

ADVANCE METHOD TO SOLVE FUZZY LINEAR PROGRAMMING PROBLEMS WITH SYMMETRIC TRIANGULAR FUZZY NUMBERS

DEEPAK GUPTA, GAURAV GUPTA and AKANSHA GUPTA

Abstract

A method for obtaining the fuzzy optimal solution to the FFLPP with symmetric triangular fuzzy numbers is described in this paper, where its constraint is in the form of an inequality. The proposed technique is highly effective in solving FFLPP in day to day life. Some numerical examples have been demonstrating its efficacy.

2010 *Mathematics subject classification*: primary 90C05, 90C70; secondary 90C31, 91A35.

Keywords and phrases: FFLPP, Symmetric triangular fuzzy number, Triangular fuzzy number, membership function and crisp product.

1. Introduction

The idea of fuzzy logic was first introduced by L.Zadeh. The FLPP when all parameters and the variable are represented is said to be FFLPP with inequality constraints. Various methods to solve FFLPP have been practiced to get the best solution of FFLPP with inequality constraints by converting FFLPP into a crisp linear programming problem. This paper is composed of the following sections: In Section -2 we looked at some basic concepts and arithmetic operations with fuzzy numbers, Section -3 deals with the formulation of FFLPP and the sorting operation of FFLPP, Section 4 interprets how to convert inequality constraints to equality constraints.

2. Preliminaries

Basic definitions

Definition 1 Let \mathbf{P} be the real line, then a fuzzy set F in \mathbf{P} is defined to be a set of ordered pairs $\{F = \{[k, M_F(k)] : k \in P\}$ where $M_F(k)$ is called the membership function for the fuzzy set. Using the membership function, each P element is assigned a membership value ranging from 0 to 1.

Definition 2 This function can be mapped to a behavior aforesaid a well known function M_F , where every value assigned to the elements of the global set K is bounded.

i.e. $M_F : K \rightarrow [0, 1]$. The prescribed assessments express the degree of association about the element in set A. The function M_F is called the membership function and the set $\{F = [\{k, M_F(k)\} : k \in K] \text{ defined by } M_F(k) \text{ for each } k \in K \}$ is called fuzzy set.

Definition 3 A fuzzy number $F = (r, s, t)$ is said to be triangular fuzzy number if its membership function is given by

$$M_F(k) = \begin{cases} 1 - \frac{m-k}{s}, & m-s \leq k < m, s > 0 \\ 1 - \frac{k-m}{t}, & m \leq k < m+t, t > 0 \\ 0, & \text{otherwise} \end{cases}$$

Definition 4 A fuzzy number $F = (m, s, s)$ is said to be symmetric triangular fuzzy number if its membership function is given by

$$M_F(k) = \begin{cases} 1 - \frac{m-k}{s}, & m-s \leq k < m, s > 0 \\ 1 - \frac{k-m}{s}, & m \leq k < m+s, s > 0 \\ 0, & \text{otherwise} \end{cases}$$

Definition 5 A Ranking function is a function $\bar{P} : \phi(P) \rightarrow P$, where $\phi(P)$ is set of fuzzy number define on the set of real numbers, which maps each fuzzy number into the real line, where natural order exists.

3. Earlier Method

Amit kumar et. al [1] have suggested the most beneficial fuzzy solution of FFLP problems with inequality constraints through representing all parameters as triangular fuzzy numbers.

Max.(or Min.) $(P \otimes Q)$ Subject to $H \otimes Q \leq, =, \geq L$ Where Q is a non-negative triangular fuzzy number and

$$P^T = [p_j]_{1 \times n}, Q = [q_j]_{n \times 1}, H = [h_{ij}]_{m \times n}, L = [l_i]_{m \times 1} \text{ and } h_{ij}, p_j, q_j, l_i \in F(R)$$

Consider Problem

$$\text{Max } (8, 6, 6) q_1 + (6, 6, 6) q_2$$

With the constraint

$$\begin{aligned} (5, 3, 3) q_1 + (3, 3, 3) q_2 &\leq (8, 6, 6) \\ (5, 5, 5) q_1 + (5, 4, 4) q_2 &\leq (6, 4, 4) \\ q_1, q_2 &\geq 0 \end{aligned}$$

Solution: The standard for of given FFLPP is

$$\text{Max } (8, 6, 6) q_1 + (6, 6, 6) q_2$$

With constraint

$$\begin{aligned}(5, 3, 3) q_1 + (3, 3, 3) q_2 + (1, 1, 1) s_1 &= (8, 6, 6) \\ (5, 5, 5) q_1 + (5, 4, 4) q_2 + (1, 1, 1) s_2 &= (6, 4, 4) \\ q_1, q_2, s_1, s_2 &\text{ are non negative triangular fuzzy numbers} \\ \text{Max } (8, 6, 6) (u_1, v_1, w_1) + (4, 4, 4) (u_2, v_2, w_2)\end{aligned}$$

With constraint

$$\begin{aligned}(4, 2, 2) (u_1, v_1, w_1) + (2, 2, 2) (u_2, v_2, w_2) + (1, 1, 1) (s_1, g_1, f_1) &= (10, 6, 6) \\ (6, 6, 6) (u_1, v_1, w_1) + (8, 6, 6) (u_2, v_2, w_2) + (1, 1, 1) (s_2, g_2, f_2) &= (8, 2, 2) \\ \text{Max } P &= (6u_1 + 4u_2, 4v_1 + 4v_2, 4w_1 + 4w_2) \\ \text{Subject to} \\ (4u_1 + 2u_2 + s_1, 2v_1 + 2v_2 + g_1, 2w_1 + 2w_2 + f_1) &= (10, 6, 6) \\ (6u_1 + 8u_2 + s_2, 6v_1 + 6v_2 + g_2, 6w_1 + 6w_2 + f_2) &= (8, 2, 2) \\ (u_1, u_2, u_3), (v_1, v_2, v_3), (s_1, g_1, f_1), (s_2, g_2, f_2) & \\ &\text{are non negative triangular fuzzy numbers.}\end{aligned}$$

The above FFLPP becomes the subsequent Crisp linear programming:

$$\begin{aligned}\text{Max } P &= \left(\frac{1}{4}(12u_1 + 8u_2 + 4v_1 + 4v_2 + 4w_1 + 4w_2)\right) \\ \text{Subject to} \\ 4u_1 + 2u_2 + s_1 &= 10 \\ 6u_1 + 8u_2 + s_2 &= 8 \\ 2v_1 + 2v_2 + g_1 &= 6 \\ 6v_1 + 6v_2 + g_2 &= 2 \\ 2w_1 + 2w_2 + f_1 &= 6 \\ 6w_1 + 6w_2 + f_2 &= 2 \\ v_1 - u_1 &\geq 0 \\ v_2 - u_2 &\geq 0\end{aligned}$$

The results to simplex methods are $U1 = (1.33, 0.33, 0.33)$, $U2 = (0, 0, 0)$ and $\text{Max } P = 10$.

4. Proposed method

This section suggests the advanced method for discovering the most useful fuzzy answer to absolutely fuzzy linear programming problems with symmetric triangular fuzzy numbers. The measures to be adopted for suggested approach are taken in various steps.

Algorithm Step 1: originate preferred problem into the subsequent entirely fuzzy linear programming problem:

$$\begin{aligned}\text{Max}(\text{min})P &= c_j u_j \\ \text{Subject to } A_{ij} u_j &\leq B_i, \\ u_j &\geq 0\end{aligned}$$

Step 2: If $A = (m, a, a)$ be a symmetric triangular fuzzy number then by using the Ranking function $\mathfrak{R}(c, A) = \frac{m+a}{2}$, the FFLPP transform into FVLPP. Step 3: Convert all the inequality constraints into equations by adding slack / surplus variable and the cost of this variable zero. Step 4: Solve the FVLPP by using simplex method / Big-M method. Let the solution be u_j . Hence the solution of FFLPP is u_j^* .

Example: -

$$\text{Max } (6, 4, 4) u_1 + (4, 4, 4) u_2$$

With constraint

$$(4, 2, 2) u_1 + (2, 2, 2) u_2 \leq (10, 6, 6)$$

$$(6, 6, 6) u_1 + (8, 6, 6) u_2 \leq (8, 2, 2)$$

$$u_1, u_2 \geq 0$$

Solution: The specified FFLPP are to be set as an example

$$\text{Max } (6, 4, 4) u_1 + (4, 4, 4) u_2 + 0s_1 + 0s_2$$

With constraint

$$(4, 2, 2) u_1 + (2, 2, 2) u_2 + (1, 1, 1) s_1 = (10, 6, 6)$$

$$(6, 6, 6) u_1 + (8, 6, 6) u_2 + (1, 1, 1) s_2 = (8, 2, 2)$$

u_1, u_2, s_1, s_2 are non negative triangular fuzzy numbers

Its equivalent FVLPP is

$$\text{Max } 5u_1 + 4u_2 + 0s_1 + 0s_2$$

With constraint

$$3u_1 + 2u_2 + s_1 = (10, 6, 6)$$

$$6u_1 + 5u_2 + s_2 = (8, 2, 2)$$

$$u_1, u_2, s_1, s_2 \geq 0$$

The mathematical calculations for simplex method are given in Table 1 and Table2.

Table 1.

Basis	u_1	u_2	s_1	s_2	RHS	$\mathfrak{R}(B) = \frac{m+a}{2}$
s_1	3	2	1	0	(10,6,6)	8
s_2	6	5	0	1	(8,2,2)	5
P	-5↑	-4	0	0		→

Table 2.

Basis	u_1	u_2	s_1	s_2	RHS	$\mathfrak{R}(B) = \frac{m+a}{2}$
s_1	0	$-\frac{3}{6}$	1	$-\frac{3}{6}$	$(\frac{36}{6}, \frac{30}{6}, \frac{30}{6})$	8
u_1	1	$\frac{5}{6}$	0	$\frac{1}{6}$	$(\frac{8}{6}, \frac{2}{6}, \frac{2}{6})$	5
P	0	$\frac{1}{6}$	0	$\frac{5}{6}$	$(\frac{40}{6}, \frac{10}{6}, \frac{10}{6})$	→

Since $P \geq 0$, the optimum solution of the FVLP problem is $u_1 = \left(\frac{8}{6}, \frac{2}{6}, \frac{2}{6}\right)$, $u_2 = (0, 0, 0)$. Therefore, the optimum solutions of FFLPP is $u_1^* = \left(\frac{8}{6}, \frac{2}{6}, \frac{2}{6}\right)$, $u_2^* = (0, 0, 0)$. moreover the Fuzzy optimum value is $\text{Max } P = \left(\frac{40}{6}, \frac{10}{6}, \frac{10}{6}\right) = 10$.

5. Results and Discussion

The outputs of the completely fuzzy linear programming problem, as chosen in the example, have been obtained using the above approach and other approaches are identical. The results of the fully fuzzy linear programming problem, chosen in the example, Obtained by using the existing method and proposed method are same.

6. Conclusion

To determine the FFLP problem's it is important to understand fuzzy optimal solution before it becomes an FVLP problem. A novel approach for the same is suggested. Using recent Ranking function, the FFLP problem is converted into an FVLP problem. The ranking method makes it easy to calculate the triangular weights of needs. In addition, when compared to the previous method, the proposed method consumes significantly lesser time. The proposed method addresses the problem of completely fuzzy linear programming in a substantially smaller number of iterations. As a result, it's a significantly more potent strategy.

Conflicts of Interest: The authors declare no conflict of interest.

References

- [1] Amit kumar ,Jagdeep kaur and Pushpinder singhr, *Fuzzy optimal solution of fully fuzzy linear programming problems with inequality constraints*, International Journal of mathematical and computer science, (2010), 37–40.
- [2] S.H. Nasser, E.Behmanesh ,F.Taleshain,M.abdlalipoor and N.A. Taghi-Nezhad, *Fully fuzzy linear programming with inequality constraints*, International Journal of Industrial Mathematics, **5**, (2013) 309-316.
- [3] A.Karpagam and P.Sumathi, *New approach to solve fuzzy linear programming problems by the Ranking Function*, Bonfring International Journal of Data mining **4** (2014) 22–25.
- [4] Amit kumar and Jagdeep kaur, *General form of linear programming problems with fuzzy parameters*, Journal of Applied Research and Technology **11** (2013) 629—635.
- [5] J. Kaur and A.Kumar, *Exact fuzzy optimal solution of fully fuzzy linear programming problems with unrestricted fuzzy variables*, Applied Intelligence **37** (2012) 145—154.
- [6] D. Gupta, P. Jain and G.Gupta, *New Ranking Function Introduced To Solve Fully Fuzzy Linear Fractional Programming Problem* **71(2)** (2021) 29—35.

Deepak Gupta, Department of Mathematics, Faculty of Engineering and Technology,
University of Lucknow, Lucknow, India
e-mail: dg61279@gmail.com

Gaurav Gupta, Department of Electrical Engineering, Faculty of Engineering and
Technology, University of Lucknow, Lucknow, India
e-mail: gauravmits@gmail.com

Akansha Gupta, Department of Electronics and Communication Engineering, Bun-
delkhand University, Jhansi, India
e-mail: akku.gupta6@gmail.com