GANITA,Vol.74(1), 2024, 161-171

MATTER CREATION FIELD MODEL OF UNIVERSE WITH VARIABLE COSMOLOGICAL CONSTANT

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Abstract

In this paper we have considered that the universe is filled with a distribution of dust in accordance with Hoyle and Narlikar [6].We examined a creation cosmological model with variable cosmological constant in the context of FRW spacetime.In order to obtain the deterministic model we made the assumption that $\Lambda = 3\alpha\frac{\dot{R}^2}{R^2} + \beta\frac{\ddot{R}}{R}$, where R is the scaling factor. We observe that the creation field grows over time but the matter density stays constant, which is kept up by the ongoing generation of new matter also we observed $\Lambda \sim \frac{1}{t^2}$ and the absence of a particle horizon. The model characterizes an expanding cosmos that is speeding up, which is similar with the findings of Riess et al.[24] and Perlmutter et al [25].

2010 *Mathematics subject classification:* 83C05, 83C15, 83F05.

Keywords and phrases: FRW Spacetime, Creation Field, Variable Λ, Singularity, Horizon.

1. Introduction

The Big-Bang model is known to have drawbacks in the following areas, though:
(1) The model contains a past singularity and may contain a future singularity as well. The singularity denotes physical and mathematical incompleteness.
(2) The Big-Bang model violates the conservation of energy, one of the most revered physics principles. we know that in the Einstein's field equation the left-hand side has zero divergence therefore the source on the right-hand side of Einstein's equations must have zero divergence. On the other hand, the big-bang model has an energy density that is positively definite. As a result, the principle of energy conservation must be broken for matter to exist. It is common practice to downplay this challenge by using phrases like " the laws of physics break down at a singularity" however the basic truth remains the same in the field of physics, despite statements like "the laws of physics break down at a singularity".
(3) Very small particle horizons are produced in the early epochs of the universe by Big-Bang models based on plausible equations of state. The "horizon problem" in cosmology is a result of this fact.
(4) Within the framework of the Big-Bang model, there is no coherent explanation for the emergence, development and properties of the universe's small-scale structures.
(5) The so-called "flatness problem" [1] can be added last to this list. The flatness

problem has certainly drawn a lot of attention recently, despite being less serious than any of the problems mentioned above especially given that inflationary models appear to be able to resolve it.

It is impossible to introduce creation of positive matter without violation of energy conservation so in order to do that there is a need of some degree of freedom which act as negative energy mode thus a natural method of producing the matter might be to introduce a system of a negative energy field at the classical level itself. The classical singularity theorem becomes invalid When the positivity of the energy density is uncertain. Thus the addition of a system of a negative energy field might be able to resolve two of the big bang model's five problems.

The Friedmann-Robertson-Walker models claim that the cosmos begins with a Big Bang, however they are unable to explain the early universe. Alternative theories of gravity have therefore been put forth.

The steady state hypothesis, developed by Bondi and Gold [5] is the most well-known theory. According to this idea, the matter density remains constant throughout the cosmos and neither its beginning nor its end on the cosmic time scale are singular. They envisaged a very slow, continuous generation of matter as opposed to the explosive creation of the conventional Friedmann-Robertson-Walker model in order to ensure the constancy of matter density.Because it provides no physical reason for the ongoing generation of matter, this hypothesis was abandoned. Hoyle and Narlikar [6] used a field theoretic technique to solve this problem by constructing a massless and chargeless scalar field. Contrary to Bondi and Gold's steady state hypothesis, there is no big-bang-style singularity in C-field theory[5].It is required to have a system that behaves as a negative energy mode if a model successfully explains the condition of positive energy without going against energy conservation. Thus, the natural explanation for how matter is created is a negative energy field. According to Narlikar [7], the C-field of negative energy is sacrificed in order to create matter. Additionally, creation field cosmological models have been investigated by Narlikar and Padmanabhan [8], Bali and Tikekar [18], Bali and Kumawat [19], Rajbali [21] and many other authors[20].

According to the present findings accelerating expansion of the cosmos is observed.Our universe is governed by an unnamed dark energy (DE), an uncommon energy with negative pressure. The vacuum energy density which is the mathematical counterpart of the cosmic constant is the simplest DE candidate. Since then gravitational theory has been driven by the cosmological constant Λ. Einstein first proposed cosmological constant in 1917, to produce the gravitational repulsion necessary to support a static world. There has been an assumption that the cosmos is expanding ever since the Hubble constant was discovered. Furthermore, Friedmann [22] succeeded in constructing an expanding cosmos without the need of cosmological constant. Einstein admitted that a term was not strictly necessary in his equations for

the gravitational field.Zel'dovich [23] revived the controversy over the value of the cosmological constant Λ by tying it to the increase in vacuum energy density brought on by quantum fluctuations. In this way, the theoretical foundation for the progressive maintenance of the cosmological constant Λ was being established. Since there was no direct astronomical evidence prior to 1998 and the empirical upper bound was so large ($\Lambda \leq 10^{-120}$ Planck units).Particle physicists hypothesised that some fundamental principle must need the empirical upper bound to have a value of exactly zero because it was so large ($\Lambda \leq 10^{-120}$ Planck units). Similar efforts have been done by two different teams, Riess et al. [24] and Perlmutter et al. [25], to use type Ia supernovae to show the pace of cosmic expansion. This discovery provided the first unambiguous demonstration that is greater than 0 using $\Lambda \sim 1.7X10^{-121}$ Planck units.
The inflationary scenario (Abers and Lee [11]) predicted that during an early exponential phase, the vacuum energy is treated as large cosmological constant, which is expected by Glashow-Salam-Weinberg and by Grand Unified Theory as mentioned by Langacker [10]. This is because the non-trivial role of vacuum generates a cosmological constant (Λ) term in Einstein's field equations. Thus, the current observations of the cosmological constant's smallness ($\Lambda \leq 10^{-56} cm^{-2}$) provide evidence in favour of the theory that the cosmological constant varies with time.According to research by Gibbons and Hawking [12], asymptotically de-Sitter space-time results from cosmological models with positive cosmological constants. Thus, a number of authors have examined the cosmological models that relate variations in the cosmological constant to the preservation of the energy-momentum tensor of matter content and the form of Einstein's field equations, including Berman [31], Abdussattar and Vishwakarma [9], Singh and Chaubey [14], Pradhan et al. [13], Bali and Jain [15], Bali and Singh [16], Bali and Tinker [17], and Ram and Verma [39].
The scientific community presently believes that a form of repulsive pressure known as DE, which operates through the cosmological constant, is the most suitable explanation for recent findings that the cosmos appears to be expanding and speeding up.

The cosmological constant, which serves as a homogeneous and isotropic source with the function $p = -\omega$ in Einstein's field equation, is the most straightforward and well-liked candidate to characterise this exotic component among the various alternatives. The cosmological constant occupies a privileged place in the hierarchy of DE models due to its good fit of the most recent astronomical evidence. Krauss and Turner [26] and Dreitlein [27] looked into the applicability of a nonzero cosmological constant in respect to the observations . The cosmological constant Λ is time-dependent, a function of temperature, and associated with the spontaneous breaking process, according to by Linde [28].Recently Tiwari and Srivastava [46],[47] discussed for the Einstein field equations with variable cosmological constant in which the cosmological constant Λ are found to be decreasing function of cosmic time. Many authors have studied the cosmological models with diminishing Λ vacuum energy density [29]- [45].

With above motivation in mind, the C-field cosmological model for a barotropoic fluid distribution with bulk isotropic pressure $p = 0$ and cosmological constant Λ with time dependence has been examined in this paper. In order to understand Λ , we must first understand the barotropic condition $p = 0$, where p is the isotropic pressure. We observed that $\Lambda \sim 1/t^2$, which is consistent with Bertolami's finding [29]. The universe is moving through an accelerating phase, according to the deceleration parameter, q less than 0. The exponential growth of the scale factor over time indicates an inflationary scenario for the model. The creation field grows over time, which is consistent with the conclusion reached by Hoyle and Narlikar [**?**]. The C-field keeps the matter density from dissipating.

2. Metric and Basic Field Equations

We consider the cosmological spacetime that Robertson-Walker describes as being homogeneous and isotropic in the form of following metric

$$ds^2 = dt^2 - R^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2] \tag{2.1}$$

where k=0,1,-1 curvature index

Hoyle and Narlikar [6] modify the Einstein field equation's solution by including a C-field and using time-dependent information provided by

$$R_i^j - \frac{R}{2} g_i^j = -8\pi G[T_{m_i}^{\ j} + T_{c_i}^{\ j}] + \Lambda g_i^j \tag{2.2}$$

where perfect fluid energy momentum tensor $T_{m_i}^{\ j}$ and creation field tensor $T_{c_i}^{\ j}$ are defined as

$$T_{m_i}^{\ j} = (\rho + p)v_i v^j - p g_i^j \tag{2.3}$$

$$T_{c_i}^{\ j} = -f(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha) \tag{2.4}$$

Where matter and creation field is coupled with coupling constant $f > 0$ and $C_i = \frac{dC}{dx^i}$. The modified Einstein field equation for FRW metric with variable Λ gives

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G[\rho - \frac{1}{2} f\dot{C}^2] + \Lambda \tag{2.5}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 8\pi G[\frac{1}{2} f\dot{C}^2 - p] + \Lambda \tag{2.6}$$

3. Solution of The Field Equations

Vanishing divergence of Einstein Tensor give rise to energy conservation equation

$$[8\pi G T_i^j + \Lambda g_i^j]_{;j} = 0 \tag{3.1}$$

this takes

$$8\pi\dot{G}[\rho - \frac{1}{2} f\dot{C}^2] + 8\pi G[\dot{\rho} - f\dot{C}\ddot{C} + 3\rho\frac{\dot{R}}{R} - 3f\dot{C}^2\frac{\dot{R}}{R} + 3p\frac{\dot{R}}{R}] + \dot{\Lambda} = 0 \tag{3.2}$$

We consider gravitational constant G to constant and isotropic pressure $p = 0$ as consider by Hoyle and Narlikar.

$$8\pi G[\dot{\rho} - f\dot{C}\ddot{C} + 3\rho\frac{\dot{R}}{R} - 3f\dot{C}^2\frac{\dot{R}}{R}] + \dot{\Lambda} = 0 \tag{3.3}$$

And also the source field equation $C^i_{;j} = 0$ become to $C = t$ for large value of r this shows that $\dot{C} = 1$ with condition $\dot{C} = 1$ the equation (5) and (6)

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G[\rho - \frac{1}{2}f] + \Lambda \tag{3.4}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 4\pi Gf + \Lambda \tag{3.5}$$

In order to find the solution of field equation we have consider [2–4] $\Lambda = 3\alpha\frac{\dot{R}^2}{R^2} + \beta\frac{\ddot{R}}{R}$ thus equation (11)and equation (12) becomes

$$8\pi G\rho = (3 - 3\alpha)\frac{\dot{R}^2}{R^2} - \beta\frac{\ddot{R}}{R} + \frac{3k}{R^2} + 4\pi Gf \tag{3.6}$$

$$(2 - \beta)\frac{\ddot{R}}{R} + (1 - 3\alpha)\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 4\pi Gf \tag{3.7}$$

This can be also written as

$$2\frac{\ddot{R}}{R} + 2\frac{(1 - 3\alpha)}{(2 - \beta)}\frac{\dot{R}^2}{R^2} + \frac{2k}{(2 - \beta)R^2} = \frac{8\pi Gf}{(2 - \beta)} \tag{3.8}$$

To find the solution of above we consider $\dot{R} = U(R)$ this imply that $\ddot{R} = UU'$ where $U' = \frac{dU}{dR}$
Thus above equation take the form

$$\frac{dU^2}{dR} + \{2\frac{(1 - 3\alpha)}{(2 - \beta)}\}\frac{U^2}{R} = \{\frac{8\pi Gf}{(2 - \beta)}\}R - \{\frac{2k^2}{(2 - \beta)}\}\frac{1}{R} \tag{3.9}$$

This gives

$$U^2 = \{\frac{4\pi Gf}{3 - 3\alpha - \beta}\}R^2 - \frac{k}{1 - 3\alpha} \tag{3.10}$$

For simplicity we considered integration constant zero thus above equation becomes

$$\frac{dR}{\sqrt{R^2 + \frac{3-3\alpha-\beta}{4\pi Gf}\frac{(-k)}{1-3\alpha}}} = \sqrt{\frac{4\pi Gf}{3 - 3\alpha - \beta}}dt \tag{3.11}$$

This becomes

$$\frac{dR}{\sqrt{R^2 + \delta^2}} = \eta dt \tag{3.12}$$

Where

$$\delta^2 = \frac{3-3\alpha-\beta}{4\pi Gf}\frac{(-k)}{1-3\alpha}, \eta = \sqrt{\frac{4\pi Gf}{3-3\alpha-\beta}} \tag{3.13}$$

The equation (3.12) becomes

$$R = \delta sinh\eta t \tag{3.14}$$

Thus

$$\frac{\dot{R}}{R} = \eta coth\eta t \tag{3.15}$$

In order to find deterministic value of R consider $\eta = 1$ .Thus we have

$$\Lambda = 3coth^2 t + \beta \tag{3.16}$$

and

$$R^2 = \frac{(-k)}{1-3\alpha} sinh^2 t \tag{3.17}$$

Now from equation (3.6) energy density

$$\rho = \frac{1}{8\pi G}(3-3\alpha)\frac{\dot{R}^2}{R^2} - \beta\frac{\ddot{R}}{R} + \frac{3k^2}{R^2} + 4\pi Gf] \tag{3.18}$$

Therefore the FRW metric now take the following form

$$ds^2 = dt^2 - (\frac{(-k)}{1-3\alpha} sinh^2 t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2] \tag{3.19}$$

Curvature Index and creation field behaviour

Case (i) :When curvature index $k = 0$

Equation (17) take the form

$$\frac{dR}{\sqrt{R^2}} = \eta dt \tag{3.20}$$

After integration it becomes

$$R = e^{\eta t} \tag{3.21}$$

Using equations (3.21) and equation (3.18) in equation(3.3) we get

$$\frac{d\dot{C}^2}{dt} + 6\eta\dot{C}^2 = 6\eta \tag{3.22}$$

this gives

$$\dot{C}^2 e^{6\eta t} = 6\eta \int e^{6\eta t} dt = e^{6\eta t} \tag{3.23}$$

From above we get

$$\dot{C} = 1 \tag{3.24}$$

Which on integration gives

$$C = t \tag{3.25}$$

This result is similar to the result used for source field equation.There the creation field varies according as cosmic time t.

Case (ii) :When curvature index $k = 1$

Equation (3.11) take the form

$$\frac{dR}{\sqrt{R^2 + (\sqrt{\frac{-1}{1-3\alpha}})^2}} = dt \tag{3.26}$$

We have considered $\eta = 1$ after integration it becomes

$$R = \sqrt{\frac{-1}{1-3\alpha}} sinht \tag{3.27}$$

Using equations (3.27) and equation (3.18) in equation(3.3) we get

$$\frac{d\dot{C}^2}{dt} + 6cotht\dot{C}^2 = 6cotht \tag{3.28}$$

This gives

$$\dot{C}^2 sinh^6 t = 6 \int cotht.sinh^6 tdt = e^{\eta t} \tag{3.29}$$

From above we get

$$\dot{C} = 1 \tag{3.30}$$

Which on integration gives

$$C = t \tag{3.31}$$

This result is similar to the result used for source field equation.There the creation field varies according as cosmic time t.

Case (iii) :When curvature index $k = -1$

Equation (3.11) take the form

$$\frac{dR}{\sqrt{R^2 + (\sqrt{\frac{1}{1-3\alpha}})^2}} = dt \tag{3.32}$$

we have considered $\eta = 1$ after integration it becomes

$$R = \frac{1}{\sqrt{1-3\alpha}} sinht \tag{3.33}$$

Using equations (3.33) and equation (3.18) in equation(3.3) we get

$$\frac{d\dot{C}^2}{dt} + 6cotht\dot{C}^2 = 6cotht \tag{3.34}$$

this gives

$$\dot{C}^2 sinh^6 t = 6\int cotht.sinh^6 tdt \tag{3.35}$$

From above we get

$$\dot{C} = 1 \tag{3.36}$$

Which on integration gives

$$C = t \tag{3.37}$$

This result is similar to the result used for source field equation.From the relation $C = t$ the creation field (C) varies according as cosmic time t.

4. Physical and Geometrical Behaviour of the model

Metric (2.1) for the previously stated constraints results in

$$ds^2 = dt^2 - (\frac{(-k)}{1-3\alpha} sinh^2 t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2] \tag{4.1}$$

where $1 - 3\alpha \neq 0$
The cosmological parameters for the model are given below
Homogeneous mass density ρ

$$8\pi G\rho = 8\pi G f + 6\alpha cosech^2 t \tag{4.2}$$

The cosmic scale factor R

$$R(t) = \sqrt{\frac{-k}{1-3\alpha}} sinht \tag{4.3}$$

Spatial volume V

$$V(t) = 2\pi^2 R^3 \tag{4.4}$$

Hubble parameter $H(t)$

$$H = \frac{\dot{R}}{R} = cotht \tag{4.5}$$

Scalar expansion Θ

$$\Theta = 3H = 3cotht \tag{4.6}$$

Cosmological constant $\Lambda(t)$

$$\Lambda(t) = 3\alpha coth^2 t + \beta \tag{4.7}$$

Deceleration parameter q

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -tanh^2 t \tag{4.8}$$

Where have considered $\eta = 1$

5. Concluding Remarks

For the every value of curvature index the homogeneous mass density is greater than zero . The creation field (C) and the spatial volume (R^3) grows with time. The model (4.1) represents an accelerating universe because the deceleration parameter q less than 0. The model (4.1) is an example of a model without singularities. The most recent astronomical observations are consistent with the cosmological constant behaviour . Additionally, the coordinate distance from the horizon (γ_H) is the furthest point a null ray could have travelled at time t initiating from the infinite past, i.e.

$$\gamma_H = \int_{-\infty}^{t} \frac{dt}{R(t)}$$

The non-singularity of space-time allowed us to extend the timings t to $-\infty$ in the past.

$$\gamma_H = \int_{-\infty}^{t} \frac{dt}{R(t)} = \int_{0}^{t} \frac{dt}{\sqrt{\frac{-k}{1-3\alpha}} sinht}$$

The model (44) is free from the horizon because the integral diverges at the lower limit. Therefore, creation field cosmology (C-field cosmology) provides an answer to a pressing issue. The horizon issue with Big-Bang cosmology. The Big-Bang cosmological model begins with a singular state, whereas the creation field model does not.

6. Material and Methods

The experiments given in the section 3 were obtained by using personal codes elaborated with software Wolfram Mathematica [48] without using any extra precision. All computations were performed on the Windows 7 .

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Conceptualisation: Ram Bharosha Tiwari, Sudhir Kumar Srivastava ; *Software:* Ram Bharosha Tiwari ; *Writing-Original Draft:* Ram Bharosha Tiwari, Sudhir Kumar Srivastava

Conflicts of Interest: The authors declare no conflict of interest.

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