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# A STUDY ON VARIATION IN THE CONCENTRATION OF CARBON MONOXIDE IN AN URBAN CITY DUE TO THE SEASONAL VARIATION WITH TEMPERATURE DEPENDENT METEOROLOGICAL PARAMETERS

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**Abstract**

The present study is an attempt to find the variation in the concentration of carbon monoxide due to the change in temperature in an urban city. We know that the season variation causes the change in temperature and, therefore, the meteorological parameters like wind speed, precipitation/humidity, diffusivity coefficients of pollutants are considered to be temperature dependent. The data regarding the meteorological parameters are considered against the average temperature for the past seven years for the individual month from January to December in the city of Lucknow (India). Using the statistical and correlation analysis we established the relationship between these parameters and the temperature. Finally, these expressions were used in the two-dimensional advective-diffusion equation, where we considered the emission of carbon monoxide from multiple point sources with appropriate boundary conditions to find the concentration level of carbon monoxide at any point $(x, z)$. The coefficient of correlation revealed that the ground level concentration holds a strong linear relationship with the temperature. The results so obtained provides an insight information about the variation in the concentration due to the change in the season and helps in determining those months in which concentration is maximum and minimum.

2010 *Mathematics subject classification:* primary 00Z99; secondary 99A00.

*Keywords and phrases:* LaTeX, concentration of carbon monoxide, temperature, correlation analysis, statistical analysis, seasonal variation..

## 1. Introduction

Carbon monoxide, a poisonous gas emitted from motor vehicles, power plants, and industries contributes on a large scale to ambient air in an urban cities. This gaseous pollutant is one of the major causes of illness and death in the U.S.A. and other countries [1], [2]. India is too facing the problem of air pollution and several programmes were launched by the Government of India like National Clean Air Programme in 2019 with an aim to improve air quality. Lucknow (Capital City of Uttar Pradesh, India) one of the fast-growing cities in India is also facing the same problem due to its large population, heavy number of motor vehicles as well as power plants and large number of industries. In a sub-tropical climate zone, Lucknow experiences a maximum temperature of $40 - 45^oC$ in summer (April – June) and a minimum temperature of $7 - 8^oC$ in winter (October – March). The rainy season is from July to September. Lucknow city is driest during April -May. Meteorological factors like wind speed, diffusivity, atmospheric stability, precipitation/humidity etc significantly impact the concentration of pollutants. Several papers revealed that these meteorological factors influence the dispersion of pollutants [3], [4], [5]. Papers [6], [7], [8] studied the concentration of gaseous and particulate matter across the years for all the seasons. In paper [6], it was found that the concentration of particulate matter $PM_{2.5}$ was especially influenced by the ambient temperature and humidity and there is an inverse relationship between the temperature and concentration of $PM_{2.5}$. The concentration of $PM_{2.5}$ has negative correlation with temperature during high humid conditions.

The paper [8] revealed that a higher level of concentration of carbon monoxide was found in the dry season (April -September) than in the rainy season (October - March). According to the paper [8], in case of high air temperature, the density of air near the surface of the earth is lower than the air above it causing an upward flow of convection carrying various air pollutants. This causes air pollutant concentration to be low, whereas the low air temperature causes the density of air near the surface of the earth to be almost the same as the air density above it. Density, making the concentration of air pollutant to be high.

In monsoon season, the concentration level becomes significantly low due to rain droplets, thus helps in reducing the concentration of pollutants [5]. In paper [9], it was found that the concentration of respirable suspended particulate matter (RSPM-PM10) and gaseous pollutants in Lucknow city reduced to a significant level in the monsoon season. The study also revealed that the pollutants (gaseous & particulate matter) coming from the industrial area situated in the outer area of the urban city also contribute in polluting the air of the city. The same finding was made in the paper [10] in which local wind carrying the pollutant emitted from the nearby industries added in the ambient air of the urban city causes the significant increase in the level of the concentration.

Considering the above studies, it is obvious that the change in temperature directly or indirectly influences the wind speed, diffusivity coefficient and precipitation. We, therefore, proposed a systematic study of the dispersion of carbon monoxide emitted from the multiple point sources in Lucknow, where the above three meteorological parameters are temperature dependent and using the method of curve fitting and correlation analysis, we obtained the relationship between the temperature and the meteorological parameter. Finally, using the two-dimensional advective-diffusion equation and the appropriate boundary conditions, we obtained the expression for the concentration of carbon monoxide (CO).

## 2. Mathematical formulation

The dispersion of air pollutants like $CO_2$, $SO_2$, $NO_2$, CO etc depends upon various factors like wind speed, diffusivity coefficients, removal factors (like precipitation, greenbelts, raindroplets). The studies in past revealed that the concentration of air pollutants varies according to the variation in seasons [7], [11]. In the present model, we considered to study the dispersion of carbon monoxide in an urban city Lucknow in Uttar Pradesh (India) under atmospheric boundary conditions, where the wind speed, diffusivity of carbon monoxide and removal parameter (as precipitation) are temperature dependent. In this regard, we have taken the data of average wind speed, diffusivity coefficient and removal factor as average precipitation in accordance with the average temperature for the past seven years (i.e. 2015 – 2022). The average temperature, average wind speed and average precipitation corresponding to each month are considered as given below in Table-1:

| **Month** | **Average Temperature ($^0$F)** | **Average Wind Speed (m/sec)** | **Average Precipitation (m)** |
|---|---|---|---|
| January | 58 | 2.72 | .020 |
| February | 65 | 3.08 | .022 |
| March | 75 | 3.48 | .0106 |
| April | 86 | 3.57 | .0076 |
| May | 91 | 3.53 | .0106 |
| June | 91 | 3.57 | .127 |
| July | 86 | 3.4 | .31 |
| August | 85 | 3.08 | .2743 |
| September | 83 | 2.68 | .162 |
| October | 78 | 2.05 | .038 |
| November | 69 | 2.10 | .0025 |
| December | 60 | 2.37 | .0076 |

Table-1

The corresponding data for diffusivity coefficient of CO are mentioned below:

| Temperature($^0$C) | 20 | 100 | 200 | 300 | 400 |
|---|---|---|---|---|---|
| Diffusion coefficient of CO ($m^2$/sec) | .0000208 | .0000315 | .0000475 | .0000662 | .0000875 |

Table- 2

In view of the above data, we considered three phases: The first phase is taken from January to May. The second phase is from June to September and third phase is from October to December. These different phases are considered so that we apply the statistical and correlation analysis accordingly and get better results.

For the three phases, the value of Pearson coefficient of correlation between the temperature and the observed wind speed are 0.9196, 0.8923, −0.8728 respectively which show a strong linear relationship between the average wind speed and the average temperature. In the first two phases, we get a positive correlation but in the third phase, we get a negative correlation which indicates that the average wind speed increases with the decrease in average temperature.

Thus, using the statistical concept of curve fitting corresponding to the above data, the relationship between the average wind speed (u) and average temperature (T) for the three phases is as follows:

First Phase (Jan -May):

$$u = 1.25T^{0.306} \tag{2.1}$$

Second Phase (June-Sept):

$$u = .0000137T^{3.6} \tag{2.2}$$

Third Phase: (Oct- Dec):

$$u = 8.898T^{-.4654} \tag{2.3}$$

The figures 1 - 3 indicate that the wind speed calculated from equation (2.1), (2.2) and (2.3) are precise to the average wind speed of the data.

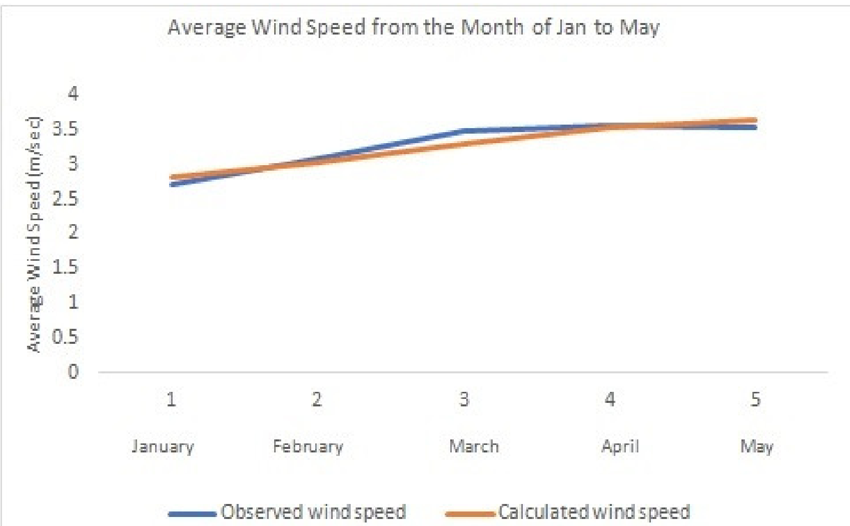

Figure: 1

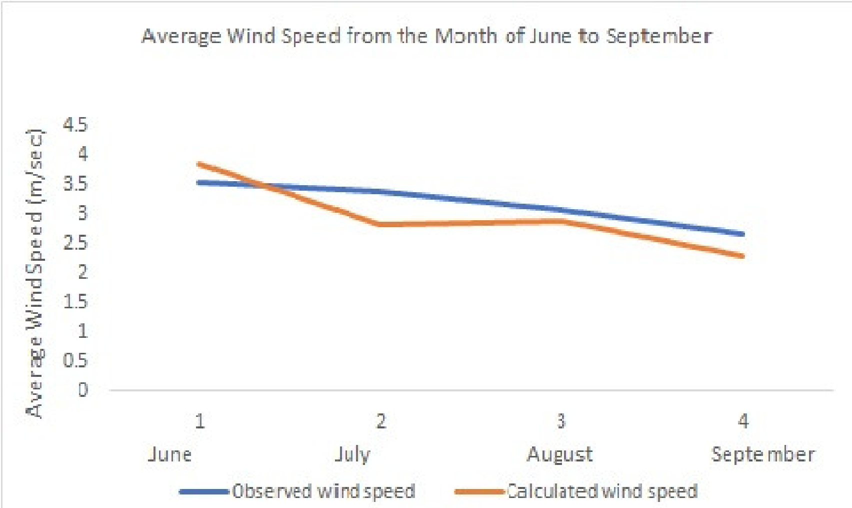


Figure: 2

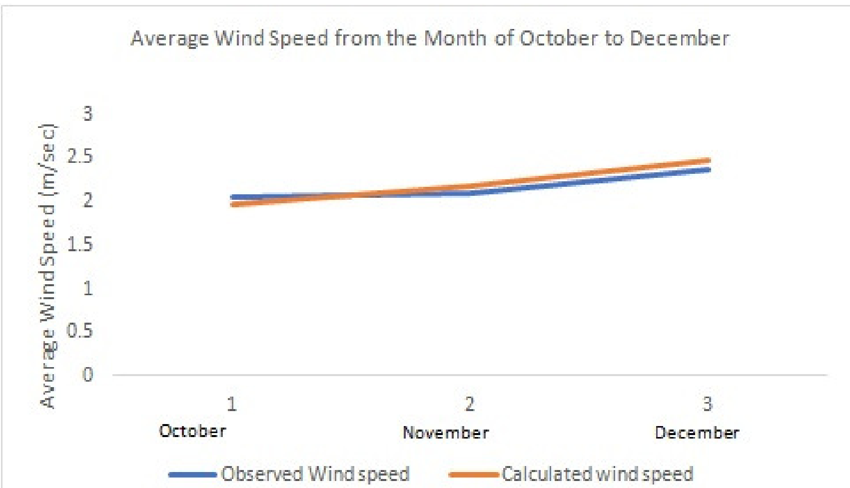


Figure: 3

The diffusivity coefficient (K) of carbon monoxide as a function of temperature (T) was found to be represented as follows:

$$K = .00000494T^{.449} \tag{2.4}$$

The Pearson coefficient of correlation between the temperature and the diffusivity from the data is found to be 0.9964, which shows that there is a very strong linear relationship between the diffusivity coefficient and the temperature. The closeness between the observed value and calculated value in the figure 4, validates the relationship between the temperature and diffusivity defined in equation (2.4).

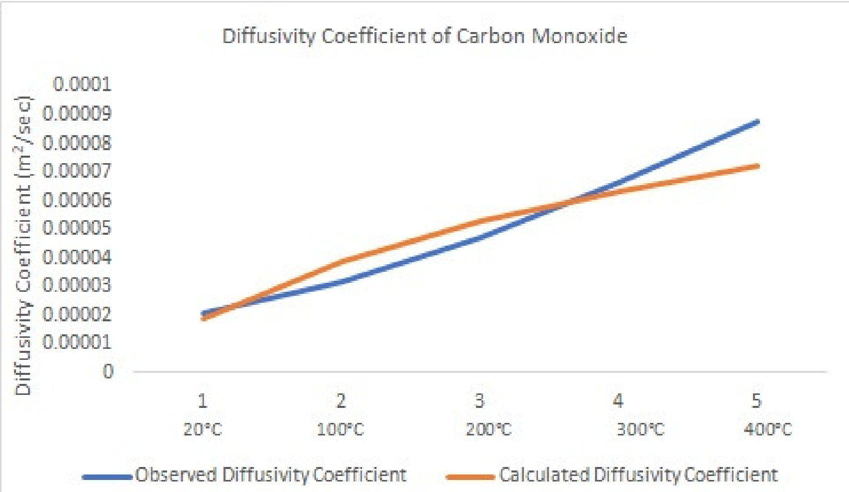


Figure: 4

In regarding with the precipitation $\alpha$, the Pearson coefficient of correlation are $-.8657$, $-.3748$, $.8358$ respectively. The trend of negative correlation between the temperature and precipitation was also observed for summer precipitation and temperature in some part of U.S.A. [12]. However, there is a weak negative correlation between the temperature and the precipitation in monsoon season.

Using the concept of curve fitting, the relationship between the precipitation and the temperature corresponding to the three phases is as follows:

First phase (Jan -May):

$$\alpha = 3.437T^{-1.724} \tag{2.5}$$

Second Phase (June-Sept):

$$\alpha = 3288516T^{-4.877} \tag{2.6}$$

Third Phase: (Oct- Dec):

$$\alpha = .0000000015T^{5.6} \tag{2.7}$$

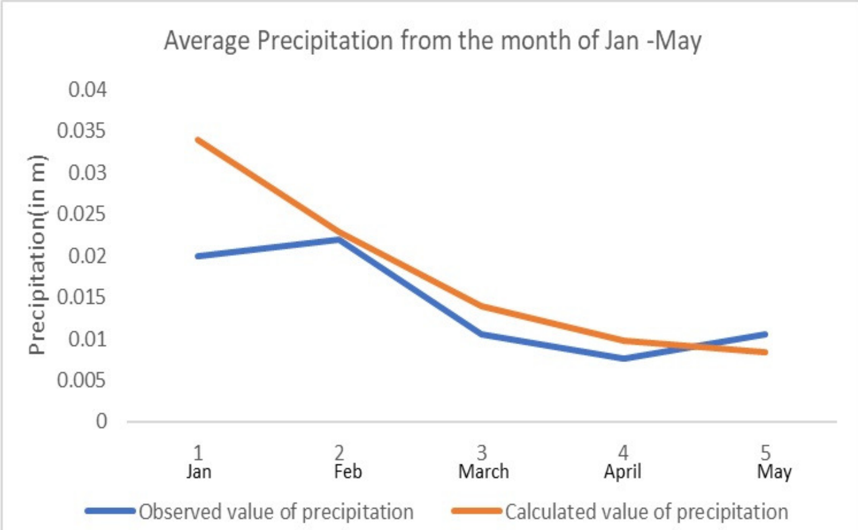


Figure: 5

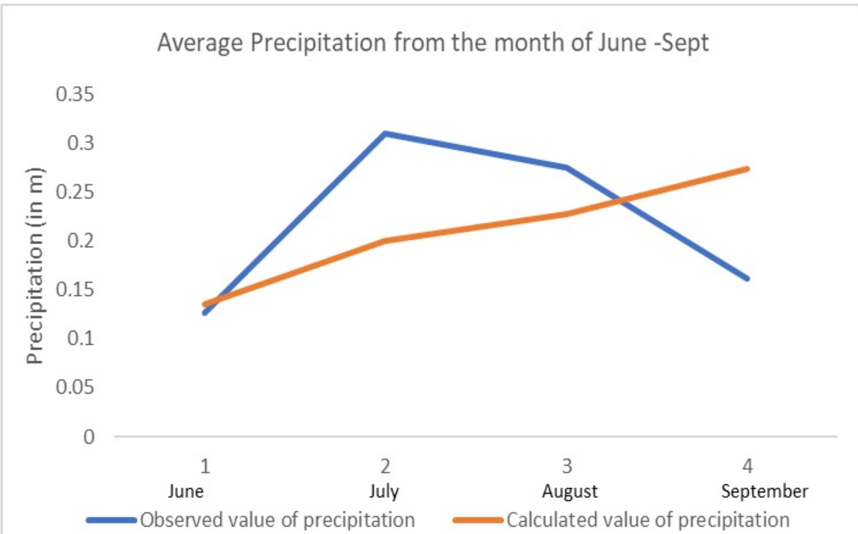


Figure: 6

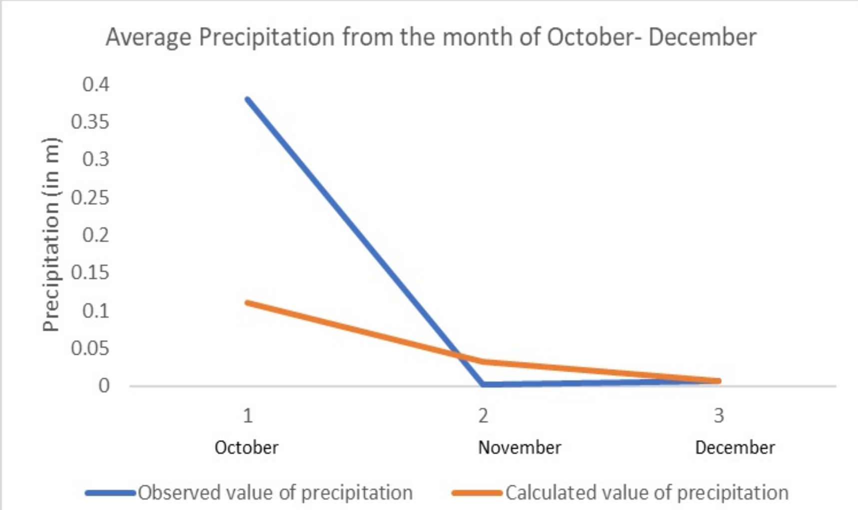


Figure: 7

These expressions are considered in the governing equation of the dispersion carbon monoxide under the atmospheric boundary layer (ABL). The two-dimensional advective diffusion equation is considered to study the variation in the concentration of carbon monoxide due to the emission from the n multiple point sources for all the seasons. Thus, the corresponding governing equation for the concentration of carbon monoxide is represented as:

$$u\frac{\partial C}{\partial x} = K\frac{\partial^2 C}{\partial z^2} - \alpha C + \sum_{i=1}^{n} Q^i \delta\left(x - x_s^i\right)\delta\left(y - y_s^i\right) \tag{2.8}$$

Here, the diffusion along the downwind direction is neglected in comparison to the advection, $C$ represents the concentration of carbon monoxide at a point (x, z), $u$ represent the wind speed, $K$ represent the diffusivity coefficient of carbon monoxide and $\alpha$ represent the removal parameter as precipitation. $Q^i$ represent the constant emission from the i$^{th}$ point source and $\delta(.)$ represent the Dirac-delta function. The pollutants so emitted from various point sources are removed from the atmosphere when they come in contact with the ground due to adsorption. Thus, we have

$$K\frac{\partial C}{\partial z} = v_d C \ at\ z = 0 \tag{2.9}$$

where $v_d$ represents the deposition velocity of carbon monoxide.

The pollutant is confined within the mixing layer and cannot penetrate through the inversion layer so, we have

$$K\frac{\partial C}{\partial z} = 0 \ at\ z = H \tag{2.10}$$

where H represent the height of an inversion layer.

## 3. Method of Solution

In order to analyse the behaviour and impact of the pollutant on the environment, we obtain the solution of the partial differential equation (2.8) along with the boundary conditions (2.9) and (2.10). Analytical approach is adopted by using the concept of Green's Function & power series solution as previously observed in several papers [3] & [10]. The solution corresponding to the partial differential equation (2.8) along with the conditions (2.9) – (2.10) is represented as.

$$C(x,z) = \sum_{i=1}^{n}\left[Q^i \sum_{k=0}^{\infty} \frac{exp\left(-\frac{(\beta_k^2 K+\alpha)}{u}\left(x - x_s^i\right)\right)}{u\int_0^H \left(sin\beta_k z + \frac{\beta_k K}{v_d} cos\beta_k z\right)^2 dz} \left(sin\beta_k z + \frac{\beta_k K}{v_d} cos\beta_k z\right)\left(sin\beta_k z_s^i + \frac{\beta_k K}{v_d} cos\beta_k z_s^i\right)\right] \tag{3.1}$$

where $\beta_k$ represents the eigenvalues and is represented as

$$\frac{\beta_k K}{v_d} = cot\beta_k H \tag{3.2}$$

The ground level concentration of the carbon monoxide i.e. at z =0 will be represented as

$$C(x,z) = \sum_{i=1}^{n}\left[Q^i \sum_{k=0}^{\infty} \frac{exp\left(-\frac{(\beta_k^2 K+\alpha)}{u}\left(x - x_s^i\right)\right)}{u\int_0^H \left(sin\beta_k z + \frac{\beta_k K}{v_d} cos\beta_k z\right)^2 dz} \left(sin\beta_k z_s^i + \frac{\beta_k K}{v_d} cos\beta_k z_s^i\right)\right] \tag{3.3}$$

The equation (3.1) is considered for the three different phases i.e. (Jan-May), (June-Sept) and (Oct-Dec). The expressions for wind speed as mentioned in equations (2.1)-(2.3), diffusivity coefficient as mentioned in equation (2.4) and precipitation (2.5) – (2.7) were substituted in the equation (3.1) & (3.3) to obtained the concentration at any point (x, z).

## 4. Result and Discussion

The corresponding data for the wind speed, diffusivity coefficient and the precipitation obtained from equation (2.1) – (2.7) for individual month are as follows:
**First Phase: (Jan -May)**
Wind Speed:

| Months | Jan | Feb | March | April | May |
|---|---|---|---|---|---|
| Calculated wind speed (m/s) | 2.82 | 3.04 | 3.30 | 3.54 | 3.63 |

Diffusivity Coefficient:

| Months | Jan | Feb | March | April | May |
| --- | --- | --- | --- | --- | --- |
| Calculated Diffusivity Coefficient ($m^2$/s) | 0.0000163 | 0.0000182 | 0.0000205 | 0.0000227 | 0.0000236 |

Precipitation:

| Months | Jan | Feb | March | April | May |
| --- | --- | --- | --- | --- | --- |
| Calculated precipitation (m) | 0.034 | 0.02289 | 0.0145 | 0.0097 | 0.0084 |

**Second Phase: (June -Sept)**

Wind Speed:

| Months | June | July | August | September |
| --- | --- | --- | --- | --- |
| Calculated wind speed (m/s) | 3.88 | 2.847 | 2.91 | 2.31 |

Diffusivity Coefficient:

| Months | June | July | August | September |
| --- | --- | --- | --- | --- |
| Calculated Diffusivity Coefficient ($m^2$/s) | 0.0000236 | 0.0000227 | 0.0000225 | 0.0000221 |

Precipitation:

| Months | June | July | August | September |
| --- | --- | --- | --- | --- |
| Calculated precipitation (m) | 0.135 | 0.205 | 0.2269 | 0.273 |

**Third Phase: (Oct -Dec)**

Wind Speed:

| Months | October | November | December |
| --- | --- | --- | --- |
| Calculated wind speed (m/s) | 1.971 | 2.182 | 2.485 |

Diffusivity Coefficient:

| Months | October | November | December |
| --- | --- | --- | --- |
| Calculated Diffusivity Coefficient ($m^2$/s) | 0.0000211 | 0.0000191 | 0.0000169 |

Precipitation:

| Months | October | November | December |
| --- | --- | --- | --- |
| Calculated precipitation (m) | 0.113 | 0.033 | 0.007 |

To understand the dynamics of the dispersion of carbon monoxide across all the seasons, we assumed two different point sources which are located at $(x, z) = (0, 50)$ and $(100, 50)$ respectively. The amount of carbon monoxide emitted from these point sources are $Q = 11.1\ kg/sec$. The deposition velocity of the carbon monoxide is considered as 0.00016m/sec.

The coefficient of correlation (r) between the ground-level concentration of carbon monoxide and the temperature for the three phases are as follows:

| Phase | r at x =0m | r at x = 200m | r at x = 400m | r at x =600m | r at x =800m | r at x =1000m |
|---|---|---|---|---|---|---|
| Jan -May | -0.9888 | 0.9771 | 0.9985 | 0.9849 | 0.9647 | 0.9417 |
| June- Sept | -0.9577 | 0.936 | 0.9265 | - | - | - |
| Oct -Dec | 0.9994 | -.9954 | -0.8745 | -0.8667 | -0.8661 | -0.866 |

The above data reveals that there is a strong linear relationship between the calculated ground level concentration and the temperature. In the first and second phase we have strong positive coefficient of correlation except at $x = 0$. This mean that the variation in the temperature and concentration goes in the same direction. The third phase has a strong negative coefficient of correlation except at $x = 0$. This mean that the variation in the concentration and the temperature goes in the opposite direction.

In figures 8, 9 and 10, the ground-level concentration (GLC) of carbon monoxide against the downwind distance were plotted. The observation reveals that the concentration of carbon monoxide increases from $x = 0$ to $x = 100m$ and then decreases along the downwind distance. This pattern of variation in the concentration level is same for all the season. The highest level of concentration was observed in the month of December, whereas minimum in the monsoon season (June – August). Further, in summer (March -May), the concentration of carbon monoxide is relatively high and persist upto greater downwind distance, whereas in the month of October, November the concentration is high near the point source and its level decreases and becomes very low (almost negligible) from $x = 400m$. The significant level of concentration in December and during summer season is observed to higher downwind distance. In remaining months, the high level of concentration persists in the periphery close to the source. Overall, we can say that the alarming condition was found to be the winter and summer season.

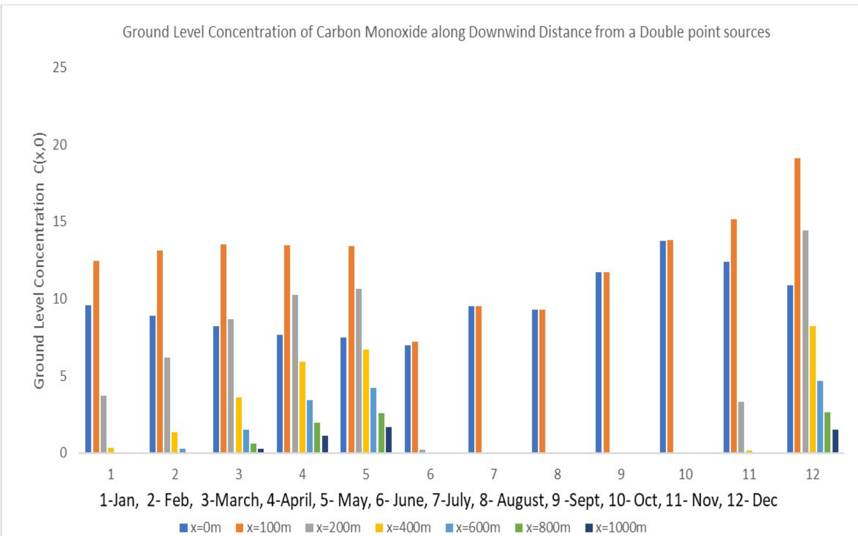

Figure: 8

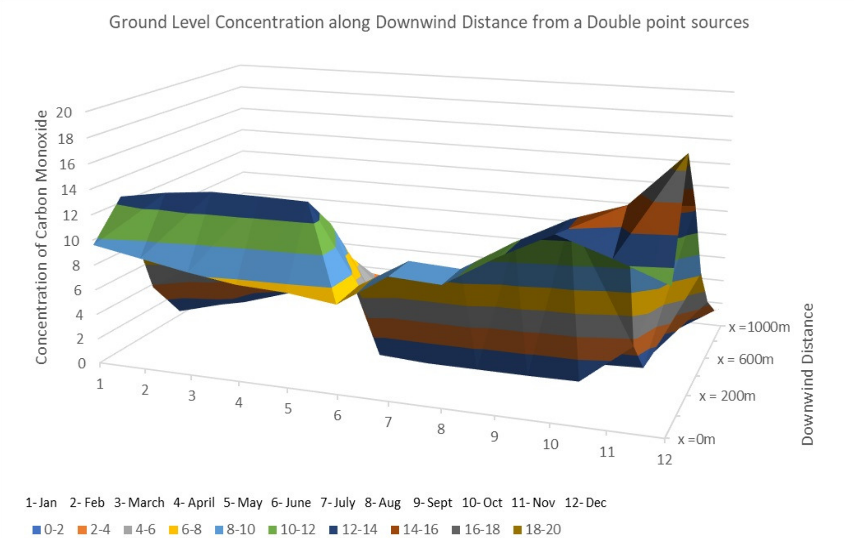


Figure: 9

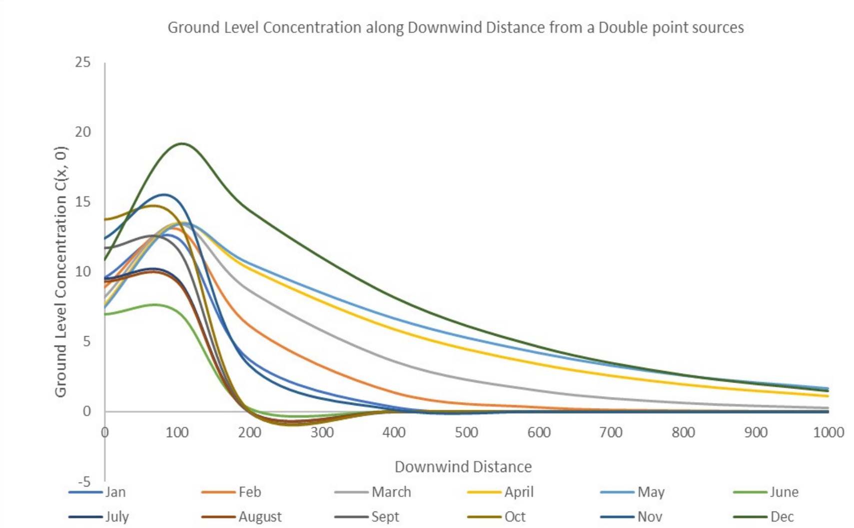


Figure:10

In figures 11 and 12, the concentration of carbon monoxide against the vertical height (upto z =250m) at $x = 200m$ was plotted. It was observed that the concentration level increase along the vertical height from the month of January to May. In monsoon season, the concentration level becomes negligible and finally from the month of November the concentration level increases and achieve its maximum limit in the month of December. The ground- level concentration is found to be highest which decrease at z =50m and then increases upto z =150m after which it starts decreasing.

From this observation we can conclude that the residential society (flats) should not be established near the industrial area as the concentration level of pollutant persist in significant level to higher altitude especially in winters and summers.

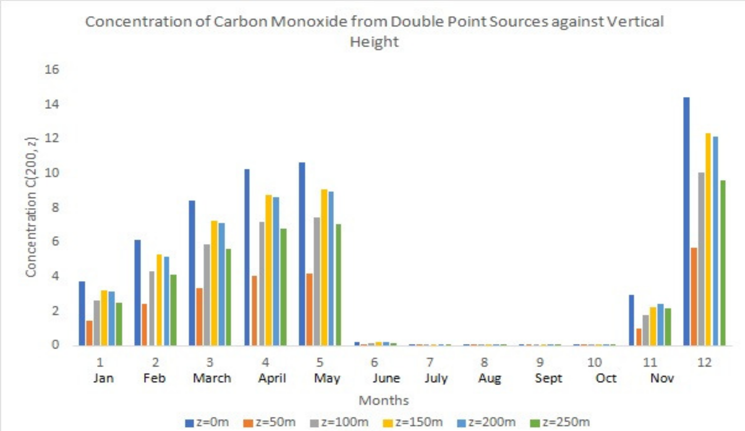


Figure: 11

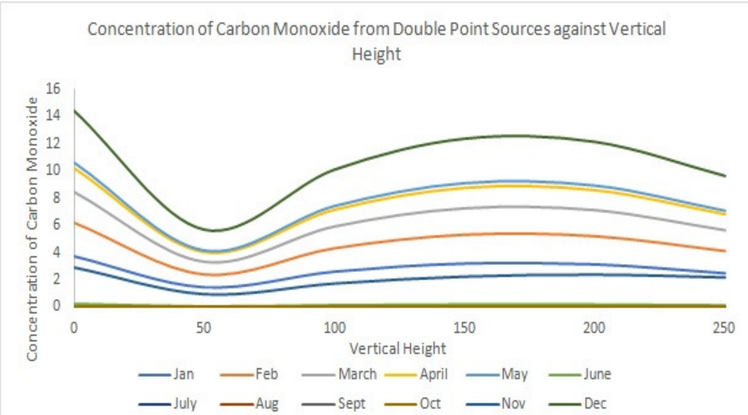


Figure:12

Figure 13 and 14 are plotted for ground-level concentration against the downwind distance from x =1000m to 3000m. The interpretation drawn from these graphs are that in summer season and in the month of December, the level of concentration persists significantly up to the greater downwind distance. This shows that during summer, large area of the region is exposed due to the emission of CO and have significant impact on the environment. Thus, we should be more vigilant and careful towards the emission of CO, especially in the month of summer. It is suggested that not only proper monitoring system is necessary but also safety measures and emergency preparedness (in case of accidental release) should exist in all those industrial plant from which emission of carbon monoxide takes place. Also, proper techniques must be applied to control the pollution caused by emission of CO.

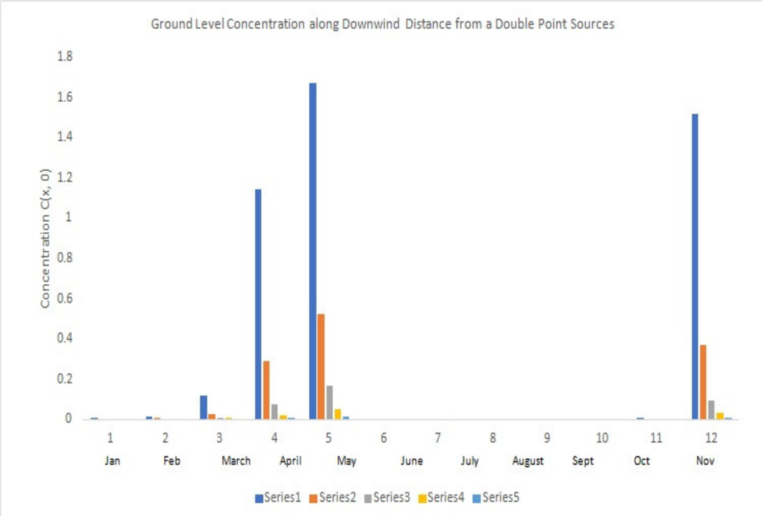


Figure: 13

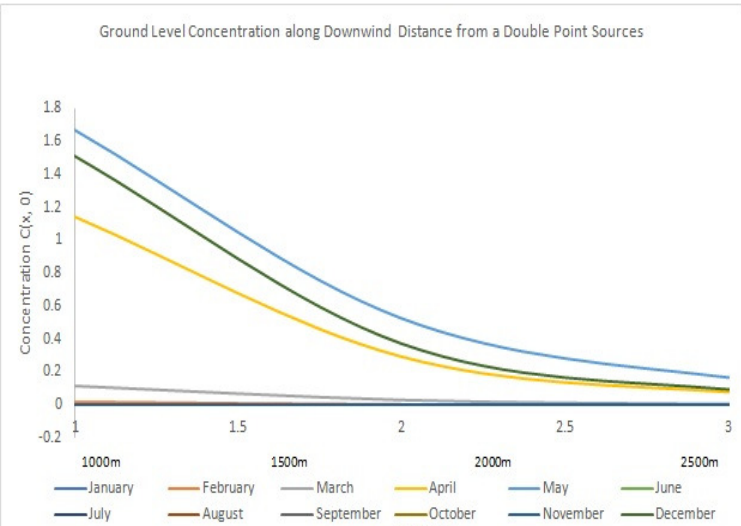


Figure: 14

## 5. Conclusion:

A two- dimensional mathematical model has been proposed to study the variation in the concentration of carbon monoxide emitted from double-point sources due to the change in temperature. The concentration of CO was evaluated throughout the year. The evaluation was done using the concept of statistical and correlation analysis. The following findings are stated below:

1. The GLC of CO is maximum in the month of December and minimum in the monsoon season as shown in 8. During summer, the GLC of CO persist to a significant level and exist upto a higher downwind distance.
2. There is a weak negative correlation between the temperature and the precipitation in monsoon season. Whereas, the strong negative correlation was observed in Ist phase (Jan – May) and strong positive correlation was observed in IIIrd phase (Oct- Dec).
3. The GLC of CO have strong positive coefficient of correlation from x = 200m during the months January to September and strong negative coefficient of correlation from x = 200m during the months October to December.
4. The concentration of CO increase along the vertical height from the month of January to May. In monsoon season, the concentration level becomes negligible. The maximum level of CO along the vertical height was observed in the month of December.
5. Figure 13 & 14, indicates that the existence of the GLC of CO was observed upto x = 3000m in the month of December & summer season. This mean that the pollutant travel to longer downwind distance. This makes us to think that when the emission quantity of carbon monoxide increases in the urban area then the level of concentration increases significantly and exist up to long distance which will be harmful for the environment and the health.

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