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INTEGRATED VENDOR-BUYER SUPPLY CHAIN MODEL WITH DETERIORATED ITEMS BY REDUCING CARBON EMISSION.

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Abstract

Carbon cap and trade and carbon offsets are common and important measures to reduce carbon emissions in many countries. In addition, through targeted capital investment in green technology, we can effectively reduce CO_2 emissions from our business activities. However, such capital investments are expensive and not all companies can afford to make these investments. Therefore, if all members of the supply chain agree to share their investment in the facility, the supply chain can reduce carbon emissions and generate more profits. In the context of carbon caps and trade and carbon tax policies, this study proposes an integrated inventory model with buyer-to-seller build-to-order policies. Fluctuating transportation costs are used as a power law function of transportation volume, with a single setup multi-delivery policy that reduces or considers proportional rate data to reduce transportation costs for co-investing funds to reduce carbon emissions. Several examples are simulated and the sensitivity analysis of the main parameters is carried out. The optimal solutions and joint total profit under various carbon emission policies are also compared. The future carbon emission control trend is expected to enable companies to share risks by co-investing and developing sustainable supply chains.

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1. Introduction

Carbon emission and carbon footprint management have increasingly been subjects of discussion in the context of supply chain management. Many existing studies have discussed carbon emission reduction policies, including carbon emissions limitation, carbon tax, carbon quotas, carbon cap and trade, and carbon offset. However, most of the studies consider carbon emissions as exogenous changes. In real life, enterprises can invest in processes, i.e., product design, production, inventory, and transportation activities, to effectively reduce carbon emissions. Greenhouse gases such as sulfur dioxide, nitrogen dioxide, and carbon dioxide have been recognized as the prime cause of global climate change, which has received significant global attention. Among these

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gases, carbon dioxide is considered as the prominent gas which motivated researchers to explore carbon reduction and mitigation strategies. Research work on this domain expands from carbon emission reporting to identifying and implementing carbon mitigation and reduction strategies.

Carbon emissions from the processes such as production, inventory management, sales, and transportation are the primary sources of greenhouse gasses. Further, factories are one of the main sources of carbon dioxide that causes environmental changes such as ozone layer depletion, the greenhouse effect, and acid rain. This destruction of nature will eventually threaten human health. Therefore, the manufacturing industry must heed the effects of the entire product life cycle on the environment. Global warming, a toxic environment, and the destruction of the ozone layer are threats to humans and animals. Ioan et al. [15] formulated the carbon emissions have long increased economic growth, authorities should formulate suitable policies to limit its impact on society. If we continue to focus only on economic development without considering the ecological impact of manufacturing, these threats will become even more serious. To reduce gas pollution emissions, developed countries have begun to discuss related threats and establish emission standards. A contract called the "Kyoto Protocol," which regulates the emission of greenhouse gases, was established in 1997 in Kyoto, Japan, and implemented in 2005. In total, 84 countries committed to this protocol. In accordance with these regulations, member governments have actively structured standards to limit greenhouse gas emissions. Pan et al.[19] presented on sustainable production-inventory model in technical cooperation on investment to reduce carbon emissions.

The supply chain model is used to mainly minimize the total cost or to maximize the total profit throughout the network under the condition that demands of all vendor and buyer have to be met. The organizations that make up the supply chain are "linked" together through physical flows and information flows. Physical flows involve the transformation, movement, storage of goods and materials. These are the most visible parts of the supply chain. But another important factor as information flows which allow the various supply chain partners to coordinate their long-term plans, and to control the day-to-day flow of goods and material up and down the supply chain. Cardenas-Barron and Sana[4] formulated a production inventory model for a two - echelon supply chain when demand is dependent on sales teams initiatives. Again Cardenas-Barron and Sana [5] established a multi - item EOQ inventory model in a two - layer supply chain while demand varies with promotional effort. Vidyadevi and Annadurai[36] established a optimization of fuzzy integrated inventory model with ordering cost reduction dependent and lead time. Taleizadeh et al. [33] discussed a joint optimization of price, replenishment frequency, replenishment cycle, and production rate in vendor managed system. EOQ models with partial back order, special selling price and special sales for perishable products were developed by Taleizadeh and Pentico [29]. Taleizadeh et al. [32] replenish-up-to multi-change-constraint inventory control system under fuzzy random lost-sale and backordered quantities. The basic economic production quantity model (EPQ) for lot size with continuous delivery was formed by Hadley and Whitin

[11]. Inventory management, production planning and scheduling was developed by Silver et al.[27]. Taleizadeh et al. [30] proposed an economic order quantity under joint replenishment policy to supply expensive imported raw materials with payment in advance.
Following this study, Taleizadeh et al.[34] proposed an EOQ model for multiple partial prepayment and partial back ordering. In those papers, both the production quantity and demand rates are finite and constant. Also there is no on-hand inventory when any replenishment cycle starts and the model focused on the make-to-order production system. Taleizadeh et al.[31] developed an EOQ model for perishable product with special sale and shortage. The finite production rate with lot-for-lot production/delivery policy was proposed by Banerjee [2]. Goyal [10] extended the model of Banerjee [2] where the shipment of delivery quantity can perform multiple times. An integrated inventory model with variable transportation cost, two - stage inspection, and defective items proposed by Sarkar et al. [25]. Sarkar and Majumder [21] proposed by integrated vendor and buyer supply chain model with vendor's setup cost reduction.
Sarker and Parijia [26] developed an inventory model to determine an optimal ordering policy for procurement of raw materials and the manufacturing batch size to minimize the total cost. To reduce setup cost and ultimately final cost of the system, a continuous investment is used by Sarkar and Majumder [23]. Mungan et al. [18] considered a dynamic delivery policy with a similar production/delivery policy.
Make-to-order is a production approach where products are not built until a confirmed order for products is received. i.e., it is a manufacturing process in which manufacturing starts only after a customer's order is received. Similarly, in make- to-stock, products are manufactured based on demand forecasts. As the accuracy of the forecasts will prevent excess inventory and opportunity loss due to stock-out, the issues are very important in real life problems. But this model will study on the make-to-order approach only. The make-to-stock production system is applicable for managing most of standard products. The shipment lot size with known or fixed on-hand inventory was proposed by Lu [16]. Hill [13] extended this model by introducing the periodic delivery quantity and variable production quantity. Both production and delivery schedule for a single- vendor multi-buyer supply chain was considered by Chan and Kingsman [6]. But the model does not contain any idea of transportation cost. The model of Ertogral et al. [7]included the cost of transportation. The major contribution regarding the transportation cost can be found in Ben-Daya et al.'s [3] and Glock's [8] model. Golhar and Sarker [9] considered periodic delivery frequency and decision variables as production quantity. Sarkar et al. [24], and used the trade- credit policy and back order price-discount in inventory model.
Inspection is the procedure to obtain the defective items. This is another important criteria in any production system to test products. When an order is received from the buyer or customer to the producer, then the producer starts the production. During the whole production process some defective items may be produced. Inspection is important to cheek that produced items regarding its perfectness. Generally, it is preferred that the inspection should properly done by the producer to remove complain from

retailer or customers, before the delivery of an item. Wee et al.[37] developed multi products single machine economic production quantity model with multiple batch size. Sarkar and Saren [22] incorporated an inspection policy with inspection errors and a warranty policy in their inventory model. For fixed lifetime products, three-stage inspections and quantity discount policy were introduced in a supply chain model by Sarkar [20]. After inspection, defective products are sent for rework. After reworking, the second stage inspection is considered and perfect items are sent for delivery to the market and defective items are disposed.

The greenhouse effect leads to climate change; therefore, identifying ways to reduce emissions of greenhouse gases (mainly carbon dioxide) is the way forward. International regulations have been established to monitor greenhouse gas emissions; a large proportion of academic studies have also considered the topics of green energy and carbon emission control. Three topics low energy consumption, low pollution, and low emission have become worldwide trends that are expected to continue in the future. Through strategic analysis of economic and social development, the concept of a low carbon economy has deeply influenced national political development, foreign trade situations, and employment situations. The amount of research on inventory management problems, such as carbon emission or carbon footprint management, has also increased in recent years. Arslan and Turkay [1] developed the traditional economic order quantity (EOQ) model to develop sustainable batch order models under different carbon emission management policies. Production lot-sizing and carbon emissions under cap-and-trade and carbon tax regulations model by He et al.[12]. Hau et al.[14] developed managing carbon foot-prints in inventory management. Analysis of the single-period problem under carbon emissions policies by Song and Leng [28]. Zhang and Xu [38] focused on the newsboy model in reference to multi-item products in limited storage spaces. Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission model by Tiwari et al. [35]. In addition, most carbon inventory management models treat carbon emissions as exogenous variables. Nevertheless, carbon emissions can be efficiently reduced by investing in green, ecological design, and green manufacturing concepts in product scheme, manufacturing, inventory, and transportation by Ma et al.[17].

Reduction in carbon emissions not only mitigates the influence of the greenhouse effect but enables enterprises to reduce other expenses. Currently, there is far too little investment in carbon reduction technologies; not every company can necessarily afford to reduce carbon emissions. Optimal savings and profits can be obtained from a supply chain system in which all members agree to share both the investments in relevant facilities and the benefits of improved carbon emission reduction.

The remainder of this article is organized as follows. Assumption and notations are presented in section2. In section3, we establish an inventory system under two stage inspection system with each policy and the existence of optimal solution is analyzed. In section 4 deals with numerical example to illustrate the results using separate algorithm and sensitivity analysis, some observations and managerial implications are presented. Finally, in section 5, the conclusions and some suggestions for future research are presented.

2. ASSUMPTIONS AND NOTATIONS

In this article, we considered the same symbols and assumptions that were used in Sarkar et al [25]. The relative assumptions are used as follows.

2.1 Notations The following notation are used to develop the model.

q - Delivery quantity to buyer

n - Number of shipments in the entire planning horizon (integer number).

q_0 - Ordering lot size (units).

T - Replenishment cycle time.

Q - Perfect item (units) to sell in a cycle time T i.e., $Q = uq_0$, where $u = (1 - \alpha + \alpha\beta)$.

r_0 - Production rate (units/unit time).

r - Production rate of perfect products i.e., $p = ur$ (units / unit time).

D - Demand rate (units / unit time).

q_p -Total production units at time t.

q_d - Total delivery units at time t.

A - Setup cost of the vendor ($/setup).

A_1 - Handling cost of the buyer ($/unit time).

A_2 - Inventory carrying cost of vendor ($ / unit / unit time).

H_1 - Inventory carrying charge of the vendor ($ / unit / unit time).

H_2 - Inventory carrying charge of the buyer ($ / unit / unit time).

$k(q)$ - Delivery cost for the vendor ($/shipment).

N_0, N_1 - Constants to adjust the transportation cost.

C_0 - Inspection cost ($/unit).

C_1 - Rework cost ($/unit).

C_2 - Disposal cost ($/unit).

α - Rate of percentage of defective items in the start of production.

β - Rate of percentage of perfect items in the rework items.

t_e - The tax rate per unit of carbon emission.

ω_b - The amount of carbon emissions of the buyer per unit of time.

ω_v - The amount of carbon emissions of vendor per unit of time.

ϵ - The technology investment for reducing carbon emissions.

m_ϵ - The propotion of reduced carbon emissions, as a function of ϵ.

2.2 Assumptions The assumptions made in the models are as follows.

1. This is a single item type integrated inventory model in which defective products are produced during production. After the first phase of inspection, α percent of the production rate of nonconforming items is detected. Therefore, there are $(1-\alpha)q_0$ complete elements in the system.
2. Defective products will be sent for rework and will be inspected a second time after rework. At this point, the β percent of the defective good product is recognized and the rest is discarded. After this phase, good items with a quantity of $\beta\alpha q_0$ are received and bad items with a quantity of $(1-\beta)\alpha q_0$ are discarded.
3. The seller will only deliver the perfect item to the market for a small amount of q $(q \leq Q)$. The perfect item was shipped to the market for a period of time $\frac{q}{D}$, where D $(D \leq q)$. Here, the demand factor of the purchaser.
4. The make-to-order policy is implemented. We do not accept reserved stock here. All products are made to order.
5. The model takes into account fixed setup costs.
6. Carbon emissions can be reduced by technology investment, and the reduced carbon emission rate is $m(\epsilon)$ $(0 < m(\epsilon) < 1)$, where $m(\epsilon)$ is an increasing function of investment with CO_2 emission technology ϵ.
7. Investing in technology to reduce CO_2 emissions and the resulting benefits are shared between sellers and buyers. In other words, the percentage of capital investment by buyers and sellers in carbon emission reduction technologies is γ and 1- γ, respectively, where $0 \leq \gamma \leq 1$.

3. Mathematical Model

This model considers a two-tiered supply chain coordination between buyers and sellers. A production system (as a vendor or supplier) is considered at the upper echelon and the customer (as a dealer or buyer) is considered at the lower echelon. When an order of q_0 quantity is received from the buyer to the vendor, then the vendor starts the production at a rate r_0 of lot size quantity q_0. To produce the quantity q_0, the machine has to maintain long-run production process, in which the production of defective items may appear. We assume that α percentage of production rate of defective items. Thus αq_0 quantities are defective items, which are sent to vendor for reworking and rest quantities $(1-\alpha)q_0$ are perfect items. At β percentage of perfect quantities are found within on the rework items αq_0 and get the perfect quantities $\beta\alpha Q_0$. Rest $(1-\beta)\alpha q_0$ defective items are disposed. Thus, the total number of perfect items in the production system are $Q = (1-\alpha+\alpha\beta)q_0 = uq_0$, which are delivered to

the buyer, where $u = (1 - \alpha + \alpha\beta)$. We assume that the buyer does not claim any type of demand for the shortage quantity $(q_0 - Q)$. In the similar manner, the production rate of perfect product is $r = ur_0$.
On the production and inspection time, the vendor deliverers perfect products to buyer at a small quantity $q(q \leq Q)$ with a fixed period $\frac{q}{D}$, where $D(D \leq q)$ is the demand rate of buyer. There are no reserved stock to meet the immediate demand, i.e., make-to-order production. Here the replenishment cycle period is $\left[0, \frac{Q}{D}\right]$. The replenishment cycle time $A = \frac{Q}{D}$ has two parts namely, one $t_1 = \left[0, \frac{Q}{p}\right]$ and another $t_2 = \left[\frac{Q}{p}, \frac{Q}{D}\right]$.

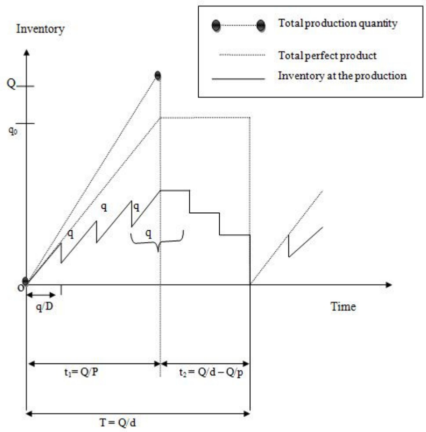

FIGURE 1. Integrated vendor-buyer production-inventory model.

Cost analysis

- The production system includes a fixed setup cost of A_1.
- At time t, the inventory of production $I_1(t)$ is the surplus of the total production $q_p(t)$ exceeds the total supply $q_d(t)$ where $0 \leq t \leq \frac{Q}{D}$. At the time t during the replenishment cycle, the total production $q_p(t)$ can be expressed as

$$q_p(t) = \begin{cases} r_0 t, & 0 \leq t \leq \frac{Q}{r}, \\ Q, & \frac{Q}{r} \leq t \leq \frac{Q}{D}. \end{cases}$$

Hence, the area formed by $q_p(t)$ on the replenishment cycle T is

$$\int_0^{\frac{Q}{D}} q_p(t)dt = \left(\frac{1}{D} - \frac{2u-1}{2ru}\right)Q^2.$$

The seller delivers Q units with equal shipments of n, and each size is q (i.e) $Q = nq$. The first delivery of the q unit is delivered at the time $\frac{q}{d}$. Therefore, k^{th} delivery time is $\frac{kq}{D} \leq t < (k+1)\frac{q}{D} \leq T$, $k = 1, 2, \ldots (n-1)$ occurs.
The total shipment quantity for the replenishment cycle T is

$\int_0^{T=\frac{Q}{D}} q_d(t)dt = \int_0^{\frac{nq}{D}} q_d(t)dt = \frac{n(n-1)q^2}{2D}$.

Hence, the total on-hand inventory under the cycle T is the subtract of the area formed by $q_p(t)$ and the total delivery quantity

i.e., $I_1 = \left(\frac{1}{D} - \frac{2u-1}{2ru}\right)Q^2 - \frac{n(n-1)q^2}{2D}$.

The total inventory carrying cost is

$H_1 I_1 = H_1\left[\left(\frac{1}{D} - \frac{2u-1}{2ru}\right)Q^2 - \frac{n(n-1)q^2}{2D}\right]$.

- Shipping cost per shipment is a power function of the delivery quantity of discounted or proportional rate data. i.e., $k(q) = N_0 + N_1 q^a$, for $N_0, N_1 \geq 0, 0 \leq a \leq 1$ and $q \geq 0$ and
  the total delivery cost is $nk(q) = n(N_0 + N_1 q^a)$.
- For the buyer, there are fixed handling cost to process the received shipments and the average on-hand inventory level is $I_2 = \frac{q}{2}$ in a single replenishment cycle $\frac{Q}{D}$. Only the increased echelon value $(H_2 - H_1)$ per unit per unit time is counted at the lower-echelon for the definition of echelon-value. This will give us the total cost of the buyer's order and storage is

$$A_2 n + \frac{Q(H_2 - H_1)q}{2D}.$$

- As the inspection cost C_0 per unit is considered on the total production quantity q_0 , the total inspection cost is in $C_0 q_0$ units. In the second step, the rework cost per unit C_1 is applied to the defective αq_0. Therefore, the total cost of reworking cost is $C_1 \alpha q_0$. The inspection is performed on the reworked item αq_0 with a fixed inspection cost of C_0 per unit, so the total inspection cost is $C_0 \alpha q_0$. At the end of the second check, there is an error in the $(1 - \beta)\alpha q_0$ set. Since C_2 is the fixed unit disposal cost of defective products, the total disposal cost is $C_2(1 - \beta)\alpha q_0$. Therefore, the total cost of inspection, rework, and disposal is $[C_0 q_0 + C_1 \alpha q_0 + C_0 \alpha q_0 + C_2(1 - \beta)\alpha q_0]$.
- Since the investment is jointly undertaken by the buyer and vendor, the fraction of the buyer's investment is γ $(0 \leq \gamma < 1)$. So the buyer's investment in the carbon emission reduction technologies per replenishment cycle is $\gamma\epsilon$ and the fraction of the vendor's investment is $1-\gamma$ $(0 \leq \gamma < 1)$, so the buyer's investment in the carbon emission reduction technologies per replenishment cycle is $(1-\gamma)\epsilon$. Hence, the total cost per unit time is

$$\begin{aligned} JTP(q,\epsilon) &= \frac{A_1 D}{nq} + \frac{H_1(D + ru - 2Du)}{2ru} nq + \frac{d(A_2 + N_0)}{q} + \frac{H_2 q}{2} \\ &+ DN_1 q^{(a-1)} + \frac{D}{u}[C_0 q_0 + C_1 \alpha q_0 + C_0 \alpha q_0 + C_2(1 - \beta)\alpha q_0] \\ &+ \gamma\epsilon + (1 - \gamma)\epsilon. \end{aligned} \tag{3.1}$$

Subsequently, the carbon emissions from each replenishment cycle of the buyer and vendor are related to the ordering, shipping, and carrying cost, which can be reduced by investing in carbon emission technologies (Lu [16]). The proportion of reduced carbon emission is $m(\epsilon)$ and thus, the carbon emission per replenishment cycle for the integrated model is

$$E_j(q,\epsilon) = [1-m(\epsilon)]\Big[\frac{A_1 D}{nq} + \frac{H_1(D+pu-2Du)}{2ru}nq + \frac{D(A_2+N_0)}{q} + \frac{H_2 q}{2} + DN_1 q^{(a-1)} + \frac{D}{u}[C_0q_0 + C_1\alpha q_0 + C_0\alpha q_0 + C_2(1-\beta)\alpha q_0]\Big]. \tag{3.2}$$

The objective is to estimate the seller's and buyer's carbon emissions of producing, filling and upgrading. Under two different carbon policies, here we maximize the total profit of the integrated system.

3.1 Carbon Cap-trade policy A cap and trade program can work in a number of ways, but here are the basics. The government sets the limit, or "cap" on emissions permitted across a given industry. It issues a limited number of annual permits that allow companies to emit a certain amount of carbon dioxide and related pollutants that drive global warming. Other pollutants that contribute to smog can also be capped. The total amount of the cap is split into allowances. Each allowance permits a company to emit one ton of emissions. The government distributes the allowances to the companies, either for free or through an auction.
The buyer and seller are subject to the ω_b and ω_v caps total carbon emigrations under the carbon cap- and- trade policy. However, products outside the boundary must be bought at the request price p_c, if carbon emigrations exceed this boundary. On the negative, if emigrations don't exceed this boundary, the remainder can be vended at the request price p_c. Assuming carbon emigration allowances can be bought and vended in the request, the total profit per unit of time under the carbon cap- and- trade policy, $JTP_{CC}(q,\epsilon)$, is

$$JTP_{CC}(q,\epsilon) = JTP(q,\epsilon) - p_c[E_j(q,\epsilon) - \omega_b - \omega_v].$$

The ideal of the policy is to determine the optimal volume, payload volume and technology investment to reduce carbon emigrations under the carbon cap- trade- policy, so as to maximize the common profit function $JTP_{CC}(q,n,\epsilon)$. We first calculate the values of q and ϵ by solving the equations $\frac{\partial JTP_{CC}(q,\epsilon)}{\partial q} = 0$ and $\frac{\partial JTP_{CC}(q,\epsilon)}{\partial \epsilon} = 0$ for given n. Then, we use Hessian matrix as follows to check the concavity of the profit function

$$H = \begin{pmatrix} \frac{\partial^2 JTP_{CC}(q,n,\epsilon)}{\partial q^2} & \frac{\partial^2 JTP_{CC}(q,n,\epsilon)}{\partial q\partial \epsilon} \\ \frac{\partial^2 JTP_{CC}(q,n,\epsilon)}{\partial \epsilon\partial q} & \frac{\partial^2 JTP_{CC}(q,n,\epsilon)}{\partial \epsilon^2} \end{pmatrix}.$$

For the value of (q,ϵ), The first and second determinants of the Hessian matrix ($|H1|$ and $|H2|$) satisfy

$$|H_1| = \frac{\partial^2 JTP_{CC}(q,n,\epsilon)}{\partial q^2}\Big|_{(q,\epsilon)} > 0,$$

and

$$|H_2| = \frac{\partial^2 JTP_{CC}(q,n,\epsilon)}{\partial q^2} \times \frac{\partial^2 JTP_{CC}(q,n,\epsilon)}{\partial \epsilon^2} - \Big[\frac{\partial^2 JTP_{CC}(q,n,\epsilon)}{\partial q\partial \epsilon}\Big]^2\Big|_{(q,\epsilon)} > 0.$$

Then the total gain per unit time has a maximum value at the point $(q\ ,\ \epsilon\ )$. Due to the difficulty of Hessian matrix, we alternate numerical analysis to verify the concavity. Next, we develop the following algorithm to get the solutions of the buyer and vendor under the carbon cap and trade policy.

Algorithm 1

Step 1: Set $n = 1$.
Step 2: Identify the values of $q_{(n)}$ and $\epsilon_{(n)}$ by setting $\frac{\partial JTP_{CC}(q,n,\epsilon)}{\partial q}$ =0 and $\frac{\partial JTP_{CC}(q,n,\epsilon)}{\partial \epsilon}$ =0.
Step 3: Substitute $q_{(n)}$ and $\epsilon_{(n)}$ into $JTP_{CC}(q, n, \epsilon)$ to obtain $JTP_{CC}(q_{(n)}, n, \epsilon_{(n)})$.
Step 4: Set $n = n + 1$, and repeat Step 2 to obtain $JTP_{CC}(q_{(n+1)}, n + 1, \epsilon_{(n+1)})$.
Step 5: If $JTP_{CC}(q_{(n+1)}, n + 1, \epsilon_{(n+1)}) < JTP_{CC}(q_{(n)}, n, \epsilon_{(n)})$, then $JTP_{CC}(q, n, \epsilon) = JTP_{CC}(q_{(n)}, n, \epsilon_{(n)})$, and hence (q, n, ϵ)= $(q_{(n)}, n, \epsilon_{(n)})$ is the optimal solution. Otherwise, return to Step 4.

3.2 Carbon tax policy Carbon taxes imposed by external regulatory agencies may provide incentives for businesses to take environmental costs into account. A simple tax table is linear and companies are required to pay a fixed amount (in C) per unit of CO2 emissions Arslan et al. [1]. Therefore, an improved model that takes Carbon tax policy is

$$JTP_{CT}(q, \epsilon) = JTP(q, \epsilon) - t_e[E_j(q, \epsilon) - \omega_b - \omega_v].$$

The purpose of this policy is to determine the optimal order quantity, shipment quantity, and technology investment to reduce carbon emissions under carbon tax regulations in order to maximize the joint profit function $JTP_{CT}(q, n, \epsilon)$.
As in this case of carbon cap and trade, identifying the closed forms of q, ϵ and assessing the concavity directly is a difficult task. Therefore, we verify the concavity by conducting a numerical analysis and then developed an algorithm to find the buyer and vendor solutions under the carbon tax regulation.

Algorithm 2

Step 1: Set $n = 1$.
Step 2: Identify the values of $q_{(n)}$ and $\epsilon_{(n)}$ by setting $\frac{\partial JTP_{CT}(q,n,\epsilon)}{\partial q}$ =0 and $\frac{\partial JTP_{CT}(q,n,\epsilon)}{\partial \epsilon}$ =0.
Step 3: Substitute $q_{(n)}$ and $\epsilon_P(n)$ into $JTP_{CT}(q, n, \epsilon)$ to obtain $JTP_{CT}(q_{(n)}, n, \epsilon_{(n)})$.
Step 4: Set $n = n + 1$, and repeat Step 2 to obtain $JTP_{CT}(q_{(n+1)}, n + 1, \epsilon_{(n+1)})$.
Step 5: If $JTP_{CT}(q_{(n+1)}, n + 1, \epsilon_{(n+1)}) < JTP_{CT}(q_{(n)}, n, \epsilon_{(n)})$, then $JTP_{CT}(q, n, \epsilon) = JTP_{CT}(q_{(n)}, n, \epsilon_{(n)})$, and hence (q, n, ϵ)= $(q_{(n)}, n, \epsilon_{(n)})$ is the optimal solution. Otherwise, return to Step 4.

4. Numerical Analysis

To demonstrate the solution methods and to perform sensitivity analysis of the optimal solutions with respect to the main parameters, we use several examples based on Lu [16].

Example 1. Let D=250 units/month, A_1= \$1700/month, $\widetilde{A_1}$=500 kg/month, A_2= \$35 /month, $\widetilde{A_2}$=15 kg/month,r_0=300 unit/month, $\widetilde{r_0}$=30 kg/month, h_1=\$300 unit/month, $\widetilde{h_1}$=20 kg/month, h_2=\$300 unit /month, $\widetilde{h_2}$=20 kg /month, c_0=\$2/units, c_1=\$2.5/units,

c_2=\$2/units, $\widetilde{c_0}$=0.02 kg/month, $\widetilde{c_1}$=0.01 kg /month, $\widetilde{c_2}$=0.03 kg/month, α=15%, β=96%, γ=0.5, a=0.8, N_0=75, N_1= 15, $\widetilde{N_0}$=10, $\widetilde{N_1}$=2, p_c=0.3/unit, w_b=5000 kg/year and w_v=5000 kg/year.
By using Algorithm 1, the optimal number of shipments and shipping quantity for the supplier under the carbon cap and trade policy are $n = 1$ and $q = 47.04$ units. The optimal order quantity of the buyer is $Q = n * q = 47.04$ units. The optimal technology investment for reducing carbon emissions are ϵ= 1.7027 and the optimal joint total profit $JTP_{CC}(q, \epsilon)$ = \$17027.

Example 2. Data are the same as in Example 1, yet it excluded t_e=0.1/ unit.
By using Algorithm 2, the optimal number of shipments and shipping quantity with carbon tax of the vendor are n=1 and q= 46.79 units. The optimal order quantity of the buyer is $Q = n * q = 46.79$ units. The optimal technology investment to reduce carbon emission is ϵ= 1.6798 and the optimal joint total profit $JTP_{CT}(q, \epsilon)$ = \$20260.

4.1 Sensitivity analysis In this section, we examine the effects of changes in the system parameters $D, A_1, A_2, h_1, h_2, p_c, \widetilde{A_1}, \widetilde{A_2}, \widetilde{h_1}$ and $\widetilde{h_2}$ on the optimal order quantity Q and reduce carbon emission ϵ, with minimum total joint cost. The optimal values of Q,ϵ and $JTP_{CC}(q, \epsilon)$ are derived, when one of the parameters changes (increases or decreases) by 25% and all other parameters remain unchanged. The results of sensitivity analysis are presented in 1. The graphical representation given in Figs 1-10.
On the basis of the results shown in Table 1, the following observations can be made:

- If q, ϵ increases while $JTP_{CC}(q,\epsilon)$ decreases with increasing value of model parameter D. Also, Q, ϵ are very sensitive, while $JTP_{CC}(q,\epsilon)$ is moderately sensitive.
- If the vendor's setup costs increase by A_1, the order quantity and overall profit decrease. However, the level of investment remains unchanged.
- If buyer's inventory cost A_2 or production rate r_0 increases, the order quantity increases, while the overall profit and the size of the investment do not change.
- If supplier's h_1 holding cost increases, the order quantity and overall profit increase, while the investment amount decreases.
- If the buyer's holding cost h_2 increases, the order quantity increases. while total profit and investment amount did not change.
- If the market price p_c increases, the order amount and total profit decrease while the investment amount does not change.
- If the CO_2 emission parameters $\widetilde{A_1}$ or $\widetilde{A_2}$ increase, the order quantity decreases, while the total profit and the investment amount do not change.
- As the CO_2 emission parameter $\widetilde{h_1}$ increases, the order quantity and investment amount increase while the overall profit increases.

Again sensitivity for the parameter α in the Example 1. The result of sensitivity analysis are presented in Table 3.

- If increase rate of percentage of defective item in start of production, the order quantity and technology investment are decrease while the over all profit increase.

TABLE 1. Sensitivity analysis of individual parameters.

Prameter	q	ϵ	$JTP_{CC}(q,\epsilon)$
D	33.07	0.8394	20265
	40.51	1.2595	20264
	52.33	2.1004	20263
	57.34	2.5212	20263
A_1	32.62	1.6798	20264
	40.34	1.6798	20263
	52.46	1.6798	20264
	57.58	1.6798	20264
A_2	46.55	1.6798	20264
	46.67	1.6798	20264
	46.92	1.6798	20264
	47.04	1.6798	20264
r_0	46.82	1.6818	20264
	46.8	1.6818	20264
	46.79	1.6794	20264
	46.79	1.6792	20264
h_1	60.28	3.3823	20264
	52.27	2.2448	20263
	42.74	1.3421	20264
	39.59	1.1174	20265
p_c	47.89	1.6798	20264
	47.35	1.6798	20263
	46.23	1.6798	20263
	45.67	1.6798	20263
$\widetilde{A_1}$	47.84	1.6798	20264
	47.32	1.6798	20264
	46.26	1.6798	20264
	45.72	1.6798	20264
$\widetilde{A_2}$	46.83	1.6798	20264
	46.81	1.6798	20264
	46.78	1.6798	20264
	46.76	1.6798	20264
$\widetilde{h_1}$	46.73	1.6742	20264
	46.76	1.677	20263
	46.82	1.6826	20262
	46.86	1.6855	20262

TABLE 2. Cost sensitiveness based on the parameter α

α	q	ϵ	$JTP_{CC}(q,\epsilon)$
0	80.27	10.2	13819
0.1	50.1	2	19249
0.2	48.6	1.85	19735
0.3	48.08	1.8	19911
0.4	47.82	1.77	20001
0.5	47.66	1.76	20056
0.6	47.56	1.75	20093
0.7	47.48	1.74	20120
0.8	47.42	1.73	20140
0.9	47.38	1.73	20156
1	47.34	1.73	20169

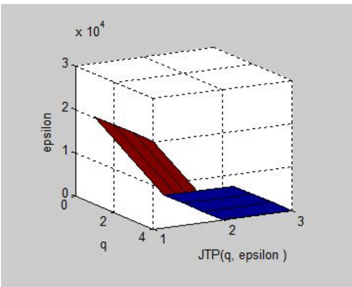

FIGURE 2. Effect of % changes in demand (D).

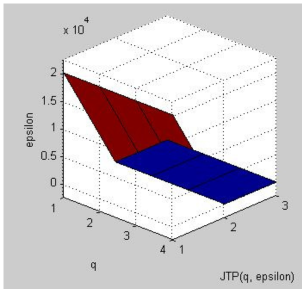

FIGURE 3. Effect of % changes in setup cost of vendor.

5. CONCLUSION

Excessive dioxide emissions contribute to world climate change. As a result, reducing carbon emissions has become a universal goal. Governments and international organizations have adopted completely different policies (such as carbon cap -and - trade and carbon taxes) to limit carbon emissions. These carbon emission restrictions have an effect on the production, replacement and transportation activities of enterprises. The main objective of this article is spot optimum production, delivery, replenishment and technology investment methods to cut back carbon emissions so as to maximize the whole profit throughout the availability chain system.

The main focus of this article is supplier-produced items that are shipped to the buyer after a two-step inspection process by multi shipment policy. The make-to-order policy was considered and two-stage inspections are conducted to make sure about the perfectness of all products which were sent to the buyer. The variable transportation cost is used as a power function in this model. By utilizing two-tier inspection policy, the supplier ensured quality of products to create the brand image in the minds of customers. Technology investment strategies to reduce carbon emissions so as to

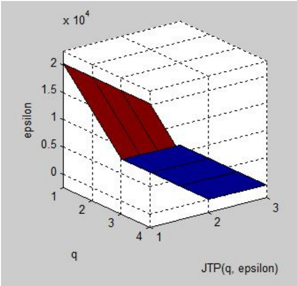

FIGURE 4. Effect of % changes in handling cost of buyer.

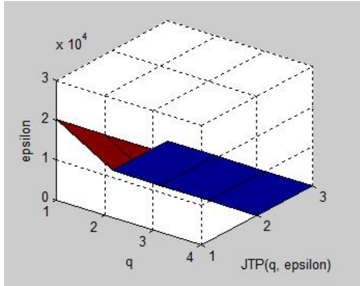

FIGURE 5. Effect of % changes in fixed carbon emissions per setup for vendor.

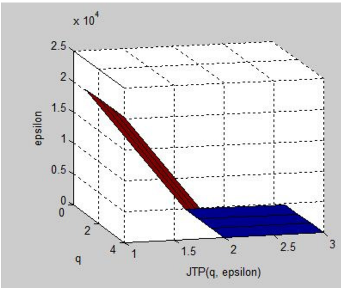

FIGURE 6. Effect of % changes in fixed carbon emissions per unit of handling cost for buyer.

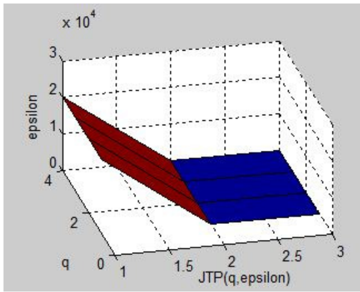

FIGURE 7. Effect of % changes in carrying cost of vendor.

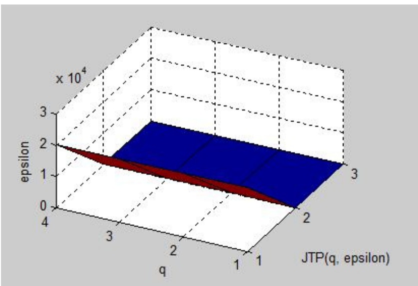

FIGURE 8. **Effect of % changes in fixed carbon emissions per unit of carrying cost of vendor.**

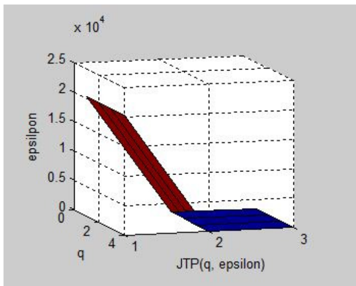

FIGURE 9. **Effect of % changes in production rate r_0.**

maximize the total profits throughout the supply chain system. The empirical results showed that when considering a CO_2 emission policy, whether be it a carbon cap-and-trade policy or a carbon tax policy .

In addition, the sensitivity of defective items at the beginning of production is represented by changes in the total costs. Finally, through a sensitivity analysis, we complied the effects of different parameter changes on the optimal solution.

A future research direction could be to take the perspective of competition between supply chain members using game theory to determine a balanced solution for all sup-

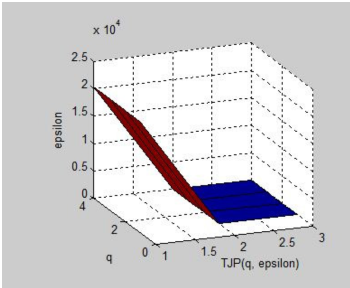

FIGURE 10. **Effect of % changes in purchased at the market price p_c.**

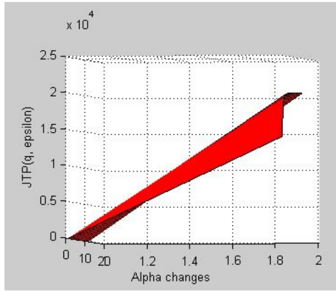

FIGURE 11. **Total cost function JTP when α varies as on Example 1.**

ply chain members. In addition, it would also be interesting to consider the issue of sharing the benefits of supply chain integration through supply chain contracts. Sometimes starting at a low production rate and then increasing the production capacity after a period of time can reduce maintenance costs (i.e) two different production rates in a production cycle or a time-dependent production rate can be considered in future work. Finally, analyzes can be performed on other general scenarios, such as out of stock, volume discount, credit business and changing demand situations.

Author contributions:

All author have read and agreed to the published version of the manuscript.

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