UNIQUENESS OF $\mathcal{L}$- FUNCTIONS RELATING TO POSSIBLE DIFFERENTIAL POLYNOMIAL SHARE WITH SOME FINITE WEIGHT

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Abstract

In this paper, we mainly investigate the few necessary criteria to establish the relationship between an $\mathcal{L}$-function and a meromorphic function concerning various polynomial shares with finite weights. We obtain some results which extend and generalize some recent results due to Rajeshwari S, Husna V and Nagarjun V [11].

1. Introduction

The Riemann zeta function serves as a prototype for $\mathcal{L}$-functions, which are important mathematical constructs in this investigation. The value distribution hypothesis was first presented by R. Nevanlinna at the starting of the nineteenth century. The renowned Nevanlinna theorem, also known as the Nevanlinna uniqueness theorem. Let f and g be two non-constant meromorphic functions and $a \in \mathbb{C} \cup \{\infty\}$ be a finite value (complex number). We say that f and g share a CM (counting multiplicities), if $f - a$ and $g - a$ have the same zeros with same multiplicities. Similarly, we say that f and g share a IM (ignoring multipliticities), if $f - a$ and $g - a$ have the same zeros ignoring multiplicities.

Value distribution of $\mathcal{L}$-functions concerns distribution of zeros of $\mathcal{L}$-functions $\mathcal{L}$ and, more generally, the a-points of $\mathcal{L}$, i.e., the roots of the equation $\mathcal{L}(a) = c$, or the points in the pre-image $\mathcal{L}^{-1} = \{a \in \mathbb{C} : \mathcal{L}(a) = c\}$, where a denotes a complex variable in the complex plane $\mathbb{C}$ and c denotes a value in the extended complex plane $\mathbb{C} \cup \{\infty\}$. $\mathcal{L}$-functions can be analytically continued as meromorphic functions in $\mathbb{C}$. It is well-known that a non-constant meromorphic function in $\mathbb{C}$ is completely determined by five such pre-images, [14] which is a famous theorem due to Nevanlinna and often referred to as Nevanlinna's uniqueness theorem.

Throughout the paper, $\mathcal{L}$-function to denote a Selberg class function are Dirichlet series with the Riemann zeta function $\zeta(s) = \sum\limits_{n=1}^{\infty} n^{-s}$ as the prototype. Such an $\mathcal{L}$-function is defined to be a Dirichlet series $\mathcal{L}(s) = \sum\limits_{n=1}^{\infty} a(n)n^{-s}$ of a complex variable $s = \sigma + it$ satisfying the following axioms [15][16]

i   Ramanujan hypothesis. $a(n) << n^{\epsilon}$ for every $\epsilon > 0$.

ii   Analytic continuation. There is a nonnegative integer k such that $(s - l)^k \mathcal{L}(s)$ is an entire function of finite order.

iii   Functional equation. L satisfies a functional equation of type $\Delta_{\mathcal{L}}(s) = \omega \overline{\Delta_{\mathcal{L}}(1 - \bar{s})}$, where $\Delta_{\mathcal{L}}(s) = \mathcal{L}(s)Q^s \prod\limits_{j=1}^{k} \Gamma(\lambda_j s + v_j)$ with positive real numbers Q, λ_j and complex numbers v_j, ω with $Rev_j \leq 0$ and $|\omega| = 1$.

iv   Euler product hypothesis $\mathcal{L}(S) = \prod_p exp\left(\sum\limits_{k=1}^{\infty} \frac{b(p^k)}{p^{ks}} \right)$ with suitable coefficients $b(p^k)$ satisfying $b(p^k) << p^{k\theta}$ for some $\theta < \frac{1}{2}$, where the product is taken over all prime numbers p.

The degree d of an $\mathcal{L}$-function $\mathcal{L}$ is defined to be
$$d = 2 \sum_{j=1}^{k} \lambda_j,$$
where k and λ_j are respecively the positive integer and the positive real number defined in axiom (iii) of the definition of $\mathcal{L}$-function. Throughout of this article, we shall use the following standard notation of Nevanlinna's Value Distribution Theory such as $T(r, f), m(r, f), N(r, f), \overline{N}(r, f)$ etc...[1]

DEFINITION 1.1. [12] A meromorphic function $b(z)$ ($\not\equiv 0, \infty$) defined in $\mathbb{C}$ is called a "small function" with respect to $f(z)$ if $T(r, b(z)) = S(r, f)$.

DEFINITION 1.2. [12] Let k be a positive integer, for any constant a in the complex plane $\mathbb{C}$. We denote

i   by $N_{k)}\left(r, \frac{1}{f-a}\right)$ the counting function of a-points of $f(z)$ with multiplicity $\geq k$.

ii   by $N_{(k}\left(r, \frac{1}{f-a}\right)$ the counting function of a-points of $f(z)$ with multiplicity $\leq k$.

DEFINITION 1.3. [12] Let a be an any value in the extended complex plane and let k be an arbitrary non-negative integer. we define
$$\Theta(a, f) = 1 - \limsup_{r \to \infty} \frac{\overline{N}\left(r, \frac{r}{f-a}\right)}{T(r, f)},$$
$$\delta_k(a, f) = 1 - \limsup_{r \to \infty} \frac{N_k\left(r, \frac{r}{f-a}\right)}{T(r, f)},$$
where
$$N_k\left(r, \frac{1}{f-a}\right) = \overline{N}\left(r, \frac{1}{f-a}\right) + \overline{N}_{(2}\left(r, \frac{1}{f-a}\right) + ... + \overline{N}_{(k}\left(r, \frac{1}{f-a}\right).$$

REMARK 1.1. *By Definition 1.3 we have*

$$0 \le \delta_k(a, f) \le \delta_{k-1}(a, f) \le \delta_1(a, f) \le \theta(a, f) \le 1.$$

Recently, Rajeshwari S, Husna V and Nagarjun V proved the following theorems

THEOREM 1.1. *[11] Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, $P(f)$ and $P(g)$ be a polynomials of degree m and let n, k be two positive integers with $t(n + m) > 3k + 8$. If $\Theta(\infty, f) > \frac{2}{n+m}$, $[f^n P(f)]^{(k)}$ and $[g^n P(g)]^{(k)}$ share $1(1, 2)$, then either $[f^n P(f)]^{(k)}[g^n P(g)]^{(k)} \equiv 1$ or $f(z)$ and $g(z)$ satisfy the algebraic equation $R(f, g) = 0$ where*

$$R(\omega_1, \omega_2) = \omega_1^m(a_m\omega_1^m + a_{m-1}\omega_1^{m-1} + ... + a_0) - \omega_2^m(a_m\omega_2^m + a_{m-1}\omega_2^{m-1} + ... + a_0).$$

THEOREM 1.2. *[11] Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, $P(f)$ and $P(g)$ be a polynomials of degree m and let n, k be two positive integers with $t(n + m) > 5k + 10$. If $\Theta(\infty, f) > \frac{2}{n+m}$, $[f^n P(f)]^{(k)}$ and $[g^n P(g)]^{(k)}$ share $1(1, 1)$, then either $[f^n P(f)]^{(k)}[g^n P(g)]^{(k)} \equiv 1$.*

We now generalise the aforementioned findings and arrive at the following theorems.

THEOREM 1.3. *Let $f(z)$ be an non-constant meromophic function. Let $\mathcal{L}$ be a $\mathcal{L}$-function, $P(f)$ and $P(\mathcal{L})$ be an polynomial of degree m and let n, k be two positive integer s$(n + m) > 3k + 6$. If $\Theta(\infty, f) > \frac{2+d}{n+m}$, $[f^n P(f)]^{(k)}$ and $[\mathcal{L}^n P(\mathcal{L})]^{(k)}$ share $1(1, 2)$, then either $[f^n P(f)]^{(k)}[\mathcal{L}^n P(\mathcal{L})]^{(k)} \equiv 1$.*

THEOREM 1.4. *Let $f(z)$ be an non-constant meromophic function. Let $\mathcal{L}$ be a $\mathcal{L}$-function, $P(f)$ and $P(\mathcal{L})$ be an polynomial of degree m and let n, k be two positive integer s$(n + m) > 5k + 8$. If $\Theta(\infty, f) > \frac{2+d}{n+m}$, $[f^n P(f)]^{(k)}$ and $[\mathcal{L}^n P(\mathcal{L})]^{(k)}$ share $1(1, 1)$, then either $[f^n P(f)]^{(k)}[\mathcal{L}^n P(\mathcal{L})]^{(k)} \equiv 1$.*

2. Lemmas

The following Lemmas are required to support our conclusion.

LEMMA 2.1. *[1] Let $f(z)$ be a non-constant mermorphic function, and $a_0, a_1, ..., a_n$ be finite complex numbers such that $a_n \neq 0$. Then*

$$T(r, a_n f^n + a_{n-1} f^{n-1} + ... + a_0) = nT(r, f) + S(r, f).$$

LEMMA 2.2. *[1] Let $f(z)$ be a non-constant meromorphic function and k be a positive integer and c a non-zero finite complex number. Then*

$$T(r, f) \le \overline{N}(r, f) + N\left(r.\frac{1}{f}\right) + N\left(r, \frac{1}{f^{(k)} - c}\right) - N\left(r, \frac{1}{f^{(k+1)}}\right) + S(r, f),$$

$$\le \overline{N}(r, f) + N_{k+1}\left(r.\frac{1}{f}\right) + \overline{N}\left(r, \frac{1}{f^{(k)} - c}\right) - N_0\left(r, \frac{1}{f^{(k+1)}}\right) + S(r, f).$$

Where $N_0\left(r, \frac{1}{f^{(k+1)}}\right)$ is the counting function which only counts those points such that $f^{(k+1)} = 0$ but nte that $f(f^{(k+1)} - c) \neq 0$.

LEMMA 2.3. *[8] Let $f(z)$ be a non-constant meromorphic function, and let k be a positive integer. Suppose that $f^{(k)} \not\equiv 0$, then*

$$N\left(r, \frac{1}{f^{(k)}}\right) \leq N\left(r, \frac{1}{f}\right) + k\overline{N}(r, f) + S(r, f).$$

LEMMA 2.4. *[10] Let $f(z)$ be non-constant meromorphic function, and let t, k be any two positive integers. Then*

$$N_t\left(r, \frac{1}{f^{(k)}}\right) \leq k\overline{N}(r, f) + N_{t+k}\left(r, \frac{1}{f}\right) + S(r, f).$$

Clearly, $\overline{N}\left(r, \frac{1}{f^{(k)}}\right) = N_1\left(r, \frac{1}{f^{(k)}}\right)$.

LEMMA 2.5. *[1] Let $f(z)$ be a transcendental meromorphic function, and let $b_1(z)$, $b_2(z)$ be two meromorphic functions such that $T(r, b_j) = S(r, f)$, $j = 1, 2, ..., n$. Then*

$$T(r, f) \leq \overline{N}(r, f) + \overline{N}\left(r, \frac{1}{f - b_1}\right) + \overline{N}\left(r, \frac{1}{f - b_2}\right).$$

LEMMA 2.6. *[9] Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions, and let $k \geq 1$, $l \geq 1$ be two positive integers. Suppose that $f^{(k)}$ and $g^{(k)}$ share $(1,l)$,*
(i) If $l = 2$ and

$$\Delta_1 = 2\Theta(\infty, f) + (k + 2)\Theta(\infty, g) + \Theta(0, f) + \Theta(0, g) + \delta_{k+1}(0, f) + \delta_{k+1}(0, g) > k + 7,$$

then either $f^{(k)}g^{(k)} \equiv 1$ or $f(z) \equiv g(z)$.
(ii) If $l = 1$ and

$$\Delta_2 = (k+3)\Theta(\infty, f) + (k+2)\Theta(\infty, g) + \Theta(0, f) + \Theta(0, g) + 2\delta_{k+1}(0, f) + \delta_{k+1}(0, g) > 2k+9,$$

then either $f^{(k)}g^{(k)} \equiv 1$ or $f(z) \equiv g(z)$.

3. The Main Results

Proof of Theorem 1.3
First we have to show that, $\mathcal{L}$ be a transcendental meromorphic function.
We denote by d the degree of $\mathcal{L}$
Then

$$d = 2\sum_{j=1}^{k} \lambda_j > 0, \tag{3.1}$$

where k λ_j are respectively the positive integer and the positive real number in the functional equation of the axiom (iii) of the definition of $\mathcal{L}$-function.
We set the function F_1 and G_1 as follows.

$$F_1 = F^{(k)}, G_1 = G^{(k)}. \tag{3.2}$$

Where $F = f^n P(f)$ and $G = \mathcal{L}^n P(\mathcal{L})$.

Clearly as $F^{(k)}, G^{(k)}$ share $(0, l)$, hence F_1, G_1 share $(1, l)$.

Noting that an $\mathcal{L}$-function has at most one pole $z = 1$ in the complex plane, we deduce by

Lemma 2.1, 2.2 and Valiron-Mokhonko's Lemma

$$(n + m)T(r, \mathcal{L}) + S(r, f) = T(r, G)$$

$$\leq \overline{N}(r, G) + N\left(r, \frac{1}{G}\right) + \overline{N}\left(r, \frac{1}{G^{(K)} - c}\right) - N\left(r, \frac{1}{G^{(k+1)}}\right) + S(r, f)$$

$$\leq \overline{N}(r, G) + N_{k+1}\left(r, \frac{1}{G}\right) + \overline{N}\left(r, \frac{1}{G_1}\right) - N_0\left(r, \frac{1}{G'_1}\right) + S(r, f)$$

$$\leq \overline{N}(r, \mathcal{L}) + (k + 1)\overline{N}\left(r, \frac{1}{G}\right) + \overline{N}\left(r, \frac{1}{G_1}\right) + S(r, f)$$

$$\leq (k + 1)(n + m)T(r, \mathcal{L}) + \overline{N}\left(r, \frac{1}{F_1}\right) + S(r, f),$$

Where $N_0\left(r, \frac{1}{G'_1}\right)$ is the countig function of those zeros of G'_1 in $|z| < r$ which is not the zeroes of G and $G_1 - 1$ in $|z| < r$. This implies

$$- k(n + m)T(r, \mathcal{L}) \leq T(r, F^{(k)}) + S(r, f). \tag{3.3}$$

By (3.1), we see that $\mathcal{L}$ is a transcendental mermorphic function. Combining this with (3.3), and the assumption of lower bound of n, we deduce that $F^{(k)}$ and so f is transcendental meromorphic function.

Now we have from Lemma 2.6

$$\Delta_1 = 2\Theta(\infty, f) + (k + 2)\Theta(\infty, g) + \Theta(0, f) + \Theta(0, g) + \delta_{k+1}(0, f) + \delta_{k+1}(0, g). \tag{3.4}$$

Consider

$$\Theta(0, F) = 1 - \varlimsup_{r \to \infty} \frac{\overline{N}\left(r, \frac{1}{F}\right)}{T(r, F)} = 1 - \varlimsup_{r \to \infty} \frac{\overline{N}\left(r, \frac{1}{f^n P(f)}\right)}{s(n + m)T(r, f)}$$

$$\geq 1 - \varlimsup_{r \to \infty} \frac{T(r, f)}{s(n + m)T(r, f)}$$

i.e.

$$\Theta(0, F) \geq 1 - \frac{1}{s(n + m)}. \tag{3.5}$$

Similarly

$$\Theta(0, G) \geq 1 - \frac{1}{s(n + m)}. \tag{3.6}$$

Consider

$$\Theta(\infty, F) = 1 - \varlimsup_{r \to \infty} \frac{\overline{N}(r, F)}{T(r, F)} = 1 - \varlimsup_{r \to \infty} \frac{\overline{N}(r, f^n P(f))}{s(n + m)T(r, f)} \geq 1 - \varlimsup_{r \to \infty} \frac{T(r, f)}{s(n + m)T(r, f)}$$

i.e.

$$\Theta(\infty, F) \geq 1 - \frac{1}{s(n+m)}. \tag{3.7}$$

Since an L-function has at most one pole at $z = 1$ in the complex plane, we have

$$N(r, \mathcal{L}) \leq log(r) + 0(1).$$

So using (3.1) we deduce that

$$\Theta(\infty, G) = 1. \tag{3.8}$$

Next, we have

$$\delta_{k+1}(0, F) = 1 - \overline{\lim_{r \to \infty}} N_{k+1} \frac{\left(r, \frac{1}{F}\right)}{T(r, F)} \leq 1 - \overline{\lim_{r \to \infty}} \frac{(k+1)\overline{N}\left(r, \frac{1}{F}\right)}{T(r, F)}$$

Therefore

$$\delta_{k+1}(0, F) = 1 - \overline{\lim_{r \to \infty}} \frac{(k+1)\overline{N}\left(r, \frac{1}{f^{(n)}P(f)}\right)}{s(n+m)T(r, f)} \geq 1 - \frac{k+1}{s(n+m)}. \tag{3.9}$$

Similarly

$$\delta_{k+1}(0, G) = 1 - \overline{\lim_{r \to \infty}} \frac{(k+1)\overline{N}\left(r, \frac{1}{\mathcal{L}^{(n)}P(\mathcal{L})}\right)}{s(n+m)T(r, \mathcal{L})} \geq 1 - \frac{k+1}{s(n+m)}. \tag{3.10}$$

From the inequalities (3.5)-(3.10), we get,

$$\Delta_1 \geq 2\left(1 - \frac{1}{s(n+m)}\right) + (k+2) + 2\left(1 - \frac{1}{s(n+m)}\right) + 2\left(1 - \frac{k+1}{s(n+m)}\right)$$
$$\geq 4\left(1 - \frac{1}{s(n+m)}\right) + (k+2) + 2\left(1 - \frac{k+1}{s(n+m)}\right). \tag{3.11}$$

On simplysing, the above experssion, we get

$$\Delta_1 \geq k + 8 - \frac{2k+6}{s(n+m)}. \tag{3.12}$$

Since $s(n+m) > 3k+6$, we ge $\Delta_1 > k+7$. Considering that $F^{(k)}$ and $G^{(k)}$ share (1,2), Then by Lemma 2.6 we deduce that either $F^{(k)}G^{(k)} \equiv 1$ or $F = G$.
Next we consider the following two cases.
Case 1. $F^{(K)}G^{(K)} \equiv 1$, that is

$$[f^n P(f)]^{(k)}[\mathcal{L}^n P(\mathcal{L})]^{(k)} \equiv 1. \tag{3.13}$$

Case 2. $F = G$, that is

$$f^n P(f) = \mathcal{L}^n P(\mathcal{L}). \tag{3.14}$$

Suppose that $f \not\equiv g$, then we consider following two cases.
(i) Let $h = \frac{f}{\mathcal{L}}$ be a constant. Then from (3.13) we get

$$f^n[a_m f^m + a_{m-1} f^{m-1} + ... + a_1 z] = \mathcal{L}^n[a_m \mathcal{L}^m + a_{m-1} \mathcal{L}^{m-1} + ... + a_1 z]. \quad (3.15)$$

i.e.

$$[a_m \mathcal{L}^{m+n}(h^{n+m} - 1) + a_{m-1}\mathcal{L}^{m+n-1}(h^{m+n-1} - 1) + ... + a_1\mathcal{L}^n(h^n - 1) = 0]. \quad (3.16)$$

Which implies $h^{d_1} = 1$.
Where $d_1 = gcd(n + m, ..., m - i + n, ..., n), a_{m-i} \neq 0$ form some $i = 0, 1, 2, ..., m$.
(ii) If h is not constant,
taking $h = \frac{f}{\mathcal{L}}$,
From (3.16) we get,

$$\mathcal{L}^{n+m}(h^{n+m} - 1) = -\mathcal{L}^n(h^n - 1). \quad (3.17)$$

Suppose h is a non-constant meromorphic function. Then by (3.17) we have

$$\mathcal{L}^m = -\frac{h^n - 1}{h^{n+m} - 1}. \quad (3.18)$$

Let $d = gcd(n, m)$. Then clearly $h^d = 1$ is the common factor of $h^n \neq 1$ and $h^{n+m} \neq 1$.
Therefore, (3.18) can we rewritten as

$$\mathcal{L}^m = -\frac{1 + h + ... + h^{n-d}}{1 + h + ... + h^{n+m-d}}. \quad (3.19)$$

By (3.19) and Lemma 2.1 we have

$$T(r, \mathcal{L}) = T\left(r, \frac{1 + h + ... + h^{n-d}}{1 + h + ... + h^{n+m-d}}\right) = (n + m - d)T(r, h) + 0(1). \quad (3.20)$$

It follow that,

$$T(r, f) = T(r, \mathcal{L}h) = \left(r, \frac{1 + h + ... + h^{n-d}}{1 + h + ... + h^{n+m-d}}h\right) = (n + m - d)T(r, h) + S(r, f). \quad (3.21)$$

On the other hand, by the second fundamental theorem of Nevanlinna we get,

$$\overline{N}(r, f) = \sum_{j=1}^{N} \overline{N}\left(r, \frac{1}{h - a_j}\right) \geq (n + m - d - 2)T(r, h) + S(r, f). \quad (3.22)$$

Here $a_j(\neq 1)$ $(j = 1, 2, ..., n)$ are $n + m - d$ distint finite complex number satisfying and $h^{n+m-d} = 1$. So we have

$$\begin{aligned}
\Theta(\infty, f) &= 1 - \overline{\lim_{r \to \infty}} \frac{\overline{N}(r, f)}{T(r, f)} \\
&\leq 1 - \overline{\lim_{r \to \infty}} \frac{(n + m - d - 2)T(r, \mathcal{L}) + s(r, \mathcal{L})}{(n + m)T(r, h)} \\
&\leq 1 - \frac{n + m - d - 2}{n + m} \\
&\leq \frac{2 + d}{n + m}.
\end{aligned} \quad (3.23)$$

Which contradicts to the assumption that $\Theta(\infty, f) > \frac{2+d}{n+m}$. Thus $F \equiv G$. Hence the proof Theorem 1.3.

Proof of Theorem 1.4

From the inequalities (3.5)-(3.10) and by Lemma 2.6, we get,

$$\begin{aligned}
\Delta_2 &\geq (k+3)\left(1 - \frac{1}{s(n+m)}\right) + (k+2) + 2\left(1 - \frac{1}{s(n+m)}\right) + 3\left(1 - \frac{k+1}{s(n+m)}\right) \\
&\geq (k+5)\left(1 - \frac{1}{s(n+m)}\right) + (k+2) + 3\left(1 - \frac{k+1}{s(n+m)}\right).
\end{aligned} \quad (3.24)$$

On simplyfying, the above expression, we get,

$$\Delta_2 \geq 2k + 10 - \frac{4k+8}{s(n+m)}. \quad (3.25)$$

Since $s(n+m) > 5k+8$, we get $\Delta_2 > 2k+9$. Considering that $F^{(k)}G^{(k)}$ share $(1, 1)$ then by Lemma 2.6, we deduce that either $F^{(k)}G^{(k)} \equiv 1$ or $F \equiv G$. Next by proceeding as in Theorem 1.3, we obtain the couclusion of Theorem 1.4. Here we omit the details.

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