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EFFECT OF POROUS MEDIUM BED ON VISCOUS FLUID FLOW DOWN RECTANGULAR INCLINED CHANNEL WITH VARYING PERMEABILITY

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Abstract

In the present investigation, viscous fluid flowing down a rectangular inclined channel with permeable beds of varying permeability is considered. We have applied the Beavers and Joseph slip boundary condition at the fluid-porous medium interface. The expressions for the velocity and the flux across the cross-section of the channel are derived and discussed. Expressions are also derived for particular cases when the width of the channel is infinite and when the beds are impermeable.

Keywords and phrases: LaTeX,Permeable beds, Permeability, Viscous fluid, Rectangular Inclined Channel..

1. Introduction

Flow problems involving porous media have important applications in various disciplines like petroleum engineering, geophysics, and agricultural engineering. The flow of viscous fluid under gravity in an inclined channel has applications, particularly in the design of drainage, flood discharge channels, and irrigation canals.

Yih [1] investigated theoretically the stability characteristics of a viscous liquid layer flowing down an open inclined plane of infinite width.Many others [2–6] further studied it with impermeable or permeable (Darcy's model) beds of infinite or finite width. Rudraiah et al.[7]studied the natural convection in an inclined channel bounded by permeable material on both sides in the presence of buoyancy force and discussed in detail the advantages of this physical configuration for laboratory experiments. Ramakrishna et al.[8] considered the flow of fluid of variable viscosity in an inclined channel bounded by two permeable layers. Vasseur et al.[9] considered the Brinkman extended Darcy model to study the flow and heat transfer characteristics in an inclined porous slot. The model used to describe the flow in the cavity accounts for Brinkman friction. It is shown that the boundary effect, though not important in low-porosity media, becomes significant in high-porosity media. Chauhan and Soni [11] is investigated stratified viscous flow down an open rectangular channel with porous medium bottom. Malashetty et al. [12] studied flow and heat transfer in an inclined channel bounded by porous layers. Liu et al. [13] studied convective Poiseuille-Couette flow in an inclined composite porous medium channel. Verma et al. [14] investigated an analytical solution for the slow flow of a fluid past a porous sphere with radial variation of permeability applying the Brinkman model and also discussed the effect of several parameters like the permeability variation, the Brinkman number, and the radius ratio on the flow characteristics and the drag force. Verma and Datta [15] discussed the flow of fluid through an annular channel that is filled with a porous medium of variable permeability and also discussed the effects of various parameters on the flow behaviour, including the permeability of the porous medium, the geometry of the channel, and the Reynolds number of the fluid. Khan et al. [16]discussed two-dimensional steady viscous flow in a rectangular converging or diverging plain wall. Verma and Singh [17] studied the flow of fluid in a composite cylindrical channel that is covered with a porous layer of variable permeability. Many researchers [18–21] explored the behaviour of nanofluids under various conditions. They studied the effects of thermal radiation, magnetic fields, and chemical reactions on the squeezing motion, shrinking surface, and stretching/shrinking sheet of different nanofluid systems. Aamir et al. [22] investigated the thermo diffusion and diffusion effects between two rectangular plain walls with a heat sink or source. Khaleel et al.[23] investigated two-dimensional gaseous slip flows in microchannels filled with porous material analytically in the presence of an electromagnetic field. Many investigators[24–29] studied boundary layer nanofluid flow in a divergent channel, MHD non-Newtonian fluid flow past a stretching sheet, and the effects of non-linear thermal radiation on unsteady electrical MHD motion of nanofluids. In this paper, the viscous incompressible fluid flow down a rectangular inclined channel with permeable beds on the upper and lower sides is considered. The flow problem is analyzed when the flow inside the bounding upper and lower permeable beds is governed, by Darcy's law with varying permeabilities of the porous material.

2. FORMULATION OF THE PROBLEM

A steady laminar flow of viscous, incompressible fluid is considered, down an inclined channel of finite depth with porous beds of varying permeability, and finite width 2b with impermeable walls normal to the surface of the channel bottom, which is taken to be inclined at an angle β $(0 \leq \beta \leq \frac{\pi}{2})$ to the horizontal. The Navier -Stokes equations for the free fluid region are

$$\nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + g\,sin\beta - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0, \tag{2.1}$$

$$g\,cos\beta - \frac{1}{\rho}\frac{\partial p}{\partial y} = 0, \tag{2.2}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = 0, \tag{2.3}$$

and the equation of continuity is

$$\frac{\partial u}{\partial x} = 0 \tag{2.4}$$

The flow in the lower porous bed is governed by the Darcy 's equation

$$Q_1 = \frac{K_1}{\mu}(-\frac{\partial p}{\partial x} + \rho g sin\beta), \tag{2.5}$$

and the flow in the upper porous bed is governed by the Darcy's equation

$$Q_2 = \frac{K_2}{\mu}(-\frac{\partial p}{\partial x} + \rho g sin\beta), \tag{2.6}$$

Here we have assumed

$$K_1 = K_{11}e^{-\theta y} \tag{2.7}$$

and

$$K_2 = K_{21}e^{-\theta y} \tag{2.8}$$

The Boundary conditions are

$$at\ z = \pm b, u = 0 \tag{2.9}$$

$$at \qquad y = 0, \frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{K_1}}(U_1 - Q_1), \tag{2.10}$$

and

$$at \qquad y = h, \frac{\partial u}{\partial y} = -\frac{\alpha}{\sqrt{K_2}}(U_2 - Q_2), \tag{2.11}$$

Where u = U_1 at y=0 , $u = U_2$ at y=h, and α is a constant, non-dimensional characteristic of the structure of the porous material.
Here, u , is the velocity in the free fluid in the channel, Q_1 and Q_2 are Darcy's velocities in the lower porous bed and upper porous bed respectively, p the pressure, μ, the viscocity , ρ, the density, ν , the kinematic viscosity, g , constant of gravity, and K_1 , K_2 are the permeabilities of the lower and upper porous beds respectively.
Taking the mean flow velocity U to be the characteristic velocity and introducing the following non-dimensional quantities

$$\begin{aligned} &x^* = \frac{x}{h}, y^* = \frac{y}{h}, z^* = \frac{z}{h}, u^* = \frac{u}{U}, Q_1^* = \frac{Q_1}{U}, Q_2^* = \frac{Q_2}{U}, K_1^* = \frac{K_1}{h^2}, \\ &K_2^* = \frac{K_2}{h^2}, U_1^* = \frac{U_1}{U}, U_2^* = \frac{U_2}{U}, p^* = \frac{p}{\rho U^2} \qquad and \quad \theta^* = \theta h \end{aligned} \tag{2.12}$$

and dropping the asterisks, equations (2.1) to (2.8) and boundary conditions (2.9) to (2.11) are transformed to, in free fluid region

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = R\frac{dp}{dx} - \frac{R}{F}sin\beta, \tag{2.13}$$

in lower permeable bed

$$Q_1 = -K_1 R(\frac{dp}{dx} - \frac{1}{F} sin\beta), \tag{2.14}$$

in upper permeable bed

$$Q_2 = -K_2 R(\frac{dp}{dx} - \frac{1}{F} sin\beta), \tag{2.15}$$

$$K_1 = K_{11} e^{-\theta y}, \tag{2.16}$$

$$K_2 = K_{21} e^{-\theta y}, \tag{2.17}$$

and boudary conditions

$$at\ z = \pm\sigma, u = 0, \tag{2.18}$$

$$at\ y = 0,\ \frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{K_1}}(U_1 - Q_1) \tag{2.19}$$

$$and\ y = 1,\ \frac{\partial u}{\partial y} = -\frac{\alpha}{\sqrt{K_2}}(U_2 - Q_2), \tag{2.20}$$

where R= $\frac{Uh}{\nu}$ is Reynolds number , F = $\frac{U^2}{gh}$ is Froude number, and $\sigma = \frac{b}{h}$, $u = U_1$ at y=0 and u =U_2 at y=1.

3. METHOD OF SOLUTION

Put

$$Z = \frac{2\sigma\xi}{\pi} - \sigma$$

in equation (2.13), we have

$$\frac{\partial^2 u}{\partial y^2} + \frac{\pi^2}{4\sigma^2}\frac{\partial^2 u}{\partial \xi^2} = A, \tag{3.1}$$

where A = $R\frac{dp}{dx}$ - $\frac{R}{F} sin\beta$,
The boundary conditions are

$$at\ \xi = 0\ and\ \xi = \pi, u = 0, \tag{3.2}$$

$$at\ y = 0,\ u = U_1, \tag{3.3}$$

and

$$at\ y = 1\ , u = U_2, \tag{3.4}$$

using finite sine - transform on equation (3.1) and boundary conditions (3.2) to (3.4), we get

$$\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\pi^2 n^2}{4\sigma^2}\bar{u} = A(\frac{1 - cosn\pi}{n}) \tag{3.5}$$

where $\bar{u} = \int_0^\pi u\, sinn\xi\, d\xi$, n is a positive integer and the boundary conditions :

$$at\ y = 0, \bar{u}(y, n) = \bar{U}_1,$$

$$at\ y = 1, \bar{u}(y, n) = \bar{U}_2.,$$

4. SOLUTION

On solving equation (3.5) under the corresponding boundary conditions and using inversion formula for finite sine- transform, we get the expression for the velocity profile in the channel, as

$$u = \frac{2}{\pi}(R\frac{dp}{dx} - \frac{R}{F}sin\beta)\Sigma_{n=1}^{\infty}(\frac{1 - cosn\pi}{n})(A_1 e^{\frac{\pi n}{2\sigma}y} + A_2 e^{-\frac{\pi n}{2\sigma}y} - \frac{4\sigma^2}{\pi^2 n^2})sinn\xi, \tag{4.1}$$

where

$A_1 = \frac{[(K_{11} - \frac{4\sigma^2}{\pi^2 n^2})A_6 - (\frac{4\sigma^2}{\pi^2 n^2} - K_{21}e^{-\theta})A_4]}{A_3 A_6 - A_4 A_5)}$

$A_2 = \frac{1}{A_4}(A_3 A_1 - K_{11} + \frac{4\sigma^2}{\pi^2 n^2})$

$A_3 = \frac{\pi n}{2\sigma}\frac{\sqrt{K_{11}}}{\alpha} - 1$

$A_4 = \frac{\pi n}{2\sigma}\frac{\sqrt{K_{11}}}{\alpha} + 1$

$A_5 = (e^{\frac{\pi n}{2\sigma}})(\frac{\pi n}{2\sigma}\frac{\sqrt{K_{21}}}{\alpha}e^{-\frac{\theta}{2}} + 1)$

$A_6 = (e^{\frac{-\pi n}{2\sigma}})(\frac{\pi n}{2\sigma}\frac{\sqrt{K_{21}}}{\alpha}e^{-\frac{\theta}{2}} - 1)$

5. PARTICULAR CASES

(i) When $K_{11} = K_{21} \rightarrow 0$, we get the velocity profile for the case when the channel beds are impermeable,

$$u = \frac{8\sigma^2}{\pi^3}(R\frac{dp}{dx} - \frac{R}{F}sin\beta)\Sigma_{n=1}^{\infty}\frac{(1 - cosn\pi)}{n^3}[(e^{-\frac{\pi n}{2\sigma}y} - 1) - (e^{-\frac{\pi n}{2\sigma}y} - 1)\frac{sinh\frac{\pi n y}{2\sigma}}{sinh\frac{\pi n}{2\sigma}}]sinn\xi \tag{5.1}$$

(ii) Taking $\sigma \rightarrow \infty$, we get the flow down an infinite inclined channel with permeable beds,

$$u = (R\frac{dp}{dx} - \frac{R}{F}sin\beta)(\frac{y^2}{2} + B_1 y + B_2), \tag{5.2}$$

where

$B_1 = [\frac{\alpha}{\sqrt{K_{11}}}B_2 + \alpha\sqrt{K_{11}}]$

$B_2 = \frac{-\sqrt{K_{11}}[\alpha\sqrt{K_{11}}(K_{21} + \alpha e^{\frac{\theta}{2}}) + \sqrt{K_{21}} + \frac{\alpha}{2}e^{\frac{\theta}{2}} + \alpha K_{21}e^{-\frac{\theta}{2}}]}{[\alpha(\sqrt{K_{21}} + \alpha e^{\frac{\theta}{2}}) + \sqrt{K_{11}}\alpha e^{\frac{\theta}{2}}]}$

(iii) When $\sigma \rightarrow \infty, K_{11} = K_{21} \rightarrow 0$, we get the flow down an inclined channel with impermeable beds on both the sides,

$$u = \frac{1}{2}(R\frac{dp}{dx} - \frac{R}{F}sin\beta)y(y - 1) \tag{5.3}$$

6. FLUX

Volume rate of flow, i.e. flux across the crosssection of the channel perpendicular to the x-axis is given by volume rate of flow, i.e. flux across the crosssection of the channel perpendicular to the x-axis is given by

$$V = \int_{-\sigma}^{\sigma}\int_{0}^{1} udydz = \frac{8\sigma^2}{\pi^3}(R\frac{dp}{dx}-\frac{R}{F}sin\beta)\Sigma_{n=1}^{\infty}\frac{(1-cosn\pi)^2}{n^3}[A_1(e^{\frac{\pi n}{2\sigma}}-1)-A_2(e^{-\frac{\pi n}{2\sigma}}-1)-\frac{2\sigma}{\pi n}] \quad (6.1)$$

Table 1. Flux across the transverse cross-section of the channel

α	K_{11}	K_{21}	V(Flux)
	0	0	0.2893946
0.01	0	10^{-4}	0.6344652
0.01	10^{-4}	10^{-4}	1.386310
1.45	0.3	0.5	1.963507
1.45	0.5	0.3	1.978251

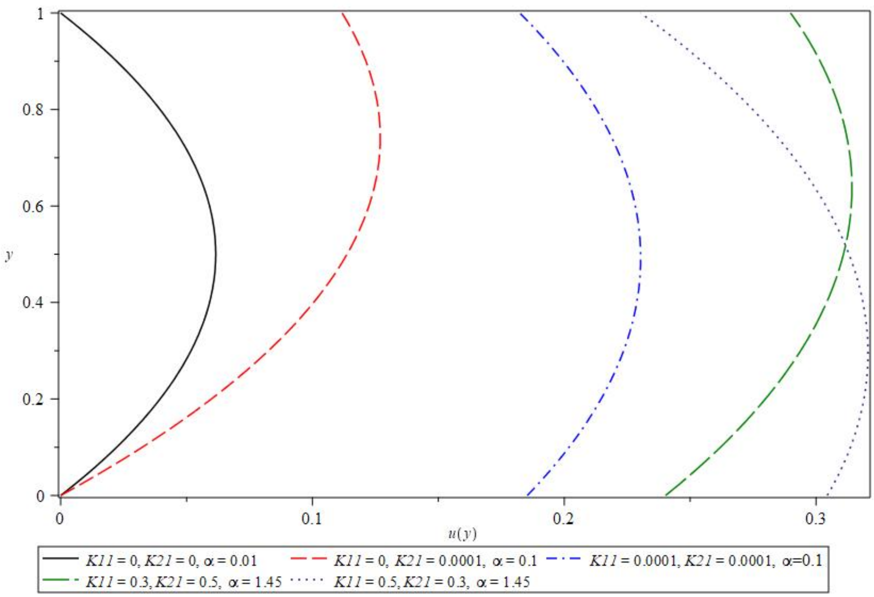

Figure 1. Velocity profiles for $\beta = \frac{\pi}{6}$, $\sigma = \frac{\pi}{2}$, $\theta = 0.1$ and $F = R = -\frac{\partial p}{\partial x} = 1$

7. Discussions

Figure 1 shows the velocity profiles at the central transverse section ($\xi = \frac{\pi}{6}$) of the channel flow for $\beta = \frac{\pi}{2}$, $\sigma = \frac{\pi}{2}$ and for different values of the permeability parameters

exponentially varying with $\theta = 0.1$ of the upper and lower porous beds, and compares the results with the case when the beds are rigid walls. we find that when the lower bed is impermeable and upper bed is permeable then the velocities at all sections are greater than when both beds are impermeable. Further, the flow is much increased in the channel when both the beds are permeable. Morever, when the permeability of the upper bed is greater than the lower bed, then the velocity near the upper bed is greater,and so is the case when the lower bed permeability is greater than the upper one, the flow near the lower bed has greater velocity.
The particular case, when $\sigma \to \infty$, that is ,when the width of the channel is infinite, is also studied, and it is found that in this case, the flow is increased by the introduction of permeability in comparison to the non permeability case. However, for the same permeability parameters of the porous beds the velocities are greater at all sections when the width is finite than when the width is infinit. it is also true when the beds are impermeable. Table 1 shows the variations of the flux across the cross-section perpendicular to the length of the channel. It is found that the flux is also greater when the beds are permeable than when they are impermeable. it is also noted that the flux is grater when k_{11}=0.5 and $K_{21} = 0.3$., Than when k_{11}=0.3 and $K_{21} = 0.5$. The results are interesting and may be used to control the flow in the inclined channel by adjusting the permeability of the beds.

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