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EFFECTS OF URBANIZATION AND INDUSTRIALIZATION ON HUMAN POPULATION: A MODELING STUDY

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Abstract

A mathematical model is formulated using the system of non-linear differential equations to see the effects of urbanization and industrialization on the human population. In the model, we have considered five variables namely; Susceptible population, Infected population, Urbanization, Industrialization and bacterial population. Conditions for the local stability of non negative equilibrium point are obtained. The verification of the system given by numerically and graphically.

Keywords and phrases: Mathematical Model,Equilibria and Susceptible and Infected populations, Urbanization, Industrilization .

1. Introduction

Industrialization and urbanization are two important factors for our environment. Basically, urbanization is the process by which large number of people become permanently cocentrated in relatively small areas forming cities. When large number of people congregate in cities, many problems occurs. The main problem that our society is faces today is the depletion of forestry resources. It is well known that forestry resources plays an important role in our life. But it being depleted by industrialization due to urbanization[8],[15]. Rapid urbanization leads to the exploitation of the natural resources for the construction industries, transport and consumption. Urbanization is the chief agent of increasing the industrialization[14],[16],[17]. Although industrialization is necessary for the growth of our country but it also have many negative impacts. Disease accounted for many death in industrial cities during industrial revolution. With a chronic lack of hygiene, little knowledge of sanitary, disease such as cholera, typhoid, typhes etc. occurs [18].

Many researchers has been developed the non linear mathematical model to investigate the effect of industrialization due to urbaization. A.K. Misra et.al. (2014) proposed a mathematical model for the depletion of forestry resources due to population pressure augmented industrialization. In this paper, they found that as the population pressure increase, the level of the industrialization increases leading to decrease in the

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cumulative biomass density of forestry biomass. They also found that if the population pressure is controlled by using some economic efforts, the decrease in cumulative biomass density of forestry resources can be made much less than the case when no control is applied. A.K. Misra et.al. (2013) also have taken a mathematical model to see the effects of population and population pressure on the forest resources and their conservation. From the model analysis they shows that as the density of population or population pressure increases, the cumulative density of forestry resources decreases and the resource may become extinct, if the population pressure become too large. They also noted that by controlling the population pressure using some economic incentives, the density of forest resources can be maintained at an equlibrium level, which is population density dependent. B.Dubey et.al.(2010) studied the effects of industrialization, population and pollution on a renewable resources. They obtained that if the densities of industrialization, population and pollution increase, then the density of the resource biomass decreases and it is settles down at the equilibrium level whose magnitude is lower than its original carraying capacity. J.B. Shukla et.al. (1997) proposed a mathematical model for the depletion and conservation of forestry resources. They conclude that the resource biomass can be maintained at a desired level by conserving the forestry resources and by controlling the growth of population and the emission rate of polluatnt in the habitat. Abhinav Tandon et. al. (2016) have proposed a mathematical model to investigate the effects of environmental pollution intensified by urbanization. They shown that the growth of population is responsible for the growing urbanization, but for very large increase of urbanization the population may not survive in the long run. Manju Agarwal et. al. (2009) presented a mathematical model for the depletion of forestry resources biomass due to industrialization pressure .

In this paper, a model for interaction between forestry biomass, wild life population and industrialization pressure is proposed and the effects of forestry biomass depletion in a forested habitat caused by industrialization pressure on the survival of the forestry biomass dependent wilidlife speciess is studied. Niharika Verma et.al.(2017) have taken a mathematical model for the increase in global warming due to CO_2 intensified by industrialization. In their analysis they shown that the reforestation is a key factor for controlling the global warming into environment. Niharika Verma et.al.(2018) also have proposed a mathematical model to study the effect of environmental tax for CO_2 on the forestry biomass depleted by industrialization. O.P.Misra et. al. (2012) studied the modeling and analysis of a single species population with viral infection in polluted environment. In their paper, they have shown that when the effect of pollution is not considered then the susceptible population never vanish and on the other hand if the effect of environmental pollution has been consider the susceptible population can vanish.

Keeping all these papers in mind, In this paper, we proposed a mathematical model to see the effect of urbanization and industrialization on human population. The organization of the paper is given as: section 1 is related to introduction. Mathematical model is introduced in section 2. In section 3, we introduced the boundedness of the system. Section 4 and 5 related to the equilibrium analysis and stability analysis respectively. Furthermore, we illustrated our results in section 6.

2. Mathematical Model

In the model, we have supposed imigration rate of human population from outside region is constant. Susceptible population become infected due to increase in bacterial population. We have taken that infected population is decreases due to some therapic treatement and assumed that urbanization is also decreases due to some government activities. Further we have taken that indusrialization is increases due to urbanization. In the view of above, a model governing the dynamics of the system under consideration proposed as follows:

$$\frac{dS}{dt} = A - \beta S I - \lambda S B - \lambda_1 S + \nu T, \tag{2.1}$$

$$\frac{dI}{dt} = \beta S I + \lambda S B - \nu T - \lambda_2 I, \tag{2.2}$$

$$\frac{dU_r}{dt} = \beta_0 N - \beta_1 U_r, \tag{2.3}$$

$$\frac{dI_n}{dt} = \gamma I_n - \frac{\gamma_0 I_n^2}{K} + \alpha U_r, \tag{2.4}$$

$$\frac{dB}{dt} = Q(I_n) - \alpha_1 B, \tag{2.5}$$

with the initial conditions and $S(t) \geq 0, I(t) \geq 0, U_r(t) \geq 0, I_n(t) \geq 0, B(t) \geq 0,$

where

$Q(I_n) = Q_0 + Q_1 I_n$, and $N = S + I$.

In the model system given by equation (2.1) to (2.5), $S(t)$ and $I(t)$ are densities of susceptible and infected population at time t. At time t, $U_r(t)$ and $I_n(t)$ represents the urbanization and Industrialization and $B(t)$ is the bacterial population. In the the model constants are given as;

A= constant imigration rate of human population from outside the region under the consideration.

β= it is the rate at which susceptible population become infected.

λ= it is the rate at which susceptible population decreases due to bacteria.

λ_1= natural death rate of susceptible population.

ν= increase in susceptible population through treatement.

λ_2= natural death rate of infected population.

β_0= Increase in urbanization due to total population (susceptible and infected population).

β_1= it is the rate at which unplanned urbanization is controlled due to some government agencies..
K= carrying capacity of industrialization.
α = it is the rate at which industrialization is increased due to urbanization.
α_1 = natural death rate of bacterial population.

3. Boundedness of the System

LEMMA 3.1. *The solution of the system given by equations (2.1) to (2.5) is bounded within the following region:*

$$w = \left\{(S, I, U, I_n, B) : 0 < N \leq \frac{A}{\lambda_1}, 0 < U_r \leq \frac{\beta_0 A}{\lambda_1 \beta_1}, 0 < I_n \leq \frac{R}{\delta}, 0 < B \leq \frac{Q_0\delta + Q_1 R}{\delta\alpha}\right\}.$$

Proof: From the equation (2.1) and (2.2),

$$\frac{dN}{dt} \leq A - \lambda_1 S - \lambda_2 I,$$
$$\leq A - \lambda_1 (S + I),$$
$$\leq A - \lambda_1 N,$$

by comparision theorem as $t \to \infty$:

$$N_{\max} = \frac{A}{\lambda_1},$$

provided $\lambda_1 = \lambda_2$.
From the equation (2.3),

$$\frac{dU_r}{dt} \leq \beta_0 N_{\max} - \beta_1 U_r,$$
$$\leq \frac{\beta_0 A}{\lambda_1} - \beta_1 U_r,$$

by comparision theorem as $t \to \infty$:

$$U_{\max} = \frac{\beta_0 A}{\lambda_1 \beta_1}.$$

From the equation (2.4),

$$\frac{dI_n}{dt} \leq \gamma I_n - \frac{\gamma_0 I_n^2}{K} + \alpha U_{r_{\max}},$$
$$\leq \gamma I_n - \frac{\gamma_0 I_n^2}{K} + \frac{\alpha\beta_0 A}{\lambda\beta_1},$$

$$\frac{dI_n}{dt} + \delta I_n \leq \frac{K\gamma^2}{2\gamma_0} - \frac{K^2\gamma^3}{4\gamma_0^2} + \frac{\alpha\beta_0 A}{\lambda\beta_1},$$

where δ is a very small quantity,
by comparision theorem as $t \to \infty$:

$$(I_n)_{\max} = \frac{R}{\delta},$$

where

$$R = \frac{K\gamma^2}{2\gamma_0} - \frac{K^2\gamma^3}{4\gamma_0^2} + \frac{\alpha\beta_0 A}{\lambda\beta_1}.$$

$$\begin{aligned}\frac{dB}{dt} &\leq Q_0 + Q_1(I_n)_{\max} - \alpha B,\\ &\leq Q_0 + \frac{Q_1 R}{\gamma\delta} - \alpha B,\end{aligned}$$

by comparision theorem as $t \to \infty$:

$$B_{\max} = \frac{Q_0\delta + Q_1 R}{\delta\alpha}.$$

This complete the proof of lemma (3.1).

4. Existence of the Equilibrium Points

After analysis of the model we found that system has only one non negative equilibrium point namely $E(S^*, I^*, U_r^*, I_n^*, B^*)$. The value of $S^*, I^*, U_r^*, I_n^*, B^*$ is given by

$$A - \beta S^* I^* - \lambda S^* B^* - \lambda_1 S^* + \nu T = 0, \tag{4.1}$$

$$\beta S^* I^* + \lambda S^* B^* - \nu T - \lambda_2 I^* = 0, \tag{4.2}$$

$$\beta_0(S^* + I^*) - \beta_1 Ur^* = 0, \tag{4.3}$$

$$\gamma I_n^* - \frac{\gamma_0 I_n^{*2}}{K} + \alpha U_r^* = 0, \tag{4.4}$$

$$Q_0 + Q_1 I_n^* - \alpha_1 B^* = 0. \tag{4.5}$$

By adding equations (4.1) and (4.2),

$$-\lambda_1 S^* - \lambda_2 I^* + A = 0,$$

$$A = \lambda_1(S^* + I^*).$$

From equation (4.3),

$$\frac{\beta_0 A}{\lambda_1} - \beta_1 U_r^* = 0,$$

$$U_r^* = \frac{\beta_0 A}{\lambda_1 \beta_1}.$$

From equation (4.4),

$$\gamma I_n^* - \frac{\gamma_0 I_n^{*2}}{K} + \alpha U_r^* = 0,$$

$$\frac{\gamma_0 I_n^{*2}}{K} - \gamma I_n^* - \frac{\alpha \beta_0 A}{\lambda_1 \beta_1} = 0,$$

$$I_n^* = \frac{\gamma + \sqrt{P}}{2\gamma_0} > 0,$$

where

$$P = \gamma^2 + \frac{4\gamma_0 \alpha \beta_0 A}{K \lambda_1 \beta_1}.$$

From equation (4.5),

$$Q_0 + Q_1 P - \alpha_1 B^* = 0,$$

$$B^* = \frac{Q_0 + Q_1 P}{\alpha_1}.$$

Now From equation (4.2),

$$\frac{\beta S^*(A - \lambda_1 S^*)}{\lambda_1} + \frac{\lambda S^*}{\alpha_1}(Q_0 + Q_1 P) - \nu T - \frac{\lambda_2(A - \lambda_1 S^*)}{\lambda_1} = 0,$$

$$S^* = \frac{D_1 + \sqrt{D}}{2\lambda_1 \beta \alpha_1} > 0,$$

provided $D > 0$,
where

$$D_1 = Q_0 \lambda_1 \lambda + Q_1 P \lambda_1 \lambda + \beta A \alpha_1 + \alpha_1 \lambda_1 \lambda_2,$$

.

$$D = (Q_0\lambda_1\lambda + \beta\alpha_1 A + Q_1 P\lambda_1\lambda + \lambda_2\lambda_1\alpha_1)^2 - 4\lambda_1\beta\alpha_1(\nu T\lambda_1\alpha_1 + \lambda_2 A\alpha_1).$$

Now from equation (4.1),

$$A - \beta\left(\frac{D_1 + \sqrt{D}}{2\lambda_1\beta\alpha_1}\right)I^* - \lambda\left(\frac{D_1 + \sqrt{D}}{2\lambda_1\beta\alpha_1}\right)\left(\frac{Q_0 + Q_1 P}{\alpha_1}\right) - \lambda_1\left(\frac{D_1 + \sqrt{D}}{2\lambda_1\beta\alpha_1}\right) + \nu T = 0,$$

$$I^* = \frac{2\lambda_1\beta\alpha_1(A + \nu T)}{D_1 + \sqrt{D}} - \left\{\lambda\left(\frac{Q_0 + Q_1 P}{\alpha_1}\right) + \lambda_1\right\} > 0,$$

provided

$$\frac{2\lambda_1\beta\alpha_1(A + \nu T)}{D_1 + \sqrt{D}} > \lambda\left(\frac{Q_0 + Q_1 P}{\alpha_1}\right) + \lambda_1.$$

5. Stability Analysis

Theorem 5.1 The interior equilibrium point is $E(S^*, I^*, U_r^*, I_n^*, B^*)$ nonlinearlly locally asymptotically stable within the region of attraction given by w provided following inequilities is satisfied;

$$a_{15}^2 < \frac{2}{3}a_{11}a_{55},$$
$$a_{25}^2 < \frac{2}{3}a_{22}a_{55},$$
$$a_{13}^2 < \frac{2}{3}a_{33}a_{11},$$
$$a_{23}^2 < \frac{2}{3}a_{22}a_{33},$$
$$a_{34}^2 < \frac{2}{3}a_{33}a_{44},$$
$$a_{45}^2 < \frac{2}{3}a_{44}a_{55}$$

where

$$a_{11} = \beta I^* + \lambda B^* + \lambda_1, a_{22} = -K_1\beta S^* + K_1\lambda_2, a_{33} = K_2\beta_1,$$
$$a_{44} = -K_3\gamma + \frac{2\gamma_0}{K}K_3 I_n^*, a_{55} = K_4\alpha_1, a_{15} = -\lambda S^*, a_{25} = K_1\lambda S^*,$$
$$a_{13} = K_2\beta_0, a_{23} = K_2\beta_0, a_{34} = K_3\alpha, a_{45} = K_4 Q_1.$$

Proof: Let us consider $S^*, I^*, U_r^*, I_n^*, B^*$ are the small perturbation around the interior equilibrium point $E(S^*, I^*, U^*, B^*)$.

So we first linearize the model by assuming $S = S^* + S_1, I = I^* + I_1, U_r = U_r^* + U_{r1}, I_n = I_n^* + I_{n1}, B = B^* + B_1$. After linearization, the model is given as:

$$\begin{aligned}
\frac{dS_1}{dt} &= -\beta S^* I_1 - \beta S_1 I^* - \lambda S^* B_1 - \lambda S_1 B^* - \lambda_1 S_1,\\
\frac{dI_1}{dt} &= \beta S^* I_1 + \beta S_1 I^* + \lambda S^* B_1 + \lambda S_1 B^* - \lambda_2 I_1,\\
\frac{dU_{r1}}{dt} &= \beta_0 S_1 + \beta_0 I_1 - \beta_1 U_{r1},\\
\frac{dI_{n1}}{dt} &= \gamma I_{n1} - \frac{2\gamma_0 I_n^* I_{n1}}{K} + \alpha U_{r1}\\
\frac{dB_1}{dt} &= Q_1 I_{n1} - \alpha_1 B_1.
\end{aligned}$$

Let us consider a positive definite function

$$V = \frac{1}{2}S_1^2 + \frac{1}{2}K_1 I_1^2 + \frac{1}{2}K_2 U_{r1}^2 + \frac{1}{2}K_3 I_{n1}^2 + \frac{1}{2}K_4 B_1^2$$

Where K_1, K_2, K_3, K_4 are positive constants taken to be suitably. After differentiating V with respect to t, we get

$$\frac{dV}{dt} = S_1\frac{dS_1}{dt} + K_1 I_1\frac{dI_1}{dt} + K_2 U_{r1}\frac{dU_{r1}}{dt} + K_3 I_{n1}\frac{dI_{n1}}{dt} + K_4 B_1\frac{dB_1}{dt}.$$

After putting the value of $\frac{dS_1}{dt}, \frac{dI_1}{dt}, \frac{dU_{r1}}{dt}, \frac{dI_{n1}}{dt}, \frac{dB_1}{dt}$

$$\begin{aligned}
\frac{dV}{dt} &= -S_1^2(\beta I^* + \lambda B^* + \lambda_1) - I_1^2(-K_1\beta S^* + K_1\lambda_2) - U_{r1}^2(K_2\beta_1) - I_{n1}^2(-K_3\gamma + \frac{2\gamma_0}{K}K_3 I_n^*)\\
&- B_1^2(K_4\alpha_1) + S_1 I_1(-\beta S^* + K_1\beta I^* + K_1\lambda B^*) + B_1 S_1(-\lambda S^*)\\
&+ B_1 I_1(K_1\lambda S^*) + S_1 U_{r1}(K_2\beta_0) + I_1 U_{r1}(K_2\beta_0) + I_{n1}U_{r1}(K_3\alpha) + B_1 I_{n1}(K_4 Q_1),
\end{aligned}$$

now choosing $K_1 = \frac{\beta S^*}{\beta I^* + \lambda B^*}$, we found that $\frac{dV}{dt}$ will be negative definite if

$$a_{15}^2 < \frac{2}{3}a_{11}a_{55}, \tag{5.1}$$

$$a_{25}^2 < \frac{2}{3}a_{22}a_{55}, \tag{5.2}$$

$$a_{13}^2 < \frac{2}{3}a_{33}a_{11}, \tag{5.3}$$

$$a_{23}^2 < \frac{2}{3}a_{22}a_{33}, \tag{5.4}$$

$$a_{34}^2 < \frac{2}{3}a_{33}a_{44}, \tag{5.5}$$

$$a_{45}^2 < \frac{2}{3}a_{44}a_{55} \tag{5.6}$$

where

$$a_{11} = \beta I^* + \lambda B^* + \lambda_1, a_{22} = -K_1\beta S^* + K_1\lambda_2, a_{33} = K_2\beta_1,$$
$$a_{44} = -K_3\gamma + \frac{2\gamma_0}{K} K_3 I_n^* a_{55} = K_4\alpha_1, a_{15} = -\lambda S^*, a_{25} = K_1\lambda S^*,$$
$$a_{13} = K_2\beta_0, a_{23} = K_2\beta_0, a_{34} = K_3\alpha, a_{45} = K_4 Q_1.$$

From equations (5.1) and (5.2) we get

$$K_4 > \max\left\{\frac{3\lambda^2 S^{*2}}{2\alpha_1(\beta I^* + \lambda B^* + \lambda_1)}, \frac{3\lambda^2 S^{*3}}{2\alpha_1(\beta I^* + \lambda B^*)(-\beta S^* + \lambda_2)}\right\},$$

From equations (5.3) and (5.4) we get

$$K_2 < \min\left\{\frac{2\beta_1(\beta I^* + \lambda B^* + \lambda_1)}{3\beta_0^2}, \frac{2\beta_1\beta S^*(-\beta S^* + \lambda_2)}{3\beta_0^2(\beta I^* + \lambda B^*)}\right\} = K_{20}$$

now suppose $K_4 = 1$, From equations (5.5) and (5.6) we get

$$\frac{3}{2}\frac{Q_1^2}{\left(-\gamma + \frac{2\gamma_0}{K} I_n^*\right)\alpha_1} < K_3 < \frac{2K_{20}\beta_1\left(-\gamma + \frac{2\gamma_0}{K} I_n^*\right)}{3\alpha^2}.$$

Hence theorem is proved.

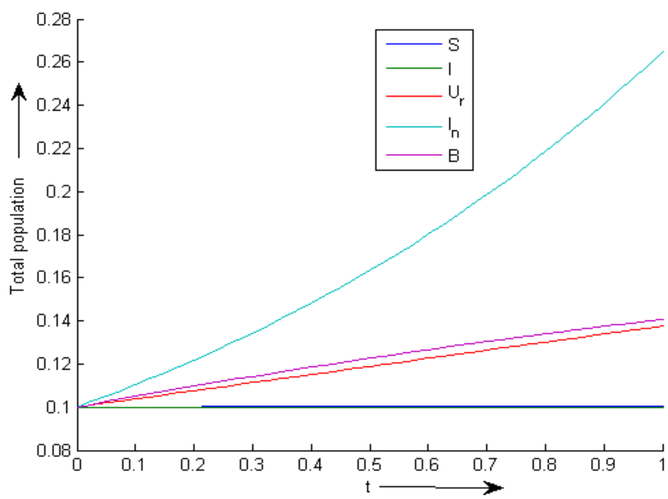

Figure 1. Time series graph

6. Numerical simulation

In this section, we introduce numerical simulation to explain the applicability of the result discussed above. We choose the following set of parameters in the model given by equation (2.1)-(2.5).

$$A = 0.001, \beta = 0.01, \lambda = 0.003, \lambda_1 = 0.002, \lambda_2 = 0.002, \nu = 0.001, T = 0.02, \beta_0 = 0.2$$
$$\beta_1 = 0.02, Q_0 = 0.1, Q_1 = 0.01, \alpha_1 = 0.1, \gamma = 1, \gamma_0 = 1, \alpha = 1.$$

For these values of the parameters the value of interior equilibrium point E is given as $S^* = 0.1004, I^* = 0.0999, U_r^* = 0.1274, I_n^* = 0.2033, B^* = 0.1313$. From these values of the parameters we can varified the conditions of the local stability given by theorem (4.1) and (4.2). Figure 1 is the time series graph. In fogure 2 we shown the variation of urbanization with respect to time t for different value of β_0 and we found that with increase in β_0 unplanned urbanization is increases. Figure 3 is drawn to show the effect of industrialization with respect to time t for different value of β_0 and we obtained that with increase in β_0 industrialization is increases. Figure 4 is drawn to show the effect of bacterial population with respect to time t for different value of β_0. Figure 5 and 6 are plotted for susceptible population and infected population respectively with respect to time t for different value of β_0 and found that with increase in β_0 susceptible population decreases and infected population is increases. So from figure 2,3, 4,5 and 6, we conclude that when urbanization is increases it leads to increase in industrialization due to which bacterial population increases and increase in bacterial population is responsible foe increase in infected population. Figure 7 is graph ofindustrialization with respect to time t for different value of α. It is clear from this figure with the increase in α industrialization is increases. Since with the increase in α industrialization is increases, bacterial population is also increases with the increase in α (from figure 8).

7. Conclusion

In this paper, we have proposed and analyzed a non-linear model to see the effect of urbanization and industrialization on human population.Analysis of model reveals that the obtained model system has only one non negative equilibrium point. The conditions for the local stability of equilibrium point are obtained using the stability theory of the differential equations. Numerical analysis has been done to illustrated the feasibility of the obtained results.

The results of model, qualitatively and numerically shows that growth of urbanization is responsible for growing industrialization. Due to increase in industrialization, bacterial population increases those are responsible for many harmful diseases. These harmful diseases effects the human population.

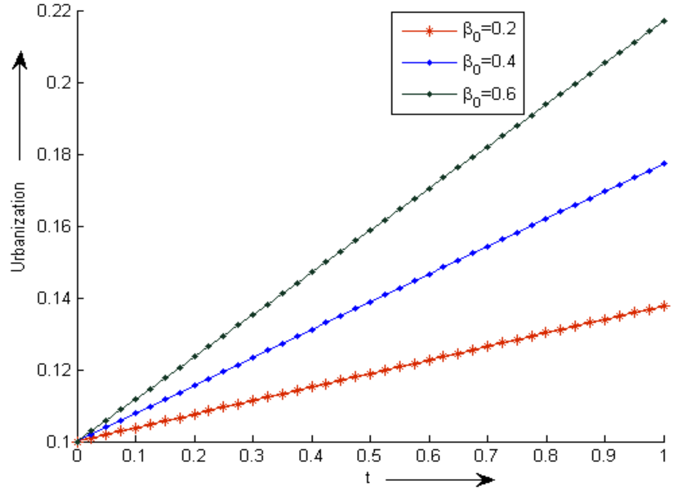

FIGURE 2. Variation of 'Urbanization' with respect to't' for different value of β_0

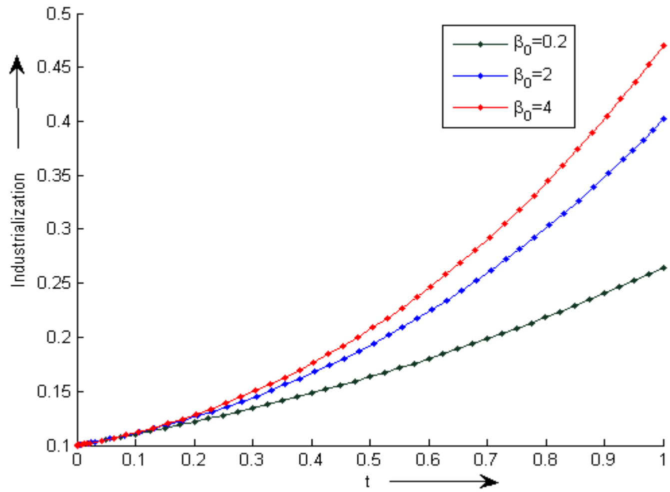

FIGURE 3. Variation of 'Industrialization' with respect to't' for different value of β_0

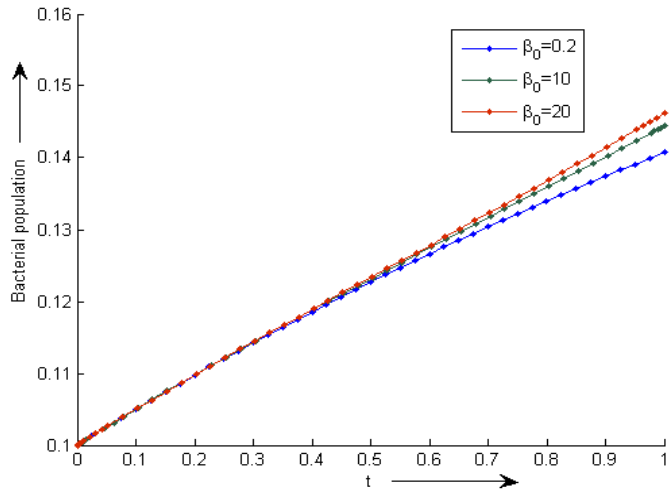

FIGURE 4. Variation of ‘Bacterial population’ with respect to‘t’ for different value of β_0

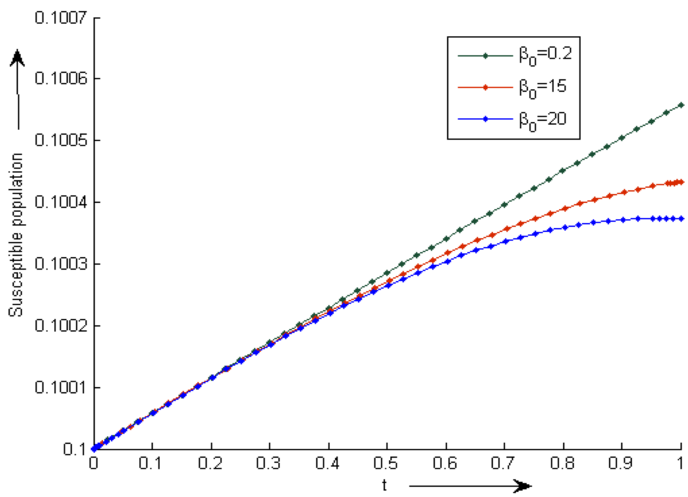

FIGURE 5. Variation of ‘Susceptible population’ with respect to‘t’ for different value of β_0

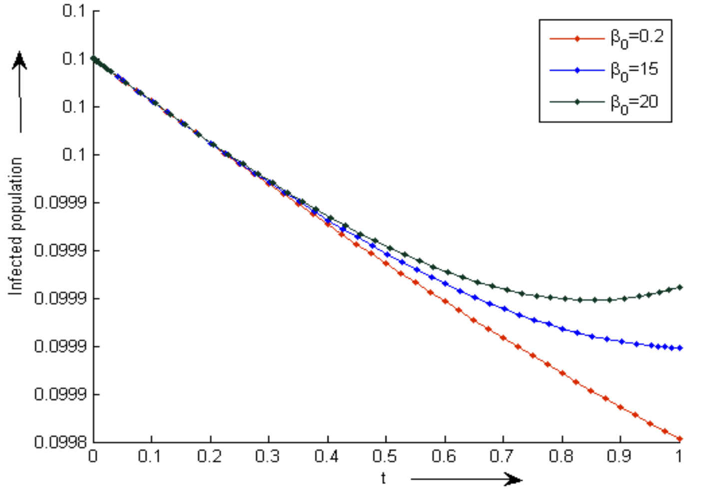

FIGURE 6. Variation of 'infected population' with respect to't' for different value of β_0

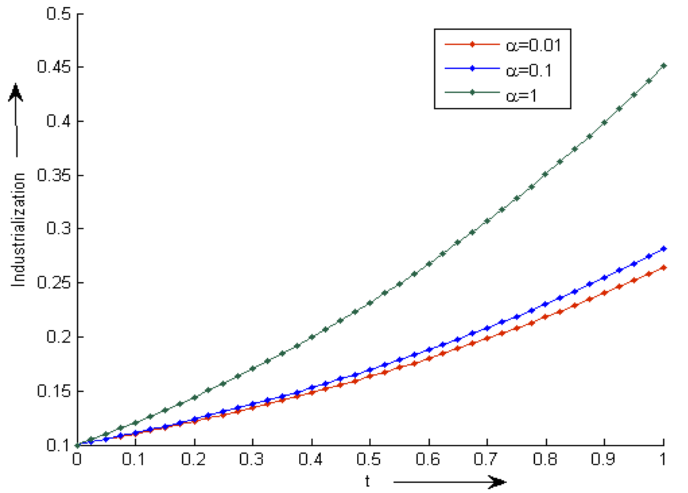

FIGURE 7. Variation of 'Industrialization' with respect to 't' for different value of α

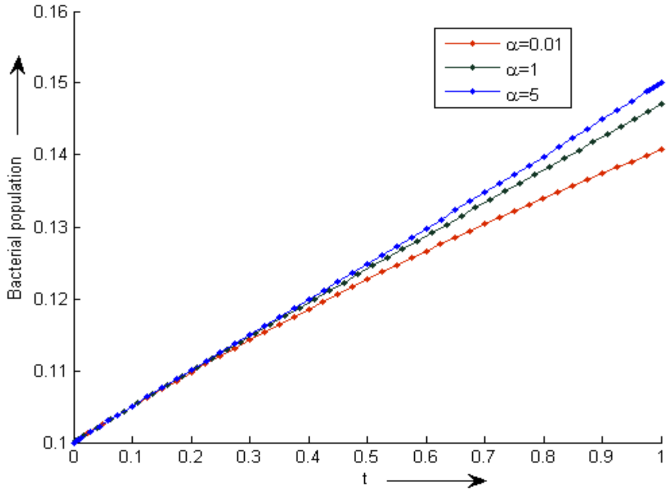

FIGURE 8. Variation of 'Bacterial population' with respect to 't' for different value of α

References

[1] Abhinav Tandon, Jyotsna Kumari, *Modeling the effects of environmental pollution intensified by urbanization on human population*, International journal of modeling simulation, and scientific computing **7** (2016) 1650013.

[2] A.K.Misra, matri verma, Ezio venturino, *Modeling the control of atmospheric carbon dioxide through reforestation: effect of time delay*, Model. earth syst. envion **1** (2015) 24.

[3] A.K.Misra, Kusum Lata, J.B.Shukla, *A mathematical model for the depletion of forestry resources due to population and population pressure augmented industrialization*,International journal of modeling, simulation and scientific computing **5** (2014) 1350022.

[4] A.K. Misra, Kusum Lata, J.B.Shukla, *Effects of population and population pressure on forest resources and their conservation* a modeling study, Eniviron. dev. sustain, springer **16** (2014) 361-374

[5] B.Dubey, A.S.Narayana *Modelling effects of industrialization, population and pollution on a renewable resource*Elsevier, Nonlinear Analysis: Real world Application **11** (2010) 2833-2848.

[6] B.Dubey, S.Sharma, P.Sinha, J.B.Shukla,*Modeling the depletion of forestry resources by population and population pressure augmented industrialization*, Applied mathematical modeling, Elsevier **33** (2009) 3002-3014

[7] J.B. Shukla, B.Dubey, *Modellling the depletion and conservation of forestry resources: effects of population and pollution*, Journal of mathematical biology **36** (1997) 71-94

[8] Jiwei He*Effect of pollution tax on population in a polluted environment*, Applied mathematical sciences **4** (2010) 2993-2998.

[9] Kaiser Manzoor *Unabated urbanization a case related to India with focus on Bangalore*, IJPSS**3** (2013) 436-459.

[10] Manju Agarwal, Tazeen Fatima and H.I.Freedman *Depletion of forestry resource biomass due to industrialization pressure: a ratio dependent model*, Journal of biological dynamics **4** (2009) 381-396.

[11] Niharika Verma, Manju Agarwal *A mathematical model for the increase in global warming due to CO_2 intensified by industrialization, Our earth*,a quaterly publication devoted to ecology and

environmental management ,**14** (2017) 2249-3832.

[12] Niharika Verma, Manju Agarwal *A mathematical model to study the effect of environmental tax for CO_2 on the forestry biomass depleted by industrialization*, International journal of ecological economics and statistics, **39** (2018) 19-40.

[13] O.P. Misra, S. chauhan*Modeling and analysis of a single species population with viral infection in polluted environment*, Journal of applied mathematics, **3** (2012).

[14] Raid Kamel Naji, Kawa Ahmed Hasan *The dynamics of prey predator ,model with disease in prey*,J. math. comput. sci. **2** (2012) 1052-1072.

[15] Suhrit K.Dey, Chanchal Pramanik, Charlie Dey, *Mathematical Modeling of the effects of urbanization and population growth on agricultural economics*, Intellectual Economics **2** (2010) 7-20.

[16] S.P.Bera, A. Maiti, G.P. Samanta *A Prey predator model with infection in both prey and predator*, Filomat**29** (2015) 1753-1767.

[17] Yanni Xiao, Lansun Chen *A ratio dependent predator-prey model with disease in the prey*, A ratio dependent predator-prey model with disease in the prey, Applied mathematics and computation, **131** (2002) 397-414.

[18] Unplanned urbanization a challenge for public health, 5 April 2010, new release India. 2010.

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