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UNSTEADY MHD FLOW THROUGH POROUS MEDIUM PAST AN EXPONENTIALLY ACCELERATED INCLINED CYLINDER WITH VARIABLE OSCILLATING WALL TEMPERATURE IN THE PRESENCE OF CHEMICAL REACTION

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Abstract

The present investigation is carried out to examine the combined effects of chemical reaction and porosity of the medium on unsteady MHD flow past an exponentially accelerated inclined cylinder with variable oscillating wall temperature and mass diffusion. The temperature of the fluid near the wall is oscillating and concentration level of fluid increase linearly with respect to time. We have used Crank-Nicolson implicit finite difference numerical method to solve our MHD flow model. The behaviour of fluid flow is discussed with the help of graphs drawn for varies parameters. The stability criterion of the finite difference scheme for constant mesh sizes are analysed and the solution of our model is consistent.

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Keywords and phrases: MHD flow, porous medium, inclined cylinder, chemical reaction.

1. Introduction

The effect of chemical reaction on fluid flow past through different shapes such as plate, sphere, cone and others have been studied among researchers in recent years. The reason is the chemical reaction effect along with influence of magnetic field on such a flow within porous media has play important role in engineering applications. Eckert and Living [3] have analysed the method for calculation of laminar heat transfer in air flow around cylinders of arbitrary cross-section concluding large temperature difference and transpiration cooling. MHD free convection about a semi-infinite vertical plate in a strong cross magnetic field was considered by Wilks [11]. Temperature field in the flow over a stretching surface with uniform heat flux was investigated by Dutta et al. [2]. Satya et al. [7] have examined the behaviour of Hall current effect on free convection MHD flow past a porous plate. Ahmed and Kalila [1] have proposed oscillatory MHD free convective flow through a porous medium with mass transfer, Soret effect and chemical reaction. Effect of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for

Hiemenz flow through porous media with radiation was developed by Seddeek et al. [8]. Vasu et al. [10] have developed radiation and mass transfer effects on transient free convection flow of a dissipative fluid past semi-infinite vertical plate with uniform heat and mass flux. Siva and Suram [9] have worked on finite element analysis of heat and mass transfer past an impulsively moving vertical plate with ramped temperature. Kumar and rizvi [4] have work on Casson fluid flow past on vertical cylinder in the presence of chemical reaction and magnetic field. Chemical reaction effect on unsteady MHD flow past an inclined plate with varies parameter was studied by Rajput and Kumar [5,6]. The fluid flow model under consideration analyses the effect of chemical reaction and porosity of the medium on MHD flow. The flow model is solved by numerically using Crank-Nicolson implicit finite-difference technique.

2. Mathematical Modelling

In this paper, consider unsteady MHD chemically reacting and electrically conducting flow on cylinder of radius r_0. Here the x-axis is taken along the axis of cylinder in the vertical direction and the radial coordinate r is taken normal tothe cylinder. Cylinder is inclined at an angle α from vertical plane. The uniform strength magnetic field B_0 is applied perpendicular to the surface of cylinder. Initially it is assume that the surface of cylinder as well as the fluid is at the same temperature T_∞. The species concentration in the fluid is considered as T_∞ for all $t \leq 0$. At time $t > 0$, the cylinder starts exponentially accelerated with acceleration parameter b and temperature of the surface T_w is oscillating with phase angle ωt. The concentration Cw near the surface is raised linearly with respect to time. Then the flow model is

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial r^2} + \frac{\upsilon}{r}\frac{\partial u}{\partial r} + g\beta(T - T_\infty)Cos\alpha + g\beta^*(C - C_\infty)Cos\alpha - \frac{\sigma B_0^2 u}{\varrho} - \frac{\upsilon u}{K} \tag{2.1}$$

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial r^2} + \frac{a}{r}\frac{\partial T}{\partial r} \tag{2.2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial r^2} + \frac{D}{r}\frac{\partial C}{\partial r} - K_c(C - C_\infty) \tag{2.3}$$

The initial and boundary conditions are as:

$$\begin{aligned}
. \quad & t \leq 0 : u = 0, T = T_\infty, C = C_\infty, \quad for\ every\ r \\
& t > 0 : u = u_0 e^{bt}, T = T_\infty + (T_w + T_\infty)Cos\omega t, C = C_\infty + (C_w + C_\infty)\frac{u_0^2 t}{r_0^2}, at\ r = r_0 \\
& u \to 0, T \to T_{\infty,} C \to C_{\infty,} \ \ as\ r \to \infty.
\end{aligned} \tag{2.4}$$

Here u is the velocity of fluid, g - the acceleration due to gravity, β - volumetric coefficient of thermal expansion, t - time,T - temperature of the fluid, β^* - volumetric coefficient of concentration expansion, C - species concentration in the fluid, ρ - the density, C_p - the specific heat at constant pressure, k - thermal conductivity of the fluid, D - the mass diffusion coefficient, T_w - temperature of the plate at $r = 0$, C_w - species concentration at the plate $r = 0$, K - the permeability parameter, K_0 - chemical reaction, σ - electrical conductivity.

The following non-dimensional quantities are introduced to transform equations (2.1), (2.2) and (2.3) into dimensionless form:

$$. \quad R = \frac{r}{r_0}, \overline{u} = \frac{u}{u_0}, \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, S_c = \frac{\upsilon}{D}, P_r = \frac{\upsilon}{a}, G_r = \frac{g\beta\upsilon(T_w - T_\infty)}{u_0^3},$$

$$M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, G_m = \frac{g\beta^* \upsilon(C_w - C_\infty)}{u_0^3}, \overline{C} = \frac{(C - C_\infty)}{(C_w - C_\infty)}, \overline{t} = \frac{t u_0^2}{\upsilon}, \overline{K} = \frac{u_0}{\upsilon^2} K,$$

$$\overline{b} = \frac{b\upsilon}{u_0^2}, \overline{\omega} = \frac{\omega\upsilon}{u_0^2}. \tag{2.5}$$

where $\overline{u}$ is the dimensionless velocity, $\overline{b}$ -dimensionless acceleration parameter, θ - the dimensionless temperature, $\overline{C}$ - the dimensionless concentration, G_r - thermal Grashof number, G_m - mass Grashof number, μ - the coefficient of viscosity, $\overline{K}$ - the dimensionless permeability parameter, P_r - the Prandtl number, S_c- the Schmidt number, M- the magnetic parameter.

The flow model in dimensionless form is:

$$\frac{\partial \overline{u}}{\partial \overline{t}} = \frac{\partial^2 \overline{u}}{\partial R^2} + \frac{1}{R}\frac{\partial \overline{u}}{\partial R} + G_r \theta Cos\alpha + G_m \overline{C} Cos\alpha - M\overline{u} - \frac{\overline{u}}{\overline{K}} \tag{2.6}$$

$$\frac{\partial \theta}{\partial \overline{t}} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{RP_r}\frac{\partial \theta}{\partial R}, \tag{2.7}$$

$$\frac{\partial \overline{C}}{\partial \overline{t}} = \frac{1}{S_c}\frac{\partial^2 \overline{C}}{\partial R^2} + \frac{1}{RS_c}\frac{\partial \overline{C}}{\partial R} - K_0 \overline{C}, \tag{2.8}$$

The corresponding boundary conditions (2.4) become:

$$\begin{aligned} . \quad & \overline{t} \leq 0 : \overline{u} = 0,\ \theta = 0,\ \overline{C} = 0 \ \ for\ every\ R, \\ & \overline{t} > 0 : \overline{u} = e^{bt}, \theta = Cos\overline{\omega}\overline{t},\ \ \overline{C} = \overline{t} \ \ \ at\ R = 0 \\ & \overline{u} \to 0, \theta \to 0, \overline{C} \to 0,\ as\ \ R \to \infty. \end{aligned} \tag{2.9}$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial R^2} + \frac{1}{R}\frac{\partial u}{\partial R} + G_r \theta\ Cos\alpha + G_m C\ Cos\alpha - Mu - \frac{u}{K} \tag{2.10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R\ P_r}\frac{\partial \theta}{\partial R}, \tag{2.11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial R^2} + \frac{1}{R\ S_c}\frac{\partial C}{\partial R} - K_0\ C, \tag{2.12}$$

The boundary conditions become

$$\begin{aligned} . \quad & t \leq 0 : u = 0, \theta = 0, C = 0 \ \ for\ every\ R, \\ & t > 0 : u = e^{bt}, \theta = Cos\omega t, C = t \ \ at\ R = 0, \\ & u \to 0, \theta \to 0, C \to 0,\ as\ R \to \infty. \end{aligned} \tag{2.13}$$

3. Method of Solution

The equations (2.10) to (2.12) are non-linear partial differential equations with boundary and initial conditions (2.13) are solved by Crank- Nicolson implicit finite difference method. The finite difference equations corresponding to equations numbers (2.10) to (2.12) are as follows:

$$\begin{aligned} u_i^{j+1} - u_i^j &= \frac{\triangle t}{2(\triangle R)^2}(u_{i+1}^j - 2u_i^j + u_{i-1}^j + u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}) + \frac{\triangle t}{4(1+(i-1)\triangle R)(\triangle R)} \\ &\quad (u_{i+1}^j - u_{i-1}^j + u_{i+1}^{j+1} - u_{i-1}^{j+1}) + \frac{\triangle t G_r Cos\alpha}{2}(\theta_i^{j+1} + \theta_i^j) \\ &\quad + \frac{\triangle t G_m Cos\alpha}{2}(C_i^{j+1} + C_i^j) - \frac{\triangle t}{2}(M + \frac{1}{K})(u_i^{j+1} + u_i^j) \end{aligned} \tag{3.1}$$

$$\begin{aligned} \theta_i^{j+1} - \theta_i^j &= \frac{\triangle t}{2P_r(\triangle R)^2}(\theta_{i+1}^j - 2\theta u_i^j + \theta_{i-1}^j + \theta_{i+1}^{j+1} - 2\theta_i^{j+1} + \theta_{i-1}^{j+1}) \\ &\quad + \frac{\triangle t}{4P_r(1+(i-1)\triangle R)(\triangle R)}(\theta_{i+1}^j - \theta_{i-1}^j + \theta_{i+1}^{j+1} - \theta_{i-1}^{j+1}) \end{aligned} \tag{3.2}$$

$$\begin{aligned} C_i^{j+1} - C_i^j &= \frac{\triangle t}{2S_C(\triangle R)^2}(C_{i+1}^j - 2Cu_i^j + C_{i-1}^j + C_{i+1}^{j+1} - 2C_i^{j+1} + C_{i-1}^{j+1}) \\ &\quad + \frac{\triangle t}{4S_C(1+(i-1)\triangle R)(\triangle R)}(C_{i+1}^j - C_{i-1}^j + C_{i+1}^{j+1} - C_{i-1}^{j+1}) \\ &\quad - \triangle t K_0(C_i^{j+1} + C_i^j) \end{aligned} \tag{3.3}$$

Where index i refers to R and j refers to time t,$\triangle t = t_{j+1} - t_j$ and $\triangle R = R_{j+1} - R_j$. Knowing the values of u, θ and C at time t, we can compute the values at time $t + \triangle t$ as follows: we substitute $i = 1, 2, ..., N-1$, where N correspond to ∞. The computation is executed for $\triangle R = 0.1, \triangle t = 0.002$ and procedure is repeated till $R = 40$.

This method provides stable solutions of the equations and requires matrix inversions which we have been done at step forward with time. The initial and boundary value of the flow model related with a finite number of spatial grid points. Therefore, the corresponding MHD flow model equations do not automatically guarantee the convergence of the mesh $\triangle t \to 0$. To achieve maximum numerically efficiency, we have used the tri-diagonal procedure to solve the two- point conditions governing the main coupled governing equations of momentum and energy.

4. Stability Analysis

The stability criterion of the finite difference scheme for constant mesh sizes are analysed by using Von-Neumann Technique. In the Fourier expansion, the general term for u, θ, C at a time arbitrarily called $t = 0$, are assumed to be of the form $e^{i\beta R}$. At a later time, these terms will become,

$$u = \zeta_1(t)\, e^{i\beta R} \tag{4.1}$$

$$\theta = \zeta_2(t)\, e^{i\beta R} \tag{4.2}$$

$$C = \zeta_3(t)\, e^{i\beta R} \tag{4.3}$$

Putting equations (4.1) to (4.3) in equations (3.1)to (3.3) under the assumption that the coefficients u, θ, C as constants over any one time step and denoting the values after one time step by ζ_1', ζ_2' and ζ_3' . After simplification, we get

$$\begin{aligned}\frac{\zeta_1' - \zeta_1}{\Delta t} \quad = \quad & \frac{G_r Cos\alpha(\zeta_2' + \zeta_2) + G_m Cos\alpha(\zeta_3' - \zeta_3)}{2} + \frac{Cos(\beta\Delta R) - 1}{\Delta R^2}(\zeta_1' + \zeta_1) \\ & -\frac{1}{2}(M + \frac{1}{K})(\zeta_1' + \zeta_1) + \frac{i\sin(\beta\Delta R)}{2(1 + (i-1)\vartriangle R)(\vartriangle R)}(\zeta_1' + \zeta_1).\end{aligned} \tag{4.4}$$

$$\frac{\zeta_2' - \zeta_2}{\Delta t} = \frac{\zeta_2' + \zeta_2}{P_r}\left[\frac{Cos(\beta\Delta R) - 1}{\Delta R^2} + \frac{i\sin(\beta\Delta R)}{2(1 + (i-1)\vartriangle R)(\vartriangle R)}\right] \tag{4.5}$$

$$\frac{\zeta_3' - \zeta_3}{\Delta t} = \frac{\zeta_3' + \zeta_3}{S_C}\left[\frac{Cos(\beta\Delta R) - 1}{\Delta R^2} + \frac{i\sin(\beta\Delta R)}{2(1 + (i-1)\vartriangle R)(\vartriangle R)}\right] - K_0(\zeta_3' + \zeta_3) \tag{4.6}$$

Equations (4.4) to (4.6) can be rewritten as,

$$(1 + \gamma_1)\zeta_1' = (1 + \gamma_1)\zeta_1 + \frac{G_r Cos\alpha}{2}(\zeta_2' + \zeta_2) + \frac{G_m Cos\alpha}{2}(\zeta_3' + \zeta_3) \tag{4.7}$$

$$(1 + \gamma_2)\zeta_2' = (1 - \gamma_2)\zeta_2 \tag{4.8}$$

$$(1 + \gamma_3)\zeta_3' = (1 - \gamma_3)\zeta_3 \tag{4.9}$$

Here,

$$\begin{aligned}. \quad \gamma_1 &= \frac{1 - Cos(\beta\Delta R)(\Delta t)}{(\Delta R)^2} + \frac{M(\Delta t)}{2} - \frac{(\Delta t)i\sin(\beta\Delta R)}{2(1 + (i-1)\vartriangle R)(\vartriangle R)} \\ \gamma_2 &= \frac{1 - Cos(\beta\Delta R)(\Delta t)}{P_r(\Delta R)^2} - \frac{(\Delta t)i\sin(\beta\Delta R)}{2P_r(1 + (i-1)\vartriangle R)(\vartriangle R)} \\ \gamma_3 &= \frac{1 - Cos(\beta\Delta R)(\Delta t)}{S_C(\Delta R)^2} - \frac{(\Delta t)i\sin(\beta\Delta R)}{2S_C(1 + (i-1)\vartriangle R)(\vartriangle R)} + K_0(\Delta t).\end{aligned}$$

Eliminating ζ_2' and ζ_3' in equation (4.7) by using (4.8) and (4.9). The resultant equation is of the form,

$$(1 + \gamma_1)\zeta_1' = (1 - \gamma_1)\zeta_1 + \frac{G_r Cos\alpha}{(1 + \gamma_2)}\zeta_2 + \frac{G_m Cos\alpha}{(1 + \gamma_3)}\zeta_3 \tag{4.10}$$

Equations (4.8) to (4.10) can be written in matrix form as,

$$\begin{pmatrix} \zeta_1' \\ \zeta_2' \\ \zeta_3' \end{pmatrix} = \begin{pmatrix} \frac{(1-\gamma_1)}{(1+\gamma_1)} & A_1 & A_2 \\ 0 & \frac{(1-\gamma_2)}{(1+\gamma_2)} & 0 \\ 0 & 0 & \frac{(1-\gamma_3)}{(1+\gamma_3)} \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{pmatrix} \tag{4.11}$$

In above matrix, the value of A_1 and A_2 are given as

$$A_1 = \frac{G_r Cos\alpha}{(1+\gamma_1)(1+\gamma_2)}, A_2 = \frac{G_m Cos\alpha}{(1+\gamma_1)(1+\gamma_2)}.$$

According to stability analysis of the finite difference scheme, the modulus of each Eigen value of the amplification matrix does not exceed unity. Since the matrix equation (4.11) in triangular form, the Eigen values are its diagonal elements. Therefore, the Eigen values of the amplification matrix are $\frac{(1-\gamma_1)}{(1+\gamma_1)}$, $\frac{(1-\gamma_2)}{(1+\gamma_2)}$, and $\frac{(1-\gamma_3)}{(1+\gamma_3)}$. Since the real part of γ_1 is greater than or equal to zero, thus $\frac{(1-\gamma_1)}{(1+\gamma_1)} \leq 1$, similarly $\frac{(1-\gamma_2)}{(1+\gamma_2)} \leq 1$ and $\frac{(1-\gamma_3)}{(1+\gamma_3)} \leq 1$ Hence, the finite difference scheme is unconditionally stable.

The Crank-Nicolson scheme has a truncation error of $O\left(\Delta t^2 + \Delta R^2\right)$, i.e. the temporal truncation error is significantly small. It tends to zero as Δt and ΔR tend to zero. So, the scheme is compatible. Stability and compatibility ensures convergence.

5. Result and Discussions

The focus of this paper to explain the study of flow behaviour of MHD fluid for different parameters are shown graphically in Figures 1 to 9. It is observed from Figure 1, when the mass Grashof number is increased then the velocity is increased throughout the boundary layer region. From Figure 2, it is deduced that velocity of flow increases with thermal Grashof number G_r. It is noticed from Figure 3, velocity of fluid decline when the surface angle (α) is increased. It is observed that when acceleration parameter is increased then the velocity is increased (Figure 4). Further, we observe that when permeability parameter K is increased then the velocity is increased (Figure 5), which is obvious due to the fact that increase in porosity helps in free movement of the particles of fluid. It is deduced that when angular frequency ω is increased then the velocity gets decreased (Figure 6). It is observed from Figure 7 that the effect of increasing values of the parameter M results in decreasing the fluid velocity. It is due to the facts of transverse magnetic field that acts as Lorentz's force which retards the motion of flow. It is deduced that when chemical reaction parameter Ko is increased then the velocity is decreased (Figure 8). Further from Figure 9, the numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. It is noticed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of P_r are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of P_r. Hence in the case of smaller Prandtl number as the thermal boundary later is thicker and the rate of heat transfer is reduced.

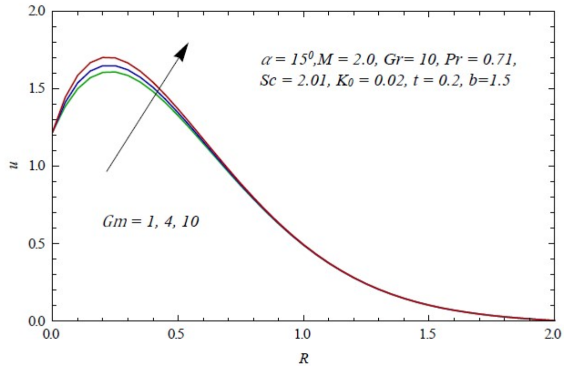

FIGURE 1. Velocity u for different values of G_m

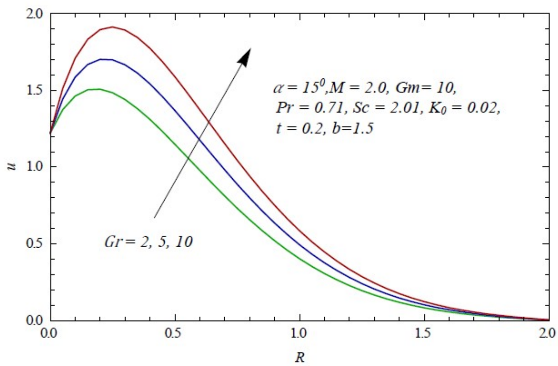

FIGURE 2. Velocity u for different values of G_r

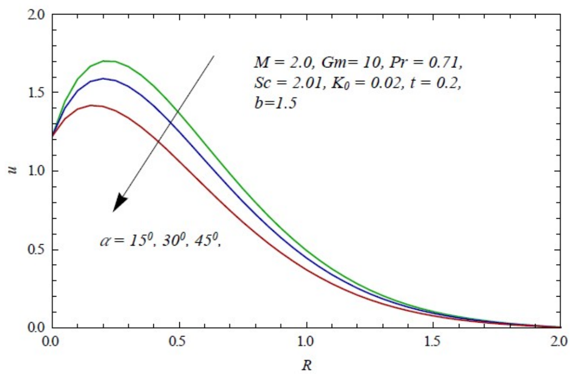

FIGURE 3. Velocity u for different values of α

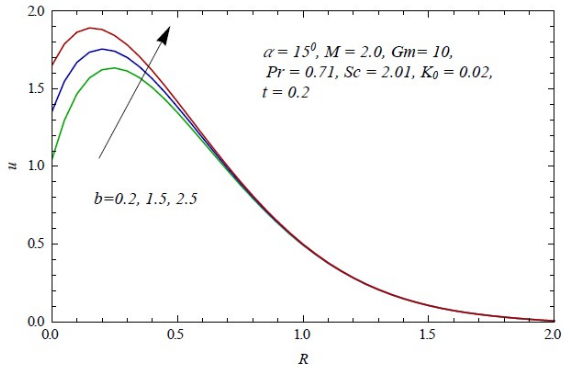

FIGURE 4. Velocity u for different values of b

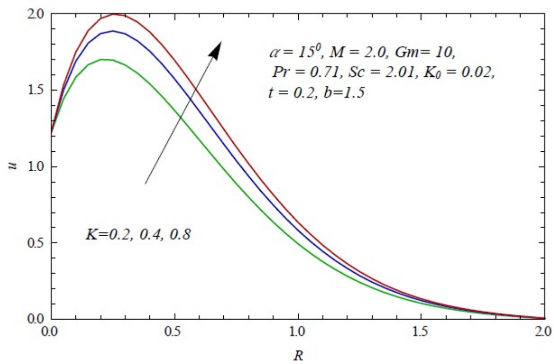

FIGURE 5. Velocity u for different values of K

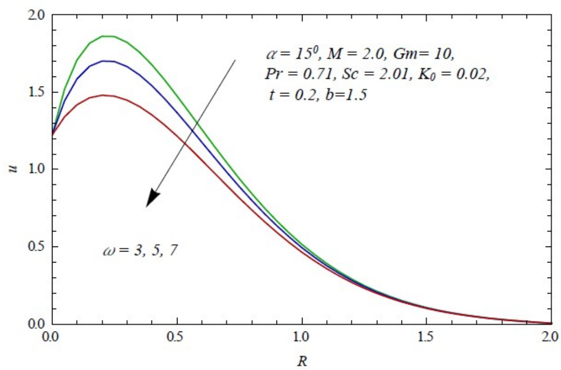

FIGURE 6. Velocity u for different values of ω

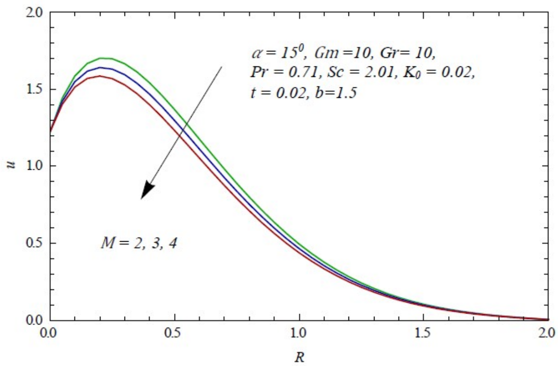

FIGURE 7. Velocity u for different values of M

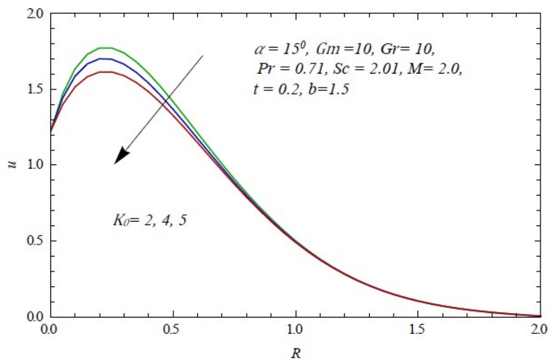

FIGURE 8. Velocity u for different values of Ko

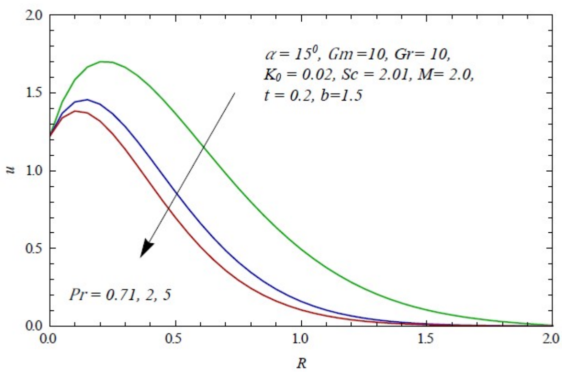

FIGURE 9. Velocity u for different values of P_r

6. Conclusion

A numerically study has been done for the model under consideration by converting the governing partial differential equations into non-dimensional form. The model consists of equations of motion, diffusion and energy equation. To investigate the solutions obtained, standard sets of the values of the parameters have been taken. The numerically result obtained is discussed with the help of graphs and table. We found that the numerically result obtained is in concurrence with the actual flow behaviour of MHD fluid.

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