 2,\ p_2 = 0 = q_2$, the multivariable $H-$function breaks up into product of two $H-$functions of single variable and hence we obtain

$$P(t) = t^{\rho-1}\ H^{(m',n')}_{(p',q')}\ [z_1 t^{\sigma_1}]\ H^{(m'',n'')}_{(p'',q'')}\ [z_2 t^{\sigma_2}]$$

and thus, the charge $q(t)$ given by the equation (3.1) reduces to

$$q(t) = J(t) + \frac{E_0}{wL} e^{-Rt/2L} t^{\rho+1} \sum_{l,k=0}^{\infty} \frac{(-1)^k (R/2L)^l t^{l+2k}}{l!} H^{0,1:(m',n');(m'',n'')}_{1,1:[p',q'];[p'',q'']}$$

$$\left[z_1 t^{\sigma_1}, z_2 t^{\sigma_2} \middle| \begin{matrix} (1-\rho;\sigma_1,\sigma_2) : \left(a'_j,\alpha'_j\right)_{1,p'} : \left(a''_j,\alpha''_j\right)_{1,p''} \\ (-1-l-\rho-2k;\sigma_1,\sigma_2) : \left(b'_j,\beta'_j\right)_{1,q'} : \left(b''_j,\beta''_j\right)_{1,q''} \end{matrix} \right] \tag{3.2}$$

Where $H-$function involved to (3.2) is the $H-$function of two variables, the conditions appropriate to (1.4) are satisfied and $J(t)$ is given by (2.3).

(iii) On taking $m'' = 0, q'' = 1, n'' = 0, p'' = 0, \sigma_2 = 1, b''_1 = 0, \beta''_1 = 1$ the $H-$function $H^{(m'',n'')}_{(p'',q'')}\ [z_2 t^{\sigma_2}]$ reduces to $H^{1,0}_{0,1}\ [z_2 t] = e^{-z_2 t}$, as a result, we get

$$P(t) = t^{\rho-1}\ e^{-z_2 t}\ H^{(m',n')}_{(p',q')}\ [z_1 t^{\sigma_1}]$$

and thus charge $q(t)$ given by (3.2) reduces to

$$q(t) = J(t) + \frac{E_0}{wL} e^{-Rt/2L} t^{\rho+1} \sum_{l,k=0}^{\infty} \frac{(-1)^k (R/2L)^l t^{l+2k}}{l!} H^{0,1:(m',n');(1,0)}_{1,1:[p',q'];[0,1]}$$

$$\left[z_1 t^{\sigma_1}, z_2 t^{\sigma_2} \middle| \begin{matrix} (1-\rho;\sigma_1,1) : \left(a'_j,\alpha'_j\right)_{1,p'} \\ (-1-l-\rho-2k;\sigma_1,1) : \left(b'_j,\beta'_j\right)_{1,q'} : (0,1) \end{matrix} \right] \tag{3.3}$$

Where the $H-$function involve in (3.3) is again the $H-$function and $J(t)$ is given by (2.3).

(iv) On taking $m'' = 1, q'' = 2, n'' = p'' = 0, \sigma_2 = 2, b''_1 = 1/2, b''_2 = 0, \beta''_1 = 1, \beta''_2 = 1$, and replacing z_2 by $z_2^2/4$, $P(t)$ reduces to

$$P(t) = t^{\rho-1}\ H^{(m',n')}_{(p',q')}\ [z_1 t^{\sigma_1}] H^{(1,0)}_{[0,2]} \left[\frac{(z_2 t)^2}{4} \middle|_{(\frac{1}{2},1),(0,1)} \right] \tag{3.4}$$

Since,

$$H^{(1,0)}_{[0,2]} \left[\frac{z_2^2}{4} \middle|_{(\frac{1}{2},1),(0,1)} \right] = \pi^{-1/2} \sin z$$

Using the above result, equation (3.4) reduces

$$P(t) = \left[\frac{1}{\sqrt{\pi}} \sin(z_2 t) t^{\rho-1}\, H^{(m',n')}_{(p',q')} \left| z_1 t^{\sigma_1} \right. \right]$$

Thus, the charge $q(t)$ given by equation (3.1) reduces to

$$q(t) = J(t) + \frac{1}{\sqrt{\pi}} \frac{E_0}{wL} e^{-Rt/2L} t^{\rho+1} \sum_{l,k=0}^{\infty} \frac{(-1)^k (R/2L)^l t^{l+2k}}{l!} H^{0,1:(m',n');(1,0)}_{1,1:[p',q'];[0,1]}$$

$$\left[z_1 t^{\sigma_1}, \frac{(z_2 t)^2}{4} \left| \begin{matrix} (1-\rho;\sigma_1,2) : \left(a'_j,\alpha'_j\right)_{1,p'} \\ (-1-l-\rho-2k;\sigma_1,1), \left(\frac{1}{2},1\right) : \left(b'_j,\beta'_j\right)_{1,q'} : (0,1) \end{matrix} \right. \right] \tag{3.5}$$

provided that the conditions appropriate to (1.4) are satisfied and $J(t)$ is given by the equation (2.3).

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