INTUITIONISTIC FUZZY SET P-CONNECTED MAPPINGS

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Abstract

A New class of mappings called intuitionistic fuzzy set P-connected mappings in intuitionistic fuzzy topological spaces has defined and studied. The relationships among, intuitionistic fuzzy set connected, intuitionistic fuzzy set s-connected, intuitionistic fuzzy continuous, intuitionistic fuzzy precontinuous and intuitionistic fuzzy semi continuous mappings are investigated.

1. Introduction

After the introduction of fuzzy sets by Zadeh [15] in 1965 and fuzzy topology by Chang [5] in 1968, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1–3] as a generalization of fuzzy sets. In the last 40 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [6] introduced the concept of intuitionistic fuzzy topological spaces. Recently Thakur and Thakur [12] introduced the concepts of intuitionistic fuzzy P-connectedness and intuitionistic fuzzy P-connectedness between intuitionistic fuzzy sets in intuitionistic fuzzy topological spaces. In the present paper using the concept of intuitionistic fuzzy P-connectedness between intuitionistic fuzzy sets we introduce a new class of mappings called intuitionistic fuzzy set P-connected mappings which contains the class of intuitionistic fuzzy set connected mappings [10] and independent to the class of intuitionistiic fuzzy set s-connected mappings [13]. Several properties of intuitionistic fuzzy set P-connected mappings have been obtained.

2. Preliminaries

DEFINITION 2.1. [2] Let X be a nonempty fixed set. An intuitionistic fuzzy set A in X is a structure A={ $<$x,μ_A(x),γ_A(x)$>$: x$\in$ X }, where the functions μ_A:X$\to$ I and γ_A: X$\to$

I denote the degree of membership (namely $\mu_A(x)$)and the degree of non membership (namely $\gamma_A(x)$) of each element x∈ X to the set A, respectively, and $0\leq \mu_A(x)+\gamma_A(x)\leq 1$ for each x∈X.

DEFINITION 2.2. [2] Let X be a nonempty set and the intuitionistic fuzzy sets A and B be in the form A={<x,$\mu_A(x),\gamma_A(x)$> : x∈ X }, B={<x,$\mu_B(x),\gamma_B(x)$> : x∈ X }. And let {A_i :i∈J} be an arbitrary family of intuitionistic fuzzy sets in X. Then:

(i)   A⊆ B if ∀ x ∈ X , $\mu_A(x)\leq\mu_B(x)$ and $\gamma_A(x)\geq \gamma_B(x)$;
(ii)   A = B if A⊆ B and B⊆ A;
(iii) A^c= {< x,$\gamma_A(x), \mu_A(x)$> : x∈ X};
(iv)   ∩A_i = {<x,∧$\mu_{Ai}(x)$,∨$\gamma_{Ai}(x)$> : x∈ X};
(v)   ∪A_i = {<x,∨$\mu_{Ai}(x)$,∧$\gamma_{Ai}(x)$> : x∈ X};
(vi) $\tilde{\mathbf{0}}$ = {<x, 0, 1> : x∈X}
(vii) $\tilde{\mathbf{1}}$ = {<x, 1, 0> : x∈ X}.

DEFINITION 2.3. [6] Let X and Y be two nonempty sets and f : X→ Y be a function. Then

(i)   If B = {< y, $\mu_B(y)$, $\gamma_B(y)$: y ∈Y}is an intuitionistic fuzzy set in Y, then the preimage of B under f denoted by $f^{-1}(B)$, is the IFS in X defined by
        $f^{-1}(B)$ ={ <x, f $^{-1}(\mu_B)(x)$, f $^{-1}(\gamma_B)(y)$> : x∈ X};
(ii)   If A = {<x,$\gamma_A(x)$, $\nu_A(x)$> : x∈ X} is an intuitionistic fuzzy set in X, then the image of A under f denoted by f(A) is the intuitionistic fuzzy set in Y defined by
        f(A) = {<y, f(γ_A)(y), f(ν_A)(y)> : y∈ Y}, where f(ν_A) = 1 - f(1-ν_A).

DEFINITION 2.4. [6] Two intuitionistic fuzzy sets A and B of X are said to be q-coincident (AqB for short) if and only if there exits an element x∈ X such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x)<\mu_B(x)$.

LEMMA 2.5. *[6] For any two intuitionistic fuzzy sets A and B of X, ⌉(AqB)⇔ A⊂ B^c.*

DEFINITION 2.6. [6] An intuitionistic fuzzy topology on a nonempty set X is a familyτ of intuitionistic fuzzy sets in X satisfying the following axioms:

(T1) $\tilde{\mathbf{0}}, \tilde{\mathbf{1}}$ ∈τ.
(T2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
(T3) ∪$G_i \in \tau$ for any arbitrary family {G_i:i∈J}⊆τ.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set in X. The complement A^c of an intuitionistic fuzzy open set A is called an intuitionistic fuzzy closed set in X.

DEFINITION 2.7. [7] An intuitionistic fuzzy set A in an intuitionistic fuzzy topological space X is called:

(i)   Intuitionistic fuzzy pre open if A⊆ int(cl(A)).
(ii)   Intuitionistic fuzzy pre closed if its compliment is intuitionistic fuzzy pre open.
(iii)   Intuitionistic fuzzy semiopen if A ⊆ cl(int(A)).

(iv)   Intuitionistic fuzzy semi closed if its compliment is intuitionistic fuzzy semiopen.

REMARK 2.8. *[7] Every intuitionistic fuzzy open set is intuitionistic fuzzy pre open (resp. intuitionistic fuzzy semi open). But the converse may not be true.*

DEFINITION 2.9. [11, 12, 14] An intuitionistic fuzzy topological space (x,τ) is called intuitionistic fuzzy c_5-connected (resp. intuitionistic fuzzy P-connected, intuitionistic fuzzy s-connected) if there no proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy open (resp intuitionistic fuzzy preopen , intuitionistic fuzzy semiopen) and intuitionistic fuzzy closed (resp. intuitionistic fuzzy preclosed, intuitionistic fuzzy semiclosed).

DEFINITION 2.10. [10–12] An intuitionistic fuzzy topological space (X,τ) is said to be intuitionistic fuzzy connected (resp. intuitionistic fuzzy P-connected , intuitionistic fuzzy s-connected) between intuitionistic fuzzy sets A and B if there is no intuitionistic fuzzy closed open (resp. intuitionistic fuzzy preclosed preopen, intuitionistic fuzzy semiclosed semiopen) set F in X such that A⊂F and ⌉(FqB).

LEMMA 2.11. *[10] An intuitionistic fuzzy topological space (X,τ) is intuitionistic fuzzy connected between intuitionistc fuzzy sets A and B if and only if there is no intuitionistic fuzzy closed open set F in X such that $A⊂F ⊂ B^c$.*

LEMMA 2.12. *[12] An intuitionistic fuzzy topological space (X,τ) is intuitionistic fuzzy P-connected between intuitionistc fuzzy sets A and B if and only if there is no intuitionistic fuzzy preclosed preopen set F in X such that $A⊂F ⊂ B^c$.*

DEFINITION 2.13. [6, 7] A mapping f :(X, τ)→(Y, Γ) is said to be :
(i)   Intuitionistic fuzzy continuous if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy open in X.
(ii)   Intuitionistic fuzzy pre continuous if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy pre open set in X.
(iii) Intuitionistic fuzzy semi continuous if the inverse image of every intuitionistic fuzzy open set of Y is intuitionistic fuzzy semi open set in X.

REMARK 2.14. *[6, 7] Every Intuitionistic fuzzy continuous mapping is intuitionistic fuzzy pre continuous (resp. Intuitionistic fuzzy semi continuous) but the converse may not be true. The concepts of intuitionistic fuzzy pre continuous and intuitionistic fuzzy semi continuous mappings are independent.*

DEFINITION 2.15. [10, 11] A mapping f :(X, τ)→(Y, σ) is said to be :
(i)   intuitionistic fuzzy set connected provided that: if X is intuitionistic fuzzy connected between intuitionistic fuzzy sets A and B, f(X) is intuitionistic fuzzy connected between f(A) and f(B) with respect to relative intuitionistic fuzzy topology.
(ii)   intuitionistic fuzzy set s-connected provided that: if X is intuitionistic fuzzy s-connected between intuitionistic fuzzy sets A and B, f(X) is intuitionistic fuzzy

connected between f(A) and f(B) with respect to relative intuitionistic fuzzy topology.

REMARK 2.16. *[10, 11] Every Intuitionistic fuzzy continuous mapping is intuitionistic fuzzy set connected and every intuitionistic fuzzy set connected mapping is intuitionistic fuzzy set s-connected but the converse may not be true.*

LEMMA 2.17. *[11] A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is intuitionistic fuzzy set connected if and only if $f^{-1}(F)$ is a intuitionistic fuzzy closed open set of X for every intuitionistic fuzzy closed open set F of f(X).*

3. Intuitionistic Fuzzy Set P-Connected Mappings

DEFINITION 3.1. A mapping f : $(X, \tau) \rightarrow (Y,\sigma)$ is said to be intuitionistic fuzzy set P-connected provided that: if X is intuitionistic fuzzy P-connected between intuitionistic fuzzy sets A and B, f(X) is intuitionistic fuzzy connected between f(A) and f(B) with respect to relative intuitionistic fuzzy topology.

THEOREM 3.2. *A mapping $f : (X, \tau) \rightarrow (Y,\sigma)$ is intuitionistic fuzzy set P- connected if and only if $f^{-1}(F)$ is a intuitionistic fuzzy pre closed pre open set of X for every intuitionistic fuzzy closed open set F of f(X).*

PROOF. Necessity: Let F be a intuitionistic fuzzy closed open set of f(X). Suppose $f^{-1}(F)$ is not intuitionistic fuzzy pre closed pre open set of X. Then X is intuitionistic fuzzy P connected between $f^{-1}(F)$ and $(f^{-1}(F))^c$. Therefore f(X) is intuitionistic fuzzy connected between $f(f^{-1}(F))$ and $f((f^{-1}(F))^c)$. But, $f((f^{-1}(F)) = F \cap f(X) = F \cap f((f^{-1}(F))^c) = f(X) \cap F^c = F^c$ imply that F is not intuitionistic fuzzy pre closed pre open set in X, a contradiction.

Sufficiency:Let X be intuitionistic fuzzy P- connected between intuitionistic fuzzy sets A and B. Suppose f(X) is not intuitionistic fuzzy connected between f(A) and f(B). Then by Lemma 2.11 there exist an intuitionistic fuzzy closed open set F in f(X) such that $f(A) \subset F \subset (f(B))^c$. By hypothesis $f^{-1}(F)$ is intuitionistic fuzzy pre closed pre open set of X and $A \subset f^{-1}(F) \subset B^c$. Therefore X is not intuitionistic fuzzy P-connected between A and B, a contradiction. Hence f is intuitionistic fuzzy set P-connected.   □

THEOREM 3.3. *If f: $(X,\tau) \rightarrow (Y,\sigma)$ is intuitionistic fuzzy set P-connected, the $f^{-1}(F)$ is an intuitionistic fuzzy pre closed preopen set of X for any intuitionistic fuzzy closed open set of Y.*

PROOF. Obvious.   □

THEOREM 3.4. *If f: $(X,\tau) \rightarrow (Y,\sigma)$ is intuitionistic fuzzy set connected, then f is intuitionistic fuzzy set P-connected.*

REMARK 3.5. *The converse of Theorem 3.4 may not be true. For,*

EXAMPLE 3.6. *Let X = {a, b}, Y = {p, q} and V = {< p, 0.5, 0.5 >, < q , 0.5, 0.5 >} be an intuitionistic fuzzy set on Y . Let τ = { 0̃, 1̃} and σ = { 0̃, V, 1̃} be the intuitionistic fuzzy topologies on X and Y respectively. Then the mapping f :(X,τ)→(Y,σ) defined by f(a) = p, f(b) = q is intuitionistic fuzzy set P-connected but not intuitionistic fuzzy set connected. Also f is not intuitionistic fuzzy set s-connected.*

Consider the following example,

EXAMPLE 3.7. *Let X ={a, b}, Y = {p, q} and U = {< a, 0.4, 0.6 >, <b, 0.4, 0.5 >}, V = {< p, 0.5, 0.5 >, <q , 0.5, 0.5 >} be the intuitionistic fuzzy sets of X and Y respectively. Let τ = { 0̃, U, 1̃} and σ ={ 0̃, V, 1̃} be the intuitionistic fuzzy topologies on X and Y respectively. Then the mapping f :(X,τ) (Y,σ) defined by f(a) = p, f(b) = q is intuitionistic fuzzy set s-connected but not intuitionistic fuzzy set P-connected.*

REMARK 3.8. *Example 3.6 and Example 3.7 reveals that the concepts of intuitionistic fuzzy set P-connected and intuitionistic fuzzy set s-connected mappings are independent.*

THEOREM 3.9. *If f: (X,τ)→(Y,σ) is intuitionistic fuzzy precontinuous then f is intuitionistic fuzzy set P-connected.*

REMARK 3.10. *The converse of Theorem 3.9 may not be true. For,*

EXAMPLE 3.11. *Let X = {a, b}, Y = {p, q} and U = {< a, 0.4, 0.6 >, <b, 0.4, 0.5 >}, V = {< p, 0.5, 0.5 >, < q , 0.4, 0.5 >} be the intuitionistic fuzzy sets of X and Y respectively. Let τ = { 0̃, U, 1̃} and σ = { 0̃, V, 1̃} be the intuitionistic fuzzy topologies on X and Y respectively. Then the mapping g :(X,τ) →(Y,σ) defined by g(a) = p, f(b) = q is intuitionistic fuzzy set P-connected but not intuitionistic fuzzy precontinuous.*

REMARK 3.12. *The concepts of intuitionistic fuzzy set connected and intuitionistic fuzzy precontinuous mappings are independent. For the mapping f in Example 3.6, is intuitionistic fuzzy precontinuous but not intuitionistic fuzzy set connected and the mapping g in Example 3.11 is intuitionistic fuzzy set connected but not intuitionistic fuzzy precontinuous.*

REMARK 3.13. *Every intuitionistic fuzzy semi continuous mappiung is intuitionistic fuzzy set s-connected but the converse may not be true. For,*

EXAMPLE 3.14. *Let X = {a, b}, Y = {p, q} and V = {< p, 0.4, 0.6 >, <q, 0.4, 0.5 >} be the intuitionistic fuzzy sets of Y. Let τ = { 0̃, 1̃} and σ = { 0̃, V, 1̃} be the intuitionistic fuzzy topologies on X and Y respectively. Then the mapping g:(X,τ)→(Y,σ) defined by f(a) = p, f(b) = q is intuitionistic fuzzy set s-connected but not intuitionistic fuzzy semi continuous.*

REMARK 3.15. *The concepts of intuitionistic fuzzy set connected and intuitionistic fuzzy semi continuous mappings are independent. For the mapping f in Example 3.7, is intuitionistic fuzzy semi continuous but not intuitionistic fuzzy set connected and the mapping g in Example 3.14 is intuitionistic fuzzy set connected but not intuitionistic fuzzy semi continuous.*

Thus we reach at the following diagram of implications:

Intuitionitic fuzzy $\Leftarrow$ Intuitionitic fuzzy $\Rightarrow$ Intuitionitic fuzzy
semi continuous   continuous   pre continuous
$\Downarrow$ $\Downarrow$ $\Downarrow$
Intuitionistic fuzzy $\Leftarrow$ Intuitionistic fuzzy $\Rightarrow$ Intuitionistic Fuzzy
set s-connected   set connected   set P-connected
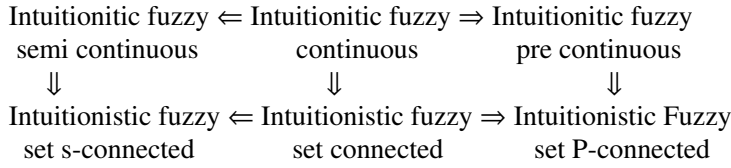
THEOREM 3.16. *Every mapping f: $(X,\tau) \to (Y,\sigma)$, such that f(X) is intuitionistic fuzzy C_5-connected is intuitionistic fuzzy set P-connected.*

PROOF. Let f(X) be intuitionistic fuzzy C_5-connected.Then no nonempty proper intuitionistic fuzzy set of f(X) which is both intuitionistic fuzzy closed and intuitionistic fuzzy open. Hence vacuously f is intuitionistic fuzzy set P-connected.   □

THEOREM 3.17. *Let f: $(X,\tau) \to (Y,\sigma)$ be an intuitionistic fuzzy set P-connected mapping. If X is intuitionistic fuzzy P-connected then f(X) is intuitionistic fuzzy C_5 - connected.*

PROOF. Suppose f(X) is not intuitionistic fuzzy C_5-connected. Then there is a non empty proper intuitionistic fuzzy set F of f(X) which is both intuitionistic fuzzy open and intuitionistic fuzzy closed. Since f is intuitionistic fuzzy set P-connected by Theorem 3.2, $f^{-1}(F)$ is a nonempty proper intuitionistic fuzzy set of X which is both intuitionistic fuzzy pre open and intuitionistic fuzzy pre closed. Consequently X is not intuitionistic fuzzy P-connected.   □

THEOREM 3.18. *Let f : $(X,\tau) \to (Y, \sigma)$ be a surjective intuitionistic fuzzy set P-connected and g : $(Y,\sigma) \to (Z, \omega)$ is an intuitionistic fuzzy set connected mapping. Then g∘f: $X \to Z$ is intuitionistic fuzzy set P-connected.*

PROOF. Let F be an intuitionistic fuzzy closed open set of g(Y). Then $g^{-1}(F)$ is an intuitionistic fuzzy closed open set of Y = f(X). And so $f^{-1}(g^{-1}(F))$ is an intuitionistic fuzzy pre closed pre open set in X. Now (g∘f)(X) = g(Y) and $(g∘f)^{-1}(F) = f^{-1}(g^{-1}(F))$, by Theorem 3.2, g∘f is intuitionistic fuzzy set P-connected.   □

THEOREM 3.19. *Let f : $(X,\tau) \to (Y,\sigma)$ be a mapping and g : $(X,\tau) \to (X x Y, \tau x\sigma)$ be the graph mapping of f defined by g(x) = (x, f(x)) for each x∈X. If g is intuitionistic fuzzy set P-connected, then f is intuitionistic fuzzy set P-connected.*

PROOF. Let F be any intuitionistic fuzzy closed intuitionistic fuzzy open set of the intuitionistic fuzzy subspace f(X) of Y. Then XxF is an intuitionistic fuzzy closed intuitionistic fuzzy open set of subspace X x f(X) of the intuitionistic fuzzy product space X x Y. Since g(X)is a subset of X x f(X), (X x F) ∩g(X) is an intuitionistic fuzzy closed intuitionistic fuzzy open set of the intuitionistic fuzzy subspace g(X) of X x Y. By theorem3.1, $g^{-1}((X x F) \cap g(X))$ is an intuitionistic fuzzy pre open and pre closed set of X. It follows from $g^{-1}((X x F) \cap g(X)) = g^{-1}(X x F) = f^{-1}(F)$ that $f^{-1}(F)$

is an intuitionistic fuzzy pre open pre closed set of X. Hence by theorem 3.2, f is intuitionistic fuzzy set P-connected.

□

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