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STRONG DOMINATION INTEGRITY OF SHADOW GRAPHS OF SOME GRAPHS

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Abstract

A network can be modeled by a graph whose nodes represent the stations and whose edges represent the communication lines. Vulnerability measures the resistivity of the network to the disruption of its operation due to the failure of certain stations or communication links. Many graph theoretic parameters have been introduced and explored to measure the vulnerability of network. The strong domination integrity of a simple connected graph is a new measure of vulnerability. Here we determine the strong domination integrity of shadow graphs of path P_n, cycle C_n, complete bipartite graph $K_{m,n}$ and bistar $B_{n,n}$.

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1. Introduction

In this paper we consider simple, finite, undirected and connected graph $G = (V(G), E(G)$. We refer to Harary [8] for basic definition, notations.

The vulnerability of network have been studied in various contexts including road transportation system, information security, structural engineering and communication network. A graph structure is vulnerable if 'any small damage produces large consequences'. Various graph theoretic parameters have been introduced and studied to describe the vulnerability of communication networks including scattering number, toughness, neighbor-connectivity, integrity, mean integrity, edge-connectivity, domination edge integrity and tenacity. Depending on network models new vulnerability measures have a important role in any failure not only on nodes also on links which have special properties.

Barefoot *et al*. [3] have introduced the concept of integrity.

Definition 1.1 The integrity of a graph G is denoted by $I(G)$ and defined by $I(G) = min\{|X| + m(G - X) : X \subset V(G)\}$ where $m(G - X)$ is the order of a maximum component of $G - X$.

A set of vertices X in graph G is an $I-$set of G if $|X| + m(G - X) = I(G)$.

Barefoot *et al*. [3, 4] have investigated many results on integrity. Various results along with variations and generalization are discussed in a survey article on integrity

by Bagga *et al.* [2].
Definition 1.2 A subset U of $V(G)$ is called dominating set if for every $x \in V - U$, there exist a $y \in U$ such that x is adjacent to y. The minimum cardinality of a minimal dominating set in G is called the domination number of G denoted as $\gamma(G)$ and the corresponding minimal dominating set is called a γ-set of G.

For concepts related to domination theory we refer to Haynes *et al.* [9].

The theory of domination plays a vital role in the analysis of network when certain part is damaged. The main aim is to determine the strength or weakness of the network in such cases. In the case of disruption of a network, the damage will be more when important nodes(dominating vertices) are under damaged. Motivated by this, Sundareswaran and Swaminathan [12] have introduced the concept of domination integrity of a graph as a measure of vulnerability.
Definition 1.3 The domination integrity of a connected graph G denoted by $DI(G)$ and defined as $DI(G) = min\{|Y| + m(G - Y) : Y \text{ is a dominating set}\}$ where $m(G - Y)$ is the order of a maximum component of $G - Y$.
A set of vertices Y in graph G is an $DI-$set of G if $|Y| + m(G - Y) = DI(G)$.

Sundareswaran and Swaminathan [12–16] have explored this concept. Vaidya and Kothari [17–19] have addressed domination integrity in the context of some graph operations. Vaidya and Shah [20–22, 26] have contributed many results. Shah[23] has discussed domination integrity for wheel related graphs. For more results on can see [1, 5, 10]. Shah and Vihol [24] have discussed total domination integrity of some wheel related graphs.
Remark 1.4 [12]

(i) $DI\,(P_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil + 1; & n = 2, 3, 4, 5 \\ \left\lceil \frac{n}{3} \right\rceil + 2; & n \geq 6 \end{cases}$

(ii) $DI\,(C_n) = \begin{cases} 3; & n = 3, 4 \\ \left\lceil \frac{n}{3} \right\rceil + 2; & n \geq 5 \end{cases}$

(iii) $DI\,(K_{m,n}) = \min\{m, n\} + 1$
Sampathkumar and Pushpa Latha[11] have introduced the concepts of strong domination and weak domination.
Definition 1.5 For a graph G and $uv \in E(G)$, we say u strongly dominates v (v weakly dominates u) if $d(u) \geq d(v)$.
Definition 1.6 A subset S of $V(G)$ is called strong (weak) dominating set sd-set (wd-set) if every $v \in V - S$ is strongly (weakly) dominated by some $u \in S$.
The strong(weak) domination number $\gamma_s(G)$ ($\gamma_w(G)$) is minimum cardinality sd-set (wd-set) of G.
Remark 1.7[7]

1. $\gamma_s(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

2. $\gamma_s(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Remark 1.8[27]

1. $\gamma_s(H_n) = \begin{cases} n; & n = 3, 4 \\ n + 1; & n \geq 5 \end{cases}$

2. $\gamma_s(CH_n) = \begin{cases} 2; & n = 3 \\ \left\lceil \frac{n}{3} \right\rceil + 1; & n \geq 4 \end{cases}$

Ganesan *et al.* [6] have introduced the concept of strong domination integrity as a new measure of vulnerability. They have discussed bounds, investigated strong domination integrity of various graphs and fuzzy graphs.

Definition 1.9 The strong domination integrity of a simple graph G denoted by $SDI(G)$ and is defined as $SDI(G) = min\{|Z| + m(G - Z) : Z \text{ is a } sd\text{-set}\}$ where $m(G - Z)$ is the order of a maximum component of $G - Z$.

A set of vertices Z in graph G is an $SDI-$set of G if $|Z| + m(G - Z) = SDI(G)$.

Remark 1.10 [6]

1. $SDI(P_n) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor + 1; & n = 2, 3, 4, 5, 7 \\ \left\lceil \frac{n}{3} \right\rceil + 2; & n \geq 8 \end{cases}$

2. $SDI(C_n) = \begin{cases} 3; & n = 3, 4 \\ \left\lceil \frac{n}{3} \right\rceil + 2; & n \geq 5 \end{cases}$

3. $SDI(K_{m,n}) = min\{m, n\} + 1$.

Shah[26] has discussed strong domination integrity of some graphs obtained from wheel.

Definition 1.11 The *shadow graph* $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex u'' in G''.

For batter understanding of definition of shadow graph, path P_8 and its shadow graph are shown in *Fig. 1* and *Fig. 2* respectively.

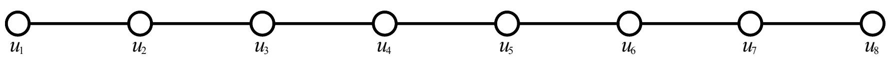

Figure 1:Path P_8

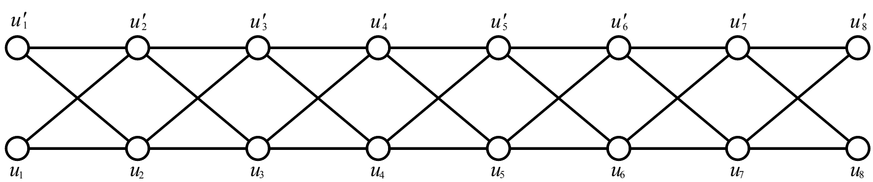

Figure 2: Shadow graph of path P_8

The present work is intended to obtain strong domination integrity of shadow graphs of path P_n, cycle C_n, complete bipartite $K_{m,n}$ and bistar $B_{n,n}$.

2. Main Results

THEOREM 2.1. *For* $2 \leq n \leq 11$, $SDI(D_2(P_n)) = 2\left\lfloor \frac{n}{2} \right\rfloor + 1$

PROOF. To construct $D_2(P_n)$, we consider P_n with $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ be the vertices of the second copy of P_n. Let $G = D_2(P_n)$.
For $n = 2$, $D_2(P_2) = C_4$, hence form remark 1.10, $SDI(D_2(P_2)) = 3$.
For $n = 3$ to 11, Let $m = \left\lfloor \frac{n}{2} \right\rfloor$, consider $U = \{y_{2i}, x_{2i}/1 \leq i \leq m\}$. U is a sd-set of G as $y_{2i-1}, x_{2i-1} \in N(y_{2i})$ and $y_{2i+1}, x_{2i+1} \in N(x_{2i})$ for $i = 1, 2, 3, 4$. Moreover $|U| = 2m$ and $m(G - U) = 1$.

In following *Fig. 3* of $D_2(P_7)$, red colored vertices represents elements of sd-set.

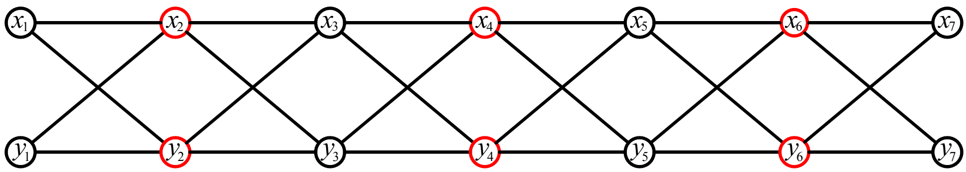

Figure 3: $D_2(P_7)$

There does not exist any sd-set U_1 of G such that $|U_1| < |U|$ and $m(G - U_1) = 1$. If we consider any sd-set U_2 of G with $m(G-U_2) \geq 2$ then $|U|+m(G-U) \leq |U_2|+m(G-U_2)$. This established the minimality of $|U| + m(G - U)$ among all sd-sets of G.
Therefore,

$$\begin{aligned} min\{|Z| + m(G - Z) : Z \text{ is a } sd\text{-set}\} &= |U| + m(G - U) \\ &= 2m + 1. \\ &= 2\left\lfloor \frac{n}{2} \right\rfloor + 1. \end{aligned}$$

Hence, $SDI(D_2(P_n)) = 2\left\lfloor \frac{n}{2} \right\rfloor + 1$, $2 \leq n \leq 11$ □

THEOREM 2.2. *For* $n \geq 12$,

$$SDI(D_2(P_n)) = \begin{cases} \frac{2n}{3} + 4; & if\ n \equiv 0 (\text{mod } 3) \\ 2\left\lfloor \frac{n}{3} \right\rfloor + 5; & if\ n \equiv 1 (\text{mod } 3) \\ 2\left\lfloor \frac{n}{3} \right\rfloor + 6; & if\ n \equiv 2 (\text{mod } 3) \end{cases}$$

PROOF. Let $V(P_n) = \{x_1, x_2, \ldots, x_n\}$ be the vertex set of the first copy of P_n and $\{y_1, y_2, \ldots, y_n\}$ be the vertex set of the second copy of P_n. Let G be the graph $D_2(P_n)$.

- If $n \equiv 0 (\text{mod } 3)$, let $m = \frac{n}{3}$, consider $U = \{y_{2+3i}, x_{2+3i} / 0 \leq i < m\}$ and $|U| = \frac{2n}{3}$.
- If $n \equiv 1 (\text{mod } 3)$, let $m = \left\lfloor \frac{n}{3} \right\rfloor$, consider $U = \{y_{2+3i}, x_{2+3i} / 0 \leq i \leq m-1\} \cup \{x_{n-1}\}$ and $|U| = 2\left\lfloor \frac{n}{3} \right\rfloor + 1$.
- If $n \equiv 2 (\text{mod } 3)$, let $m = \left\lfloor \frac{n}{3} \right\rfloor$, consider $U = \{y_{2+3i}, x_{2+3i} / 0 \leq i \leq m-1\} \cup \{x_{n-1}, y_{n-1}\}$ and $|S| = 2\left\lfloor \frac{n}{3} \right\rfloor + 2$.

In all the above cases U is a *sd*-set for G as $y_{1+3t}, y_{3+3t} \in N(y_{2+3t})$, $d(y_{2+3t}) \geq d(y_{1+3t})$, $d(y_{2+3t}) \geq d(y_{3+3t})$ and $x_{1+3t}, x_{3+3t} \in N(x_{2+3t})$, $d(x_{2+3t}) \geq d(x_{1+3t})$, $d(x_{2+3t}) \geq d(x_{3+3t})$ for $t \in \mathbb{N} \cup \{0\}$ moreover $m(G-U) = 4$.
If we consider any *sd*-set U_1 of G with $m(G-U_1) > 4$ then due to adjacency of vertices of $G = D_2(P_n)$ (i.e. to convert $G - U_1$ into disconnect graph the set U_1 must contain y_i and x_i both for fixed i) and as U_1 is *sd*-set, $|U_1| > |U|$. Hence, $|U| + m(G-U) < |U_1| + m(G-U_1)$.
If U_2 is any *sd*-set of G with $m(G-U_2) < 4$ then $|U_2| > |U|$. Hence, $|U| + m(G-U) < |U_2| + m(G-U_2)$.
This established the minimality of $|U| + m(G-U)$ among all *sd*-sets of G.
Therefore,

$$\begin{aligned} |U| + m(G-U) &= min\{|Z| + m(G-Z) : Z \text{ is a } sd\text{-set}\} \\ &= SDI(D_2(P_n)). \end{aligned}$$

Hence, for $n \geq 12$

$$SDI(D_2(P_n)) = \begin{cases} \frac{2n}{3} + 4; & if\ n \equiv 0 (\text{mod } 3) \\ 2\left\lfloor \frac{n}{3} \right\rfloor + 5; & if\ n \equiv 1 (\text{mod } 3) \\ 2\left\lfloor \frac{n}{3} \right\rfloor + 6; & if\ n \equiv 2 (\text{mod } 3) \end{cases}$$ □

THEOREM 2.3. $$SDI(D_2(C_n)) = \begin{cases} 5; & n = 3, 4 \\ 7; & n = 5, 6 \\ 9; & n = 7, 8 \\ 10; & n = 9 \\ 11; & n = 10 \end{cases}$$

PROOF. Consider two copies of C_n. Let $x_1, x_2, \ldots, x_n$ be the vertices of the first copy of C_n and $y_1, y_2, \ldots, y_n$ be the vertices of the second copy of C_n. Let $G = D_2(C_n)$.
We consider following three cases to prove the result.

Case 1: $n = 3$ to 8

For $n = 2t+1$ and $n = 2t+2$ where $t = 1$ to 3, consider $U = \{y_{1+2k}, x_{1+2k} / 0 \leq k \leq t\}$ and $|U| = 2(t+1)$. U is *sd*-set for G as $y_{2+2k}, x_{2+2k} \in N(y_{1+2k})$ and $y_n, x_n \in N(y_1)$ with $m(G-U) = 1$. If U_1 is any *sd*-set of G with $m(G-U_1) > 1$ then due to adjacency of vertices of $D_2(C_n)$ and as U_1 is *sd*-set, $|U_1| + m(G-U_1) > |U| + m(G-U)$. Moreover there does not exist any *sd*-set U_2 with $m(G-U_2) = 1$ and $|U_2| < |U|$.

Hence, $|U| + m(G - U) = 2t + 2 + 1$
$= min\{|Y| + m(G - Y) : Y \text{ is a } sd\text{-set}\}$

Therefore, $SDI(D_2(C_n)) = \begin{cases} 5; & n = 3,4 \\ 7; & n = 5,6 \\ 9; & n = 7,8 \end{cases}$

Case 2: $n = 9$

Consider $U = \{y_1, y_4, y_7, x_1, x_4, x_7\}$ then $|U| = 6$ and $m(G - U) = 4$. Clearly U is a sd-set of $D_2(C_9)$. Moreover for any other sd-set U_1 of $D_2(C_9)$ we have $|U_1| + m(G - U_1) > |U| + m(G - U) = 10$. Hence, $SDI(D_2(C_9)) = 10$.

Case 3: $n = 10$

Consider $S = \{y_1, y_3, y_5, y_7, y_9, x_1, x_3, x_5, x_7, x_9\}$ then $|U| = 10$ and $m(G - U) = 1$. Clearly U is a sd-set of $D_2(C_{10})$. Moreover for any other sd-set U_1 of $D_2(C_{10})$ we have $|U_1| + m(G - U_1) > |U| + m(G - U) = 11$. Hence, $SDI(D_2(C_{10})) = 11$.

In following *Fig. 4* of $D_2(C_{10})$, red colored vertices represents elements of sd-set.

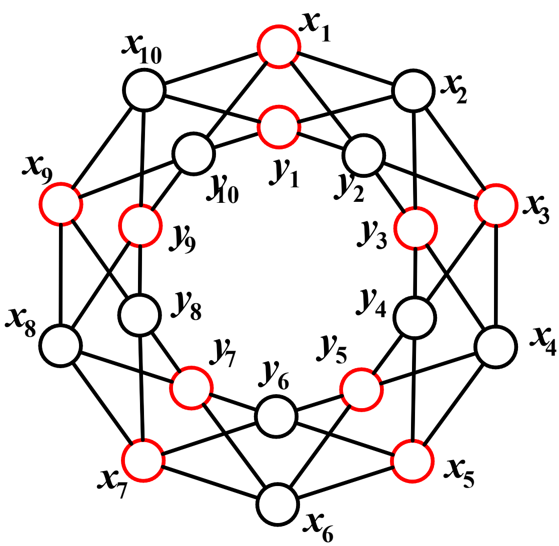

Figure 4: $D_2(C_{10})$

Therefore from above cases, $SDI(D_2(C_n)) = \begin{cases} 5; & n = 3,4 \\ 7; & n = 5,6 \\ 9; & n = 7,8 \\ 10; & n = 9 \\ 11; & n = 10 \end{cases}$ □

THEOREM 2.4. *For $n \geq 11$,*

$$SDI(D_2(C_n)) = \begin{cases} \frac{2n}{3} + 4; & if\ n \equiv 0 (\text{mod } 3) \\ \frac{2(n-1)}{3} + 6; & if\ n \equiv 1 (\text{mod } 3) \\ \frac{2(n-2)}{3} + 6; & if\ n \equiv 2 (\text{mod } 3) \end{cases}$$

Proof. Consider two copies of C_n. Let x_1, x_2,..., x_n be the vertices of the first copy of C_n and y_1, y_2,..., y_n be the vertices of the second copy of C_n. Let $G = D_2(C_n)$.

- If $n \equiv 0 (\bmod 3)$ (i.e. $n = 3m$), consider $U = \{y_{2+3i}, x_{2+3i} / 0 \leq i \leq m-1\}$ and $|U| = 2m = \frac{2n}{3}$.
- If $n \equiv 1 (\bmod 3)$ (i.e. $n = 3m+1$) or $n \equiv 2 (\bmod 3)$ (i.e. $n = 3m+2$) consider $U = \{y_{2+3i}, x_{2+3i} / 0 \leq i \leq m\} \cup \{x_{n-1}\}$. Then $|U| = 2m + 2 = \frac{2(n-1)}{3} + 2$ for $n \equiv 1 (\bmod 3)$ and $|U| = 2m + 2 = \frac{2(n-2)}{3} + 2$ for $n \equiv 2 (\bmod 3)$.

In all the above cases U is a *sd*-set for G as $y_{2+3t}, x_{2+3t} \in N(y_{1+3t})$, $y_{3+3t}, x_{3+3t} \in N(y_{4+3t})$ for $t \in \mathbb{N} \cup \{0\}$ and $y_n, x_n \in N(y_1)$ and $d(y_{1+3t}) \geq d(y_{2+3t})$, $d(y_{1+3t}) \geq d(x_{2+3t})$, $d(y_{4+3t}) \geq d(y_{3+3t})$, $d(y_{4+3t}) \geq d(x_{3+3t})$ for $t \in \mathbb{N} \cup \{0\}$ moreover $m(G-U) = 4$.
If we consider any *sd*-set U_1 of G with $m(G-U_1) > 4$ then due to adjacency of vertices of $G = D_2(C_n)$ (i.e. to convert $G - U_1$ into disconnect graph the set U_1 must contain y_i and x_i both) and as U_1 is *sd*-set, $|U_1| > |U|$. Hence, $|U| + m(G-U) < |U_1| + m(G-U_1)$. If we consider *sd*-set U_2 of G with $m(G-U_2) < 4$ then $|U_2| > |U|$. Hence, $|U| + m(G-U) < |U_2| + m(G-U_2)$.
This established the minimality of $|U| + m(G-U)$ among all *sd*-sets of G.
Therefore,

$$\begin{aligned} |U| + m(G-U) &= min\{|Z| + m(G-Z) : Z \text{ is a } sd\text{-set}\} \\ &= SDI(D_2(C_n)). \end{aligned}$$

Hence, for $n \geq 11$

$$SDI(D_2(C_n)) = \begin{cases} \frac{2n}{3} + 4; & if\ n \equiv 0 (\bmod 3) \\ \frac{2(n-1)}{3} + 6; & if\ n \equiv 1 (\bmod 3) \\ \frac{2(n-2)}{3} + 6; & if\ n \equiv 2 (\bmod 3) \end{cases}$$ □

Theorem 2.5. *$SDI(D_2(K_{m,n})) = 2n + 1$, where $n \leq m$.*

Proof. Consider two copies of complete bipartite graph $K_{m,n}$ where $n \leq m$ with $A = \{x_1, x_2, \ldots, x_m\}$ and $B = \{y_1, y_2, \ldots, y_n\}$ are two partitions of first copy of $K_{m,n}$ where $C = \{x'_1, x'_2, \ldots, x'_m\}$ and $D = \{y'_1, y'_2, \ldots, y'_n\}$ are partitions of second copy of $K_{m,n}$. Let $G = D_2(K_{m,n})$.
Consider $U = \{y_1, y_2, \ldots, y_n, y'_1, y'_2, \ldots, y'_n\}$ then $|U| = 2n$ and $m(G-U) = 1$. U is *sd*-set of G as $x_i, x'_i \in N(y_1)$ for $1 \leq i \leq m$ and $d(y_1) \geq d(x_i)$, $d(y_1) \geq d(x'_i)$ for $1 \leq i \leq m$.
In following *Fig. 5* of $D_2(K_{3,2})$, red colored vertices represents elements of *sd*-set.

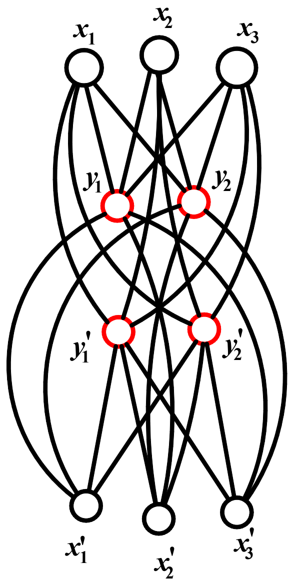

Figure 5: $D_2(K_{3,2})$

For $U = \{y_1, y_2, \ldots, y_n, y'_1, y'_2, \ldots, y'_n\}$, $m(G - U) = 1$ which is minimum. If we consider any sd-set U_1 of G with $|U_1| = t < 2n = |U|$ then $m(G - U_1) = 2(m + n) - t$.

Therefore $|U_1| + m(G - U_1) = t + 2(m + n) - t$

$$= 2(m + n)$$
$$> 2n + 1$$
$$= |U| + m(G - U).$$

Hence, $|U_1| + m(G - U_1) > |U| + m(G - U)$

Therefore $|U| + m(G - U)$ is minimum among all sd-sets of G.

Hence, $|U| + m(G - U) = 2n + 1$

$$= min\{|Z| + m(G - Z) : Z \text{ is a } sd\text{-set}\}$$
$$= SDI(D_2(K_{m,n}))$$

Therefore $SDI(D_2(K_{m,n})) = 2n + 1$, where $n \le m$. □

Theorem 2.6. $SDI(D_2(B_{n,n})) = 5$.

Proof. Consider two copies of $B_{n,n}$. Let $\{y, x, y_i, x_i, 1 \le i \le n\}$ and $\{y', x', y'_i, x'_i, 1 \le i \le n\}$ be the corresponding vertex sets of each copy of $B_{n,n}$ where y_i, x_i, y'_i, y'_i are pendant vertices. Let $G = D_2(B_{n,n})$.

Consider $U = \{y, x, y', x'\}$ then $|U| = 4$ and $m(G - U) = 1$. U is sd-set of G as $y_i, y'_i \in N(y)$, $d(y) \ge d(y_i)$, $d(y) \ge d(y'_i)$ and $x_i, x'_i \in N(x)$, $d(x) \ge d(x_i)$, $d(x) \ge d(x'_i)$ for $1 \le i \le n$. For above U, $m(G - U) = 1$ which is minimum.

In following *Fig. 6* of $D_2(B_{5,5})$, red colored vertices represents elements of sd-set.

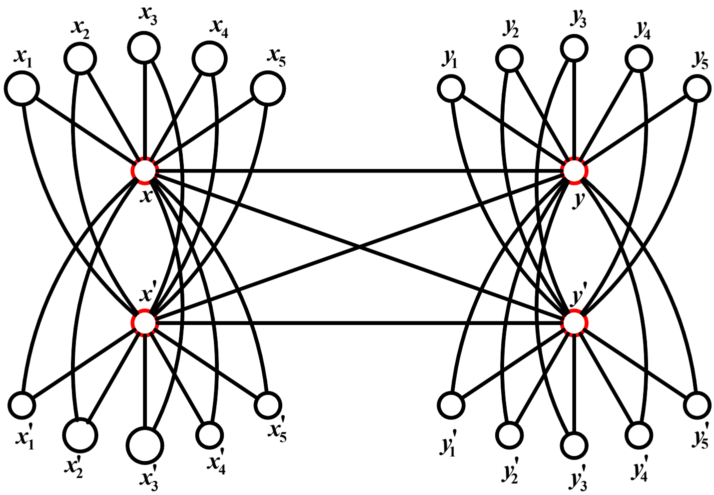

Figure 6: $D_2(B_{5,5})$

We claim that there does not exist any sd-set U_1 of G such that $|U_1| + m(G - U_1) < |U| + m(G - U)$. due to adjacency of vertices of $D_2(B_{n,n})$ if U_1 is any sd-set then $U_1 \cap U \neq \emptyset$.

Case 1: $U_1 \subset U$

If we consider any dominating set $U_1 \subset U$ then clearly $m(G - U_1) = n > 1$ and $|U_1| + m(G - U_1) > |U| + m(G - U)$

Case 2: If U_1 is any one of following sets $\{y, x, y_i\}$, $\{y, x', x_i\}$, $\{y', x', y_i\}$, $\{y', x, x_i\}$ for any fixed i then $m(G-U_1) = n > 1$. Hence, $|U_1|+m(G-U_1) = n+3 > |U|+m(G-U)$.

Therefore $|U| + m(G - U)$ is minimum among all sd-sets of G.

Therefore $|U| + m(G - U) = min\{|Z| + m(G - Z) : Z \text{ is a } sd\text{-set}\}$

$= SDI(D_2(B_{n,n}))$

Hence, $SDI(D_2(B_{n,n})) = 5$ □

3. Concluding Remarks and Open problems

We have studies vulnerability of graph network through strong domination integrity in the context of shadow graph operation. We have investigated strong domination integrity of $D_2(P_n)$, $D_2(C_n)$, $D_2(K_{m,n})$ and $D_2(B_{n,n})$. We suggest following open problems to interested readers for further research work.

1) To determine bounds or exact value of strong domination integrity of shadow graph of any graph is also interesting problem.

2) To determine strong domination integrity for other graphs and graph families.

Author contributions:

Conceptualisation: Nirav H. Shah, Nirav B. Vyas; *Writing-Original Draft:* Nirav H. Shah, Nirav B. Vyas

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