η-Ricci soliton on nearly-Sasakian manifolds

Pawan Bhatt [1] **and S.K. Chanyal** [2]

[1] *Department of Mathematics,*
Kumaun University,
Nainital-263001, India.
pawanbhatt500@gmail.com

[2] *Department of Mathematics,*
Kumaun University,
Nainital-263001, India.
skchanyal.math@gmail.com

Abstract

The present paper is devoted to the study of η-Ricci soliton on nearly-Sasakian manifold. We study Ricci-semisymmetricity and Einstein-semisymmetricity properties of η-Ricci soliton on nearly-Sasakian manifold. As a consequence, we prove that a nearly-Sasakian η-Ricci soliton can not be Ricci-semisymmetric and can not be Einstein - semisymmetric. We also prove the existance of η-Ricci soliton on the manifold in our setting. An example is given at the end to illustrate the results.

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1 Introduction

As a natural generalization of Einstein metric, a Ricci soliton is defined as a touple (g, V, λ) on a Riemannian manifold M of dimension n by the equation

$$(L_V g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) = 0. \tag{1.1}$$

Where $L_V g$ denotes the Lie derivative of the Riemannian metric g along the vector field V, λ is a constant, S is Ricci tensor and X and Y are arbitrary vector fields on TM. A Ricci soliton is known as *shrinking*, *steady* or *expanding* according as λ is negative, zero or positive respectively. A Ricci soliton with $V = 0$, is reduced to Einstein equation. In the last two decades, many geometers explore the geometry of Ricci solitons in the different settings of manifolds. Ricci soliton has been studied in contact geometry by many authors such as Sharma [18], Tripathi [19], Ashoka et al. [2, 1], Ingalahalli and Bagewadi [13], Bejan and Crasmareanu [3], Chandra et al. [6], and many others. It becomes more popular when Grigory Perelman applied Ricci solitons to solve the long standing Poincare conjecture which was posed in 1904.

On the other hand, η-Ricci soliton is a generalization of Ricci soliton and was introduced by Cho and Kimura [7] in 2009. An η-Ricci soliton is a 4-tuple (g, V, λ, μ) satisfying

$$(L_V g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) + 2\mu\eta(X)\eta(Y) = 0. \tag{1.2}$$

Where μ is a constant and other notations are same as for Ricci soliton. In particular when $\mu = 0$, η-Ricci soliton becomes Ricci soliton. Blaga obtained several results concerning η-Ricci soliton on para-Kenmotsu manifold [5] and on Lorentzian para-Sasakian manifolds [4]. Further, Sardar and De [17], Pahan [15], Haseeb and Prasad [8], Hui and Chakraborty [9] and other authors also explores η-Ricci soliton on different structure. For more detail on Ricci and η-Ricci solitons one may refer [10, 11, 12] and references therein.

The present paper organised in seven sections. After preliminaries in second section, we obtain Ricci tensor for nearly-Sasakian η-Ricci soliton in section 3, which will be used in subsequent sections. Some semisymmetric conditions such as Ricci-Semisymmetricity and Einstein-semisymmetricity has been studied in section 4 and 6 respectively. Further, Nearly-Sasakian η-Ricci soliton satisfying the condition $S \cdot R = 0$ is discussed in section 5. In seventh section we show that the second order parallel symmetric tensor of type (0,2) defines an η- Ricci soliton on the underlying manifold. Finally we give an example of 5-dimensional manifold to verify our results.

2 Preliminaries

An almost contact structure on a smooth manifold M of dimension $n(= 2m+1)$ is a triplet (ϕ, ξ, η), where ϕ is a (1,1)-tensor field, ξ is a vector field, and η is a 1-form on M satisfying

$$\phi^2 X = -X + \eta(X)\xi, \qquad \eta(\xi) = 1. \tag{2.1}$$

Equation (2.1) implies that

$$\phi(\xi) = 0, \qquad \eta(\phi X) = 0, \qquad rank(\phi) = 2n. \tag{2.2}$$

A smooth manifold M endowed with an almost contact structure is called an almost contact manifold. A Riemannian metric g on M is said to be compatible with an almost contact structure (ϕ, ξ, η), if

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.3}$$

for all $X, Y \in \chi(M)$, where $\chi(M)$ is the Lie algebra of all vector fields on M. An almost contact manifold endowed with a compatible Riemannian metric is said to be an almost contact metric manifold and is denoted by $M(\phi, \xi, \eta, g)$. The fundamental 2-form Φ on $M(\phi, \xi, \eta, g)$ is defined by $\Phi(X, Y) = g(X, \phi Y)$ for all $X, Y \in \chi(M)$. An almost contact metric manifold is said to be Sasakian manifold if

$$(\nabla_X \phi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.4}$$

where ∇ denotes covariant differentiation with respect to the Riemannian connection of g. And an almost contact metric manifold is said to be nearly-Sasakian manifold [14] if

$$(\nabla_X \phi)Y + (\nabla_Y \phi)X = 2g(X, Y)\xi - \eta(Y)X - \eta(X)Y. \tag{2.5}$$

From the above equation, we deduce that for a nearly-Sasakian structure

$$\nabla_X \xi = -\phi X. \tag{2.6}$$

Further, on a Nearly-Sasakian manifold the following relation holds:

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.7}$$

$$R(\xi, X)Y = -R(X, \xi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.8}$$

$$S(X, \xi) = (n-1)\eta(X), \tag{2.9}$$

$$Q\xi = (n-1)\xi, \tag{2.10}$$

$$S(\phi X, \phi Y) = S(X, Y) - (n-1)\eta(X)\eta(Y), \tag{2.11}$$

for all vector fields X,Y and Z on M.
We now recall some definitions which will be used later.

Definition 2.1. *An n-dimensional nearly-Sasakian manifold M is called Ricci-semisymmetric if* $R \cdot S = 0$.

Definition 2.2. *An n-dimensional nearly-Sasakian manifold M is called Einstein-semisymmetric if* $R \cdot E = 0$*. Where E is Einstein tensor, given by*

$$E(X, Y) = S(X, Y) - \frac{r}{n} g(X, Y), \tag{2.12}$$

S is Ricci tensor and r is scalar curvature of the manifold M.

Definition 2.3. *A manifold M is said to be an η-Einstein manifold if its Ricci tensor S, given by*

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$$

for all vector fields X and Y on M. Where a and b are smooth functions on M.

3 η-Ricci soliton on nearly-Sasakian manifolds

Let M be a nearly-Sasakian manifold and let (g, ξ, λ, μ) be an η-Ricci soliton on M, then by (1.2), we have

$$2S(X, Y) = -(L_\xi g)(X, Y) - 2\lambda g(X, Y) - 2\mu\eta(X)\eta(Y). \tag{3.1}$$

On a nearly-Sasakian manifold, we have from (2.6)

$$(L_\xi g)(X, Y) = 0. \tag{3.2}$$

Using (3.2) in (3.1) we get

$$S(X, Y) = -(\lambda g(X, Y) + \mu\eta(X)\eta(Y)). \tag{3.3}$$

So $(M, g, \xi, \lambda, \mu)$ is an η-Einstein manifold. Setting $Y = \xi$, we have

$$S(X, \xi) = -(\lambda + \mu)\eta(X), \tag{3.4}$$

comparing (3.4) with (2.9) we get $n = 1 - (\lambda + \mu)$. Thus we can state the following result:

Theorem 3.1. *For an η-Ricci soliton on a nearly-Sasakian manifold M, the Ricci tensor is of the form* (3.3) *and* $\lambda + \mu = 1 - n$.

4 η-Ricci soliton on Ricci-Semisymmetric nearly-Sasakian manifolds

Consider a Ricci-Semisymmetric nearly-Sasakian manifold M, the by definition (2.1), we have

$$S(R(\xi, X)Y, Z) + S(Y, R(\xi, X)Z) = 0. \tag{4.1}$$

Now, suppose that Ricci-Semisymmetric nearly-Sasakian manifold admits an η-Ricci soliton, then by (3.3), (4.1) becomes

$$-\lambda g(R(\xi, X)Y, Z) - \mu\eta(R(\xi, X)Y)\eta(Z) - \lambda g(Y, R(\xi, X)Z) - \mu\eta(Y)\eta(R(\xi, X)Z) = 0. \tag{4.2}$$

With the help of (2.8), we get

$$\begin{aligned} &-\lambda g(g(X, Y)\xi - \eta(Y)X, Z) - \mu g(X, Y)\eta(Z) + \mu\eta(X)\eta(Y)\eta(Z) \\ &-\lambda g(Y, g(X, Z)\xi - \eta(Z)X) - \mu g(X, Z)\eta(Y) + \mu\eta(X)\eta(Y)\eta(Z) = 0, \end{aligned} \tag{4.3}$$

which implies

$$-\mu[g(X,Y)\eta(Z) + g(X,Z)\eta(Y)] + 2\mu\eta(X)\eta(Y)\eta(Z) = 0. \tag{4.4}$$

Setting $Z = \xi$, we get

$$-\mu g(X,Y) + \mu\eta(X)\eta(Y) = 0. \tag{4.5}$$

Now replacing X by ϕX and Y by ϕY, we get

$$-\mu g(\phi X, \phi Y) = 0. \tag{4.6}$$

This implies $\mu = 0$. Thus we can state the following:

Theorem 4.1. *A nearly-Sasakian η-Ricci soliton can not be Ricci-semisymmetric.*

Using Theorem (3.1), we have $\lambda = 1 - n$, now by (3.3),

$$S(X,Y) = -(1-\lambda)g(X,Y).$$

So, we have the following corollary:

Corollary 4.1. *An η-Ricci soliton on a Ricci-semisymmetric nearly-Sasakian manifold of dimension $n > 1$ is shrinking Ricci soliton and is Einstein manifold.*

5 Nearly-Sasakian η-Ricci soliton satisfying $S \cdot R = 0$.

We now consider a nearly-Sasakian η-Ricci soliton satisfying

$$(S(X,Y)\cdot R)(U,V)Z = 0, \tag{5.1}$$

for any vector fields X, Y, Z, U and V on M.
This implies that

$$\begin{aligned}(X \wedge_S Y)R(U,V)Z + R((X \wedge_S Y)U,V)Z + R(U,(X \wedge_S Y)V)Z \\ +R(U,V)(X \wedge_S Y)Z = 0,\end{aligned} \tag{5.2}$$

where the endomorphism $X \wedge_S Y$ is defined by

$$(X \wedge_S Y)Z = S(Y,Z)X - S(X,Z)Y. \tag{5.3}$$

By virtue of (5.3), (5.2) becomes

$$\begin{aligned}(S(X,Y)\cdot R(U,V)Z) = S(Y,R(U,V)Z)X - S(X,R(U,V)Z)Y \\ +S(Y,U)R(X,V)Z - S(X,U)R(Y,V)Z \\ +S(Y,V)R(U,X)Z - S(X,V)R(U,Y)Z \\ +S(Y,Z)R(U,V)X - S(X,Z)R(U,V)Y.\end{aligned} \tag{5.4}$$

Putting $X = \xi$ and using (5.1), (5.4) becomes

$$\begin{aligned}S(Y,R(U,V)Z)\xi - S(\xi,R(U,V)Z)Y + S(Y,U)R(\xi,V)Z \\ -S(\xi,U)R(Y,V)Z + S(Y,V)R(U,\xi)Z - S(\xi,V)R(U,Y)Z \\ +S(Y,Z)R(U,V)\xi - S(\xi,Z)R(U,V)Y = 0.\end{aligned} \tag{5.5}$$

Taking inner product with ξ

$$\begin{aligned}S(Y,R(U,V)Z) - S(\xi,R(U,V)Z)\eta(Y) + S(Y,U)\eta(R(\xi,V)Z) \\ -S(\xi,U)\eta(R(Y,V)Z) + S(Y,V)\eta(R(U,\xi)Z) - S(\xi,V)\eta(R(U,Y)Z) \\ +S(Y,Z)\eta(R(U,V)\xi) - S(\xi,Z)\eta(R(U,V)Y) = 0.\end{aligned} \tag{5.6}$$

Using (3.3), and setting $V = Z = \xi$, we obtain

$$\begin{aligned} &-\mu[\eta(Y)\eta(U)\eta(R(\xi,\xi)\xi) - \eta(U)\eta(R(Y,\xi)\xi) + \eta(Y)\eta(R(U,\xi)\xi) \\ &-\eta(R(U,Y)\xi) + \eta(Y)\eta(R(U,\xi)\xi) - \eta(R(U,\xi)Y)] = 0. \end{aligned} \tag{5.7}$$

Making use of (2.7) and (2.8), in (5.7) we get

$$-(2\lambda + \mu)[g(U,Y) - \eta(U)\eta(Y)] = 0, \tag{5.8}$$

which implies

$$\lambda = -\frac{\mu}{2}.$$

Hence by (3.3) we have

$$S(X,Y) = \frac{\mu}{2}g(X,Y) - \mu\eta(X)\eta(Y). \tag{5.9}$$

For $Y = \xi$, we get

$$S(X,\xi) = -\frac{\mu}{2}\eta(X). \tag{5.10}$$

Comparing it with (2.9) gives $\mu = 2(n-1)$, $\lambda = 1 - n$. Therefore we have the following theorem:

Theorem 5.1. *If a nearly-Sasakian η-Ricci soliton satisfies the condition $S \cdot R = 0$, then $\mu = 2(n-1)$, $\lambda = 1 - n$ and it is an η-Einstein manifold.*

Corollary 5.1. *An η-Ricci soliton on a nearly-Sasakian manifold M of dimension $n > 1$ satisfying the condition $S \cdot R = 0$, is shrinking.*

6 η-Ricci soliton on Einstein-Semisymmetric nearly-Sasakian manifolds

Consider an Einstein-Semisymmetric nearly-Sasakian manifold M, then by definition (2.2), we have

$$E(R(X,Y)Z,W) + E(Z,R(X,Y)W) = 0. \tag{6.1}$$

Putting $X = \xi$ and using (2.12), we have

$$S(R(\xi,Y)Z,W) + S(Z,R(\xi,Y)W) - \frac{r}{n}[g(R(\xi,Y)Z,W) + g(Z,R(\xi,Y)W)] = 0. \tag{6.2}$$

Now suppose that Einstein-Semisymmetric nearly-Sasakian manifold admits an η-Ricci soliton, then by (3.3) we obtain

$$\begin{aligned} &-\Big(\lambda + \frac{r}{n}\Big)[g(R(\xi,Y)Z,W) + g(Z,R(\xi,Y)W)] \\ &-\mu[\eta(R(\xi,Y)Z)\eta(W) + \eta(Z)\eta(R(\xi,Y)W)] = 0. \end{aligned} \tag{6.3}$$

Using (2.8), we get

$$-\mu[g(Y,Z)\eta(W)) + \eta(Z)g(Y,W) - 2\eta(Y)\eta(Z)\eta(W)] = 0. \tag{6.4}$$

Setting $W = \xi$, we get

$$\mu[g(Y,Z) - \eta(Y)\eta(Z)] = 0. \tag{6.5}$$

Which again implies $\mu = 0$. Therefore we can state the following:

Theorem 6.1. *A nearly-Sasakian η-Ricci soliton can not be Einstein-semisymmetric.*

Again using Theorem (3.1), we have $\lambda = 1 - n$, and by (3.3),

$$S(X,Y) = -(1 - \lambda)g(X,Y).$$

So, we have the following corollary:

Corollary 6.1. *An η-Ricci soliton on a Einstein-semisymmetric nearly-Sasakian manifold of dimension $n > 1$ is shrinking Ricci soliton and is Einstein manifold.*

7 Second order parallel symmetric tensor of type (0,2) and η-Ricci soliton

Consider the symmetric tensor α of type (0,2) given by

$$\alpha(X,Y) = L_\xi g(X,Y) + 2S(X,Y) + 2\mu\eta(X)\eta(Y) \tag{7.1}$$

which is parallel with respect to Levi-Civita connection associated to metric g. Then we have

$$\alpha(\xi,\xi) = -2\lambda. \tag{7.2}$$

By Ricci identity, we have

$$\nabla^2\alpha(X,Y,Z,W) - \nabla^2\alpha(X,Y,W,Z) = 0. \tag{7.3}$$

Putting $X = Z = W = \xi$ and using symmetricity of α we get

$$\alpha(\xi, R(\xi,Y)\xi) = 0. \tag{7.4}$$

Now by (2.8), we get

$$\eta(Y)\alpha(\xi,\xi) - \alpha(\xi,Y) = 0, \tag{7.5}$$

$$\text{or,} \quad g(Y,\xi)\alpha(\xi,\xi) - \alpha(\xi,Y) = 0. \tag{7.6}$$

Covariant differentiation with respect to X of (7.6) gives

$$\begin{aligned} g(\nabla_X Y,\xi)\alpha(\xi,\xi) + g(Y,\nabla_X\xi)\alpha(\xi,\xi) + 2g(Y,\xi)\alpha(\nabla_X\xi,\xi) \\ -\alpha(\nabla_X\xi,Y) - \alpha(\xi,\nabla_X Y) = 0. \end{aligned} \tag{7.7}$$

Replacing $\nabla_X Y$ by Y in (7.6) we get

$$g(\nabla_X Y,\xi)\alpha(\xi,\xi) - \alpha(\xi,\nabla_X Y) = 0. \tag{7.8}$$

Now using (2.6) and (7.8), (7.7) becomes

$$-g(Y,\phi X)\alpha(\xi,\xi) - 2g(Y,\xi)\alpha(\phi X,\xi) + \alpha(\phi X,Y) = 0. \tag{7.9}$$

Replacing X by ϕX and using (2.1), we have

$$\begin{aligned} &g(X,Y)\alpha(\xi,\xi) - \eta(X)\eta(Y)\alpha(\xi,\xi) + 2\eta(Y)\alpha(X,\xi) \\ &-2\eta(X)\eta(Y)\alpha(\xi,\xi) - \alpha(X,Y) + \eta(X)\alpha(\xi,Y) = 0. \end{aligned} \tag{7.10}$$

Again using(7.5), we get

$$\alpha(X,Y) = \alpha(\xi,\xi)g(X,Y). \tag{7.11}$$

By the virtue or (7.2), (7.11) becomes

$$\alpha(X,Y) = -2\lambda g(X,Y). \tag{7.12}$$

This implies that

$$L_\xi g(X,Y) + 2S(X,Y) + 2\mu\eta(X)\eta(Y) + 2\lambda g(X,Y) = 0 \tag{7.13}$$

defines an η-Ricci soliton on M, hence we can state the following theorem:

Theorem 7.1. *Let M be an nearly-Sasakian manifold, if the symmetric tensor field α of type (0,2) is parallel with respect to Levi-Civita connection of g, then the equation* (7.1) *defines an η-Ricci soliton on M.*

Further, if $\mu = 0$, then $\alpha(X,Y) = L_\xi g(X,Y) + 2S(X,Y)$. It follows that $L_\xi g(X,Y) + 2S(X,Y) = -2\lambda g(X,Y)$. Also $(L_\xi g)(X,Y) = 0$ and $S(X,\xi) = (n-1)\eta(X)$ implies that for $\mu = 0$, we have $\alpha(\xi,\xi) = 2(n-1)$, so by (7.2) we get $\lambda = 1 - n$. Thus we have the following corollary:

Corollary 7.1. *Let M be a nearly-Sasakian manifold, if the symmetric tensor field $\alpha(X,Y) = L_\xi g(X,Y) + 2S(X,Y)$ of type (0,2), is parallel with respect to Levi-Civita connection of g, then for $\mu = 0$ the equation* (7.1) *defines a Ricci soliton on M, which is shrinking when $n > 1$.*

8 Example

Consider the 5-dimensional almost contact manifold $M = \{(x, y, z, u, v) \in \mathbb{R}^5\}$, where (x, y, z, u, v) are standard coordinates in $\mathbb{R}^5$, with the almost contact structure (ϕ, ξ, η, g) defined as

$$\phi e_1 = e_2, \quad \phi e_2 = -e_1, \quad \phi e_3 = 0, \quad \phi e_4 = e_5, \quad \phi e_5 = -e_4.$$

$$\xi = e_3, \quad g(e_i, e_j) = \delta_{ij}, \quad \eta(Z) = g(e_3, Z) \text{ for all } Z \in \chi(M).$$

Where $\{e_1, e_2, e_3, e_4, e_5\}$ be a linearly independent frame field on M given by,

$$e_1 = 2\Big(y\frac{\partial}{\partial z} - \frac{\partial}{\partial x}\Big), \quad e_2 = \frac{\partial}{\partial y}, \quad e_3 = -2\frac{\partial}{\partial z} = \xi, \quad e_4 = 2\Big(v\frac{\partial}{\partial z} - \frac{\partial}{\partial u}\Big), \quad e_5 = -2\frac{\partial}{\partial v}.$$

We can easily calculate the Lie brackets as

$$[e_1, e_2] = [e_4, e_5] = 2e_3, \text{ and } [e_i, e_j] = 0 \text{ otherwise.}$$

Where ∇ is Levi-Civita connection of g [16]. Now, by Koszuls formula, we can calculate,

$$\nabla_{e_1} e_1 = 0, \quad \nabla_{e_1} e_2 = e_3, \quad \nabla_{e_1} e_3 = -e_2, \quad \nabla_{e_1} e_4 = 0, \quad \nabla_{e_1} e_5 = 0,$$
$$\nabla_{e_2} e_1 = -e_3, \quad \nabla_{e_2} e_2 = 0, \quad \nabla_{e_2} e_3 = e_1, \quad \nabla_{e_2} e_4 = 0, \quad \nabla_{e_2} e_5 = 0,$$
$$\nabla_{e_3} e_1 = -e_2, \quad \nabla_{e_3} e_2 = e_1, \quad \nabla_{e_3} e_3 = 0, \quad \nabla_{e_3} e_4 = 0, \quad \nabla_{e_3} e_5 = e_4,$$
$$\nabla_{e_4} e_1 = 0, \quad \nabla_{e_4} e_2 = e_3, \quad \nabla_{e_4} e_3 = -e_5, \quad \nabla_{e_4} e_4 = 0, \quad \nabla_{e_4} e_5 = e_3,$$
$$\nabla_{e_5} e_1 = 0, \quad \nabla_{e_5} e_2 = 0, \quad \nabla_{e_5} e_3 = 0, \quad \nabla_{e_5} e_4 = 0, \quad \nabla_{e_5} e_5 = 0.$$

It is now easy to verify that (M, ϕ, ξ, η, g) satisfies the equation (2.5). We now calculate Riemann curvature tensor $R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$, as

$$R(e_1, e_2)e_1 = 3e_2, \quad R(e_1, e_3)e_1 = -e_3, \quad R(e_2, e_4)e_1 = -e_5, \quad R(e_2, e_5)e_1 = e_4,$$
$$R(e_4, e_5)e_1 = 2e_2, \quad R(e_1, e_2)e_2 = -e_1, \quad R(e_1, e_4)e_2 = e_5, \quad R(e_2, e_3)e_2 = -e_3,$$
$$R(e_4, e_5)e_2 = -2e_1, \quad R(e_1, e_3)e_3 = e_1, \quad R(e_2, e_3)e_1 = -e_3, \quad R(e_3, e_4)e_3 = -e_4,$$
$$R(e_4, e_5)e_4 = 2e_5, \quad R(e_1, e_2)e_5 = -2e_4, \quad R(e_1, e_4)e_5 = e_2, \quad R(e_2, e_4)e_5 = e_1,$$
$$R(e_4, e_5)e_5 = -2e_4, \quad R(e_1, e_4)e_5 = -e_2.$$

And the Ricci tensor S(X,Y), as

$$S(e_1, e_1) = -2, \quad S(e_2, e_2) = 3, \quad S(e_3, e_3) = 4, \quad S(e_4, e_4) = 4, \quad S(e_5, e_5) = -1.$$

Thus from (3.3) we get $\lambda = -1$ and $\mu = -3$. Hence $(\phi, \xi, \eta, g, \lambda, \mu)$ defines an η-Ricci soliton on M.

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