A two warehouse inventory model for deteriorating Items with multivariate demand and backlogging

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Abstract

In this paper, a two-warehouse inventory policy has been developed with selling price and stock dependent demand. In today's competitive business transactions, to induce more purchases, a certain time period is given by the supplier, to the retailer to settle the account. The holding cost is linear time dependent and different for different warehouse. Allowable shortages with partial backlogging. Finally, two numerical examples are presented to validate the model.

Subject Classification: 90B05

Keywords: Inventory,Two Warehouses,Non-instantaneous Deteriorating Items,Shortages.

1 Introduction

From a home to a big company or large shopping malls, inventory management is required which is important for running a home or a large business smoothly. Inventory management not only controls the quantity of stock but also helps in keeping stock replenishment, time required for replenishment, forecast, physical location for inventory, cost of inventory etc. With good inventory management, any businessman or manager can earn more profits by reducing costs and your inventory will always be balanced and you will never run out of stock. If we ignore the impact of spoilage in today's business environment then it would be a big mistake. Often, in the definition of perishable, we assume that if after a time interval the product or product is not in its original state or it is not usable. It is also true that the rate of deterioration of all things is not same, some start deteriorating as soon as they come in storage, some slowly and some do not deteriorate at all; they deteriorate after a time interval like wheat, Toys, glass products, etc. These types of items are called non-instantaneous perishable items. It is often seen that the prices of all items like pulses, wheat etc. are low on the season. If the seller buys these items at a lower price on the season and sells them in the off-season when the price is higher, then he will get more profit. Since he has limited capacity to storage the items, due to which he rents the warehouse of Unlimited Capacity, which is a real situation today. The proposed model consists of two warehouses. In the last few years, many scholars have come up with different hypotheses on their inventory model. the remarkable work also has been published in the area of two warehouse system, viz. Hartley (1976) was the first who considered two warehouses in his model. S. R. Singh (2016) presented their model with backlogging and preservation technology investment. U. B. Gothi (2016) used constant rate of deterioration with permissible delay in payment and inflation under quadratic demand. Palanivel M (2017) considered inflation and permissible delay in payment. M. Srinivasa Reddy (2018) considered quadratic demand function of time. Sachin goal (2018) developed two warehouse inventory model in inflationary environment investigated and time varying holding cost. Ajay Singh Yadav (2019) discussed the model with variable holding cost. Md. Al-Amin Khan (2019) discussed advanced payment scheme with constant backlogging rate. R.D. Patel (2019) developed the policy for single buyer single vendor with different rate of facilities in both the warehouse with price dependent demand. A good number of works in the field of non-instantaneous items has been presented. First of all, Wu (2006) establish the phenomena of non-instantaneous and developed the model with partial backlogging and stock dependent demand. Sunil Tiwari (2016) explored the trade credit policy with inflation in two warehouse environment. Ali. Akbar. Shaikh et. Al. (2019) has discussed an inventory model with price and stock dependent demand. Chandra K. Jaggi (2017) applied the LIFO FIFO despatched policy for two warehouses. Udayakumar (2018) considered two warehouses with permissible delay in payments. Vipin Kumar (2019) assumed price and stock dependent demand for non-instantaneous items.

This paper is a profit minimized optimal replenishment policy for two-warehouse, where demand is selling price and stock dependent. The holding cost is linear function of time. Inflation and trade credit also considered in this model. Allowable shortages with partial backlogging. two numerical examples are presented to validate the model.

2 Assumptions and Notations

2.1 Assumptions

Some specific assumptions used to develop the model are

1. Time horizon is infinite.
2. Demand rate is stock and selling price dependent and defines as

$$D(p, I(t)) = \begin{cases} \alpha + \beta I(t) - p, \text{if } I(t) \geq 0. \\ \alpha - p, \text{if } I(t) \leq 0. \end{cases}$$

3. $\theta(t) = \theta t$ is the time dependent rate of deterioration, where θ is deterioration parameter.
4. Shortages are allowed
5. $B(t) = \frac{1}{1+\delta x}$ is the backlogging rate function, where $0 < \delta < 1$ is the backlogging parameter.
6. The RW is of infinite capacity and OW has limited.
7. Replenishment rate is infinite

2.2 Notations

The following are the notations

1. $I_r(t)$,$I_o(t)$ and $I_B(t)$ are the inventory level in RW, OW and back order level during time period [0,t_1], [t_1,t_2] and [t_2,T] respectively at any time t.
2. Q : the order quantity;
3. Z : is the total inventory level
4. p : is selling price
5. A : Ordering cost;
6. $h_1 = h + \tau t$ and $h_2 = f + \sigma t$ are the holding cost of RW and OW respectively (per unit per time unit) and
7. C_s : the shortage cost
8. C_d : the deterioration cost

9. T : The total cycle length of the model;
10. t_d : The point where is no deterioration
11. t_1 : The point where inventory level approaches to zero in RW;
12. t_2 : The point where inventory level approaches zero in OW.
13. TC : Total cost per unit time;

3 Mathematical Model Formulation

The inventory model starts with a total of Q units, after completing the back order, out of the remaining Z units, the W units move to OW and the Z-W units are placed in RW. Since items are non-instantaneous, so there is no deterioration till time period t_d in both the warehouses and deterioration starts after that. As shown in the fig. No. 1, two cases are made here, one in which the time of no deterioration period is less than the time to start the consuming the items from OW (i.e.$t_d < t_1$) and second is when no deterioration period is greater than the time to start the consuming the items from OW (i.e.$t_d > t_1$).

3.1 Case I $t_d \leq t_1$

Since the items are non-instantaneous, therefore in the time interval $(0, t_d)$ the inventory stock depletes due to demand only in the RW. The inventory level in the RW become to zero due to joint effect of deterioration and demand in the time interval (t_d, t_1) and in the same interval, the units depletes due to deterioration only in OW. After the units are exhausted in RW. In the time interval (t_1, t_2) the units of OW are approaches to zero level due to combined effect of demand and deterioration during. In the last phase of the inventory cycle which starts from t_2, the shortages occur duration of the interval (t_2, T) and backlogged. This inventory cycle is represented graphically by Fig. 1 and mathematically by differential equations (3.1)-(3.5)

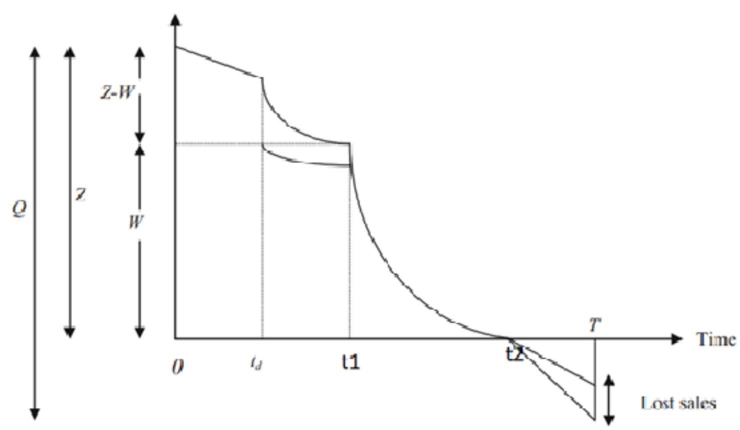

Fig. 1: Graphical Representation of Inventory Cycle.

$$\frac{dI_r(t)}{dt} = -(\alpha + \beta I(t) - p) \qquad 0 < t < t_d \tag{3.1}$$

(3.2) $$\frac{dI_r(t)}{dt} + \theta t I_r(t) = -(\alpha + \beta I(t) - p) \qquad t_d < t < t_1$$

(3.3) $$\frac{dI_o(t)}{dt} + \theta t I_o(t) = -0 \qquad t_d < t < t_1$$

(3.4) $$\frac{dI_o(t)}{dt} + \theta t I_o(t) = -(\alpha + \beta I(t) - p) \qquad t_1 < t < t_2$$

(3.5) $$\frac{dI_B(t)}{dt} = -\frac{1}{1+\delta(T-t)}(\alpha - p) \qquad t_2 < t < T$$

Under the boundary conditions $I_r(0) = Z - W, I_r(t_1) = 0, I_o(t_d) = W, I_o(t_2) = 0 and I_B(t_2) = 0$ Also $I_r(t)$ and $I_o(t)$ are continuous at $t = t_d$ and $t = t_1$

(3.6) $$I_r(t) = \frac{1}{\beta}(\alpha - p)(e^{-t\beta} - 1) + (Z - W)e^{-t\beta} \qquad 0 < t < t_d$$

(3.7) $$I_r(t) = (\alpha - p)[(t_1 - t) + \frac{1}{2}\beta(t_1^2 - t^2) + \frac{\theta}{6}(t_1^3 - t^3)]e^{-t\beta + \frac{1}{2}\theta t^2} \qquad t_d < t < t_1$$

(3.8) $$I_o(t) = e^{-\frac{1}{2}\theta(t^2 - t_d^2)} \qquad t_d < t < t_1$$

(3.9) $$I_o(t) = (\alpha - p)[(t_2 - t) + \frac{1}{2}\beta(t_2^2 - t^2) + \frac{\theta}{6}(t_2^3 - t^3)]e^{-t\beta + \frac{1}{2}\theta t^2} \qquad t_1 < t < t_2$$

(3.10) $$I_B(t) = \frac{\alpha - p}{\delta}[In(1 + \delta(T - t)) - In(1 + \delta(T - t_2))] \qquad t_2 < t < T$$

Using the continuity at $t = t_d$ of $I_r(t)$ in equation (3.6) and (3.7), we have

(3.11) $$Z = (\alpha - p)[(t_1 - t_d) + \frac{1}{2}\beta(t_1^2 - t_d^2) + \theta(t_1^3 - t_d^3)]e^{-\frac{1}{2}\theta t_d^2} + \frac{1}{\beta}(1 - e^{t_d\beta}) + W$$

Using the continuity at $t = t_1$ of $I_o(t)$ in equation (3.8) and (3.9) , we get

(3.12) $$W = (\alpha - p)[(t_2 - t_1) + \frac{1}{2}\beta(t_2^2 - t_d^1) + \frac{\theta}{6}(t_2^3 - t_1^3)]e^{-t_1\beta}$$

The maximum level of negative inventory is obtained by putting $t = T$ in equation (3.10), we get

(3.13) $$I_B(t) = \frac{\alpha - p}{\delta}In[1 + \delta(T - t_2)]$$

Therefore, the total ordering quantity is

(3.14) $$Q = Z + I_B(T)$$

Different costs per cycle associated with the model are

1. The ordering cost

$$OC = A$$

2. The holding cost in rented warehouse
$HC_{RW} = \int_0^{t_d}(f+\sigma t)I_r(t)dt + \int_{t_d}^{t_1}(f+\sigma t)I_r(t)dt$

$$= \frac{1}{2\beta^3}(-2(e^{-t_1 b\beta}(p-\alpha)+(W-Z)\beta)(f\beta+\sigma)+t_d(p-\alpha)\beta(2f+t_d\sigma)$$
$$+2e^{-t_d\beta}[(p-\alpha)+(W-Z)\beta](f\beta+\sigma+t_d\beta\sigma)$$
$$+\frac{1}{1680}t_d(p-\alpha)[70f(3(2+t_d\beta)(2t_d+t_d^2\beta-2t_1(2+t_1\beta)))+(9t_d^3-24t_1^3+6t_d^4\beta-12t_d^3\beta-2t_d^2t_1(2+t_1\beta))\theta$$
$$+2t_d^2(t_d^3-2t_1^3)theta^2+t_d(350t_d^4\beta\theta+120t_d^5\theta^2+168t_d^3(\beta^2+3\theta)$$
$$-105t_d^2(-6\beta+b(2+t_1\beta)\theta+2t_1^3\theta^2)$$
$$-420t_1(2+t_1(\beta+2t_1\theta)-280t_d(-2+t_1\beta(2+t_1\beta(2+t_1(\beta+2t_1\theta)))\sigma]$$

3. The holding cost in owned warehouse
$HC_{OW} = \int_0^{t_d}(f+\sigma t)I_r(t)dt + \int_{t_d}^{t_1}(f+\sigma t)I_r(t)dt$

$$=(t_dh+\frac{1}{2}t_d^2\tau)W+[(\frac{1}{3}t_d^3+t_1+t_dt_1^2-\frac{4}{3}t_1^3)hW$$
$$+(-\frac{1}{2}t_d^2+\frac{1}{4}t_d^4+\frac{1}{2}t_1^2+\frac{1}{2}t_d^2t_1^2-\frac{3}{2}t_1^4)W\tau-\frac{1}{1680}(t_1-t_2)^2(p-\alpha)]$$
$$+[70h(3(2+(t_1+t_2)\beta^2)+(6t_1^3\beta+3t_1^2(3+4t_2^2\beta(15+8t_2\beta))+t_2^2(19+8t_2\beta))\theta)$$
$$+2(t_1^2+t_1t_2+t_2^2)^2\theta^2+14(12t_1^3\beta^2+3t_1^2\beta(15+8t_2\beta)$$
$$+4t_1^3(18+25t_2\beta)+3t_1^2t_2(38+45t_2\beta))\theta+2t_1(20+t_2\beta(25+8t_2\beta)+t_2(20+t_2\beta(25+8t_2\beta)))$$
$$+30(4t_1^5+4t_1^4t_2+12t_1^3t_2^2+9t_1^2t_2^3+6t_1t_2^4+t_2^5)\theta^2)\tau]$$

4. The shortage cost per cycle

$$SC = -C_s\int_{t_2}^{T} I_B(t)dt$$
$$=\frac{C_S}{\beta}\frac{\alpha-p}{\delta}[\delta(T-t_2)-In(1+\delta(T-t_2))]$$

5. The deterioration cost per cycle

$$DC = C_d[\mu\int_{t_d}^{t_1} I_r(t)dt + \pi\int_{t_1}^{t_2} I_o(t)dt]$$

$$=C_d[\mu(-t_d+t_1+\frac{1}{3}t_d^3\theta-at_1^2+\frac{2}{3}t_1^3\theta)W]$$
$$+\frac{1}{120}C_d(b-c)^2\pi(p-\alpha)[(6t_1^3(1+4\beta)+3t_1^2(4t_d(1+4\beta)-5)+t_d^2(4t_d(1+9\beta)-85)+$$
$$2t_1t_d(4c(1+9\beta)-25))\theta+(3t_1^2+t_d(4+3t_d)+t_1(8+6t_d))\beta-4(3+t_1+2t_d)+2(t_1^2+t_1t_d+t_d^2)^2\theta^2)]$$

Therefore, the total average cost per unit time
$TC_1(t_1,T,P) = \frac{1}{T}[OC+HC_RW+HC_OW+SC+DC]$

3.2 Case II $t_d \geq t_1$

The following are the differential equations to represent the inventory status

$$\frac{dI_r(t)}{dt} = -(\alpha + \beta I_r(t) - p) \qquad 0 < t < t_d \tag{3.15}$$

$$\frac{dI_o(t)}{dt} = -(\alpha + \beta I_o(t) - p) \qquad t_1 < t < t_d \tag{3.16}$$

$$\frac{dI_o(t)}{dt} + \theta t I_o(t) = -(\alpha + \beta I_o(t) - p) \qquad t_d < t < t_2 \tag{3.17}$$

$$\frac{dI_B(t)}{dt} = -\frac{1}{1 + \delta(T - t)}(\alpha - p) \qquad t_2 < t < T \tag{3.18}$$

Under the boundary conditions $I_r(t_1) = 0, I_o(t_1) = W, I_o(t_2) = 0$ and $I_B(t_2) = 0$ Also $I_o(t)$ are continuous at $t = t_d$

$$I_r(t) = \frac{\alpha - p}{\beta}(e^{(t_1 - t)\beta} - 1) \qquad 0 < t < t_d \tag{3.19}$$

$$I_o(t) = \frac{\alpha - p}{\beta}(e^{(t_1 - t)\beta} - 1) + We^{(t_1 - t)\beta} \qquad 0 < t < t_d \tag{3.20}$$

$$I_o(t) = (\alpha - p)[(t_2 - t) + \frac{1}{2}\beta(t_2^2 - t^2) + \frac{\theta}{6}(t_2^3 - t^3)]e^{-t\beta + \frac{1}{2}\theta t^2} \qquad t_1 < t < t_2 \tag{3.21}$$

$$I_B(t) = \frac{\alpha - p}{\delta}[In(1 + \delta(T - t)) - In(1 + \delta(T - t_2))] \qquad t_2 < t < T \tag{3.22}$$

Using the continuity at $t = t_d$ of $I_r(t)$ in equation (3.6) and (3.7), we have

$$Z = (\alpha - p)[(t_1 - t_d) + \frac{1}{2}\beta(t_1^2 - t_d^2) + \theta(t_1^3 - t_d^3)]e^{-\frac{1}{2}\theta t_d^2} + \frac{1}{\beta}(1 - e^{t_d \beta}) + W \tag{3.23}$$

Using the continuity at $t = t_1$ of $I_o(t)$ in equation (3.8) and (3.9) , we get

$$W = (\alpha - p)[(t_2 - t_1) + \frac{1}{2}\beta(t_2^2 - t_d^1) + \frac{\theta}{6}(t_2^3 - t_1^3)]e^{-t_1 \beta} \tag{3.24}$$

The maximum level of negative inventory is obtained by putting $t = T$ in equation (3.10), we get

$$I_B(t) = \frac{\alpha - p}{\delta}In[1 + \delta(T - t_2)] \tag{3.25}$$

Therefore, the total ordering quantity is

$$Q = Z + I_B(T) \tag{3.26}$$

Different costs per cycle associated with the model are

1. The ordering cost

$$OC = A$$

2. The holding cost in rented warehouse
$HC_{RW} = \int_0^{t_d}(f+\sigma t)I_r(t)dt + \int_{t_d}^{t_1}(f+\sigma t)I_r(t)dt$

$$= \frac{\alpha - p}{\beta}[(f\beta+\sigma)(\frac{e^{t_1\beta-1}}{\beta^2} - \frac{t_1}{\beta} - \frac{t_1^2\sigma}{2})]$$

3. The holding cost in owned warehouse
$HC_{OW} = \int_0^{t_d}(h+\tau t)Wdt + \int_{t_d}^{t_1}(h+\tau t)I_o(t)dt + \int_{t_1}^{t_2}(h+\tau t)I_o(t)dt$

$$\begin{aligned}
&= t_1hW + \frac{1}{2}t_1^2W\tau + \frac{1}{2\beta}(p-\alpha)(t_d(2h+t_d\tau) - t_1(2h+t_1\tau)) \\
&+ \frac{\alpha + W\beta - p}{\beta^3}((h\beta+\tau+t_1\beta\tau) - e^{-(t_d+t_1)\beta}(h\beta+\tau+t_d\beta\tau)) \\
&+ \frac{1}{1680}(p-\alpha)840t_dt_2(2+t_2+2t_2^2\theta) + 120t_d^7\theta^2\tau + 70t_d^6\theta(2h\theta+\tau+4\beta\tau) \\
&+ 420t_d^2(-h(2+t_2\beta(2+t_2+2t_2^2\theta)) + t_2(2+t_2+2t_2^2\theta)\tau) \\
&+ 84t_d^5(h\theta(1+4\beta) + 2(\beta-\theta)\tau) \\
&- 105t_d^4(2h(\theta-\beta) + (2-4\beta+t_2\theta(2+t_2+2t_2^2\theta)\tau) \\
&- 140t_d^3(h(2-4\beta+t_2\theta(2+t_2+2t_2^2\theta) + 2(2-4\beta+t_2\theta(2+t_2+2t_2^2\theta)\tau \\
&+t_2^2[14ht_2(-85+2t_2(2+5t_2\theta)-40)-60+\beta(20+3t_2((5+12t_2\theta))+t_2(28\beta(5+2t_2(2+5t_2\theta)-210) \\
&- 280 + t_2\theta(5t_2(7+18t_2\theta) - 462)]\tau
\end{aligned}$$

4. The shortage cost per cycle

$$SC = -C_s\int_{t_2}^{T} I_B(t)dt$$

$$= \frac{C_S}{\beta}\frac{\alpha - p}{\delta}[\delta(T-t_2) - In(1+\delta(T-t_2))]$$

5. The deterioration cost per cycle

$$DC = C_d[\pi\int_{t_d}^{t_2} I_o(t)dt]$$

$$= \frac{1}{120}C_d(b-c)^2\pi(p-\alpha)[5(3t_d^2+(4+3t_2)(t_2+t_d))\beta + 10(t_d^2+t_dt_2+t_d^2)^2\theta^2 - 20(3+t_d+2t_2)$$
$$(6t_d^2 + (1+4\beta) + 3t_d^2(4t_2(1+4\beta)-5) + t_2^2(4t_2(1+9\beta)-85) + 2t_dt_2(4t_2(1+9\beta)-25))\theta)]$$

Therefore, the total cost per unit time

$$TC_2(t_1, T, P) = \frac{1}{T}[OC + HC_RW + HC_OW + SC + DC]$$

Therefore, the total cost per unit time during the cycle $(0, T)$ is given by

$$TC(t_1, T, P) = \begin{cases} TC_1(t_1, T, P), & \text{if } t_d \le t_1. \\ TC_2(t_1, T, P), & \text{if } t_d > t_1. \end{cases}$$

4 Optimality

In this section we proposed a algorithm to calculate the optimum value of t_1, p and T which minimize the $TC(t_1, p, T)$
The necessary condition for minimization given by the following expressions

$$\frac{\partial TC_i(t_1, p, T)}{\partial t_1} = 0, \frac{\partial TC_i(t_1, p, T)}{\partial p} = 0, \frac{\partial TC_i(t_1, p, T)}{\partial T} = 0$$

By the help of above equations, optimum value of t_1, p and T can be solved.
By using Hessian matrix, the sufficient condition for minimization $TC(t_1, p, T)$

$$H = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$H_1 = [a_{11}] > 0$$

$$H_2 = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} > 0$$

$$H_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} > 0$$

Where minors are H_1, H_2 and H_3 of the Hessian matrix H. To obtain the optimal value of Q, Z and TC, we can use these optimal values.

5 Numerical Examples

Example 1
A=250,$\alpha = 0.08, \beta = 0.3, \delta = 0.02, f = 0.4, h = 0.6, \tau = 0.002, \sigma = 0.003, W = 200, C_d = 12, C_s = 15, \mu = 0.02. \pi = 0.001, t_d^* = 0.25, t_2^* = 0.52$
$p^* = 12.2729, t_1^* = 0.2900, T * 0.8211, Z^* = 112.242, B^* = 32.2900, Q^* 412.12, TC_1^* = 578.2921$

Example 2 A=250,$\alpha = 0.08, \beta = 0.3, \delta = 0.02, f = 0.4, h = 0.6, \tau = 0.002, \sigma = 0.003, W = 200, C_d = 12, C_s = 15, \mu = 0.02. \pi = 0.001, t_d^* = 0.25, t_2^* = 0.57$
$p^* = 15.1216, t_1^* = 0.2901, T* = 0.7112, Z^* = 80.1231, B^* = 29.2310, Q^* 389.91, TC_1^* = 598.9192$

6 Conclusion

In this paper, we developed an inventory models with two-warehouse in which items are deteriorating but non-instantaneous in nature. Demand is multivariate and deterministic and depends on selling price and stock simultaneously. Also, Shortages are allowed and partially backlogged. Moreover, the holding cost and rate of deterioration both are assumed time dependent. Model is illustrated with some numerical examples to validate the result.

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