*GANITA, Vol.74(1), 2024, 259-267*

# UNIQUENESS OF A MEROMORPHIC FUNCTION AND ITS LINEAR $q$-DIFFERENCE POLYNOMIAL SHARING VALUES PARTIALLY

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**Abstract**

In this paper, we prove uniqueness of a meromorphic function sharing values partially with its linear $q$-difference polynomial which improves and generalizes the results of Chen, Yi and L*ü*, L*ü*.

2010 *Mathematics subject classification:* primary 30D35; secondary 39A05.
*Keywords and phrases:* Nevanlinna theory, Meromorphic function, Linear $q$-difference polynomial, Uniqueness, Partially shared value.

## 1. Introduction and results

In this paper, the term meromorphic, will always mean meromorphic in the complex plane $\mathbb{C}$. We shall use the standard notations in Nevanlinna's value distribution theory (see, e.g. [17, 27]).

Recently the topic of difference analogues to Nevanlinna's theory is initiated by Halburd and Korhonen ([15]), Chiang and Feng and other authors (e.g.[3]-[6]) investigated uniqueness of meromorphic functions sharing values with its shift and difference operator. Many researchers have focused on complex difference and difference equations ([20]-[22]). Nowadays $q$-difference is becoming an interesting topic in Nevanlinna Theory ([23]-[24]).

In this paper, the following definitions are used.

Definition 1.1 ([27]). Let $f(z)$ and $g(z)$ be two meromorphic functions in the complex plane $\mathbb{C}$. If $f(z)-a$ and $g(z)-a$ assume the same zeros with the same multiplicities, then we say that $f(z)$ and $g(z)$ share the value $a$ CM, where $'a'$ is a complex number.

Definition 1.2 ([5]). Let $f(z)$ and $g(z)$ be two non-constant meromorphic functions and $a \in \mathbb{C} \cup \{\infty\}$. Denote the set of all zeros of $f(z)-a$ by $E(a, f)$, where a zero with multiplicity $m$ is counted $m$ times. If $E(a, f) \subset E(a, g)$, then we say $f(z)$ and $g(z)$ partially share the value $'a'$ CM. Note that $E(a, f) = E(a, g)$ is equal to $f(z)$ and

The first author is supported by Ref. No. F.510/3/DRS-III/2016(SAP-I) Dated:29th Feb.2016 and the second author is supported by URS fellowship, Department of Mathematics, Karnatak University, Dharwad, Ref No. KU/Sch/URS/2017-18/468.

$g(z)$ share $'a'$ CM. Therefore, it is clear that the condition partially shared value CM is general than the condition shared value CM.

Definition 1.3. If $f(z)$ is a transcendental meromorphic function of zero order, then

$$L_q(z, f) = b_k(z)f(q_k z + c_k) + \cdots + b_0(z)f(q_0 z + c_0) \tag{1.1}$$

is called linear $q$-difference polynomial of $f(z)$, where $b_0(z), b_1(z), \cdots, b_k(z)(\not\equiv 0)$ are small functions of $f(z)$, let $q_0, \cdots, q_k$ are non-zero complex constants and $c_0, \cdots, c_k$ are complex constants and $k$ is a non negative integer.

In 2013, Chen, Yi [4], considered a meromorphic function which share three distinct values together with its first-order difference operator and obtained the following result.

Theorem 1.4. *Let $f(z)$ be a transcendental meromorphic function such that its order of growth $\rho(f)$ is not an integer or infinite, and let $\eta \in \mathbb{C}$ be a constant, such that $f(z+\eta) \not\equiv f(z)$. If $\Delta f = f(z+\eta) - f(z)$ and $f(z)$ share three distinct values $a, b, \infty$ CM, then $f(z+\eta) \equiv 2f(z)$.*

In 2016, Lü, Lü [25] proved that the above Theorem 1.4 still holds for meromorphic functions of finite order. That is

Theorem 1.5. *Let $f(z)$ be a transcendental meromorphic function of finite order, and let $\Delta f = f(z+c) - f(z)(\not\equiv 0)$, where $c \neq 0$ is a finite number. If $\Delta f$ and $f(z)$ share three distinct values $e_1, e_2, \infty$ CM, then $\Delta f \equiv f$.*

In this paper, we consider above theorems for linear $q$-difference polynomials and proved the following Theorem.

Theorem 1.6. *Let $f(z)$ be a transcendental meromorphic function of zero order and let $L_q(z, f)(\not\equiv 0)$ is of the form (1.1) with $b_0(z) + \cdots + b_k(z) \equiv 0$. If $L_q(z, f)$ and $f$ share the value 1 CM and satisfy $E(0, f) \subset E(0, L_q(z, f))$ and $E(\infty, f) \supset E(\infty, L_q(z, f))$, then $L_q(z, f) \equiv f$.*

Remark 1.7. *As a particular cases of Theorem 1.6, we deduce several interesting corollaries as follows.*

If

$$L_q(z, f) = \Delta_q^m f(z) = \begin{cases} f(qz) - f(z) = \Delta_q f(z) & \text{if} \quad m = 1, \\ \Delta_q^{m-1}\left(\Delta_q f(z)\right) = \sum_{r=0}^{m} (-1)^r \binom{m}{r} f(q^{m-r}z) & \text{if} \quad m \geq 2, \end{cases} \tag{1.2}$$

where $q \in \mathbb{C} \setminus \{0, 1\}$ and $m$ be a positive integer in the Theorem 1.6, then Theorem 1.6 reduces as the following Corollary 1.8.

Corollary 1.8. *Let $f(z)$ be a transcendental meromorphic function of zero order and $\Delta_q^m f(z)(\not\equiv 0)$ be of the form (1.2). If $\Delta_q^m f$ and $f$ share the value 1 CM and satisfy $E(0, f) \subset E(0, \Delta_q^m f)$ and $E(\infty, f) \supset E(\infty, \Delta_q^m f)$ then $\Delta_q^m f \equiv f$.*

If $m = 1$ in Corollary 1.8, then we have the following Corollary 1.9.

Corollary 1.9. *Let $f(z)$ be a transcendental meromorphic function of zero order and $\Delta_q f(z)(\not\equiv 0)$ be of the form (1.2). If $\Delta_q f$ and $f$ share the value 1 CM and satisfy $E(0, f) \subset E(0, \Delta_q f)$ and $E(\infty, f) \supset E(\infty, \Delta_q f)$ then $\Delta_q f \equiv f$.*

If the linear $q$-difference polynomial $L_q(z, f)$ in Theorem 1.6 is replaced by

$$L(z, f) = b_k(z)f(z + c_k) + \cdots + b_0(z)f(z + c_0), \tag{1.3}$$

where $c_0, c_1, \cdots, c_k$ are non-zero complex constants, then we have the following Corollary 1.10

Corollary 1.10. *Let $f(z)$ be a transcendental meromorphic function of zero order and let $L(z, f)(\not\equiv 0)$ is of the form (1.3) with $b_0(z) + \cdots + b_k(z) \equiv 0$. If $L(z, f)$ and $f$ share the value 1 CM and satisfy $E(0, f) \subset E(0, L(z, f))$ and $E(\infty, f) \supset E(\infty, L(z, f))$, then $L(z, f) \equiv f$.*

If the linear difference polynomial $L(z, f)$ in Corollary 1.10 is replaced by

$$L^*(z, f) = b_k(z)f(z + kc) + \cdots + b_0(z)f(z), \tag{1.4}$$

where $c$ is non-zero complex constant, then we have the following Corollary 1.11

Corollary 1.11. *Let $f(z)$ be a transcendental meromorphic function of zero order and let $L^*(z, f)(\not\equiv 0)$ is of the form (1.4) with $b_0(z) + \cdots + b_k(z) \equiv 0$. If $L^*(z, f)$ and $f$ share the value 1 CM and satisfy $E(0, f) \subset E(0, L^*(z, f))$ and $E(\infty, f) \supset E(\infty, L^*(z, f))$, then $L^*(z, f) \equiv f$.*

If

$$L^*(z, f) = \Delta_c^m f(z) = \begin{cases} f(z + c) - f(z) = \Delta_c f(z) & \text{if} \quad m = 1, \\ \Delta_c^{m-1}\left(\Delta_c f(z)\right) = \displaystyle\sum_{r=0}^{m} (-1)^r \binom{m}{r} f(z + (m - r)c) & \text{if} \quad m \geq 2, \end{cases} \tag{1.5}$$

where $c \in \mathbb{C} \setminus \{0\}$ and $m$ be a positive integer in the Corollary 1.11, then Corollary 1.11 reduces as the following Corollary 1.12.

Corollary 1.12. *Let $f(z)$ be a transcendental meromorphic function of zero order and $\Delta_c^m f(z, f)(\not\equiv 0)$ be of the form (1.5). If $\Delta_c^m f$ and $f$ share the value 1 CM and satisfy $E(0, f) \subset E(0, \Delta_c^m f)$ and $E(\infty, f) \supset E(\infty, \Delta_c^m f)$ then $\Delta_c^m f \equiv f$.*

If $m = 1$ in Corollary 1.12, then we have the following Corollary 1.13.

Corollary 1.13. *Let $f(z)$ be a transcendental meromorphic function of zero order and $\Delta_c f(z, f)(\not\equiv 0)$ be of the form (1.5). If $\Delta_c f$ and $f$ share the value 1 CM and satisfy $E(0, f) \subset E(0, \Delta_c f)$ and $E(\infty, f) \supset E(\infty, \Delta_c f)$ then $\Delta_c f \equiv f$.*

## 2. Some preliminary results

To prove our main results, we require the following Lemmas.

LEMMA 2.1. ([23], Theorem 2.1) *Let $f(z)$ be a non-constant zero-order meromorphic function and $q$ is nonzero complex constant. Then*

$$m\left(r, \frac{f(qz+c)}{f(z)}\right) = S(r, f)$$

*on a set of logarithmic density 1.*

LEMMA 2.2. *Let $f(z)$ be a meromorphic function of zero order and $L_q(z, f)$ is as defined in (1.1). Then we have*

(i) $m\left(r, \frac{L_q(z,f)}{f(z)}\right) = S_q(r, f),$

(ii) *Further if $b_0(z) + \cdots + b_k(z) \equiv 0$, then $m\left(r, \frac{L_q(z,f)}{f-a}\right) = S_q(r, f)$, for any complex constant $a$ on a set of logarithmic density 1.*

*Proof:* (i) *Using Lemma 2.1, we deduce that*

$$\begin{aligned} m\left(r, \frac{L_q(z,f)}{f(z)}\right) &= m\left(r, \frac{b_k(z)f(q_kz+c_k)+\cdots+b_0(z)f(q_0z+c_0)}{f(z)}\right) \\ &\leq m\left(r, \frac{b_k(z)f(q_kz+c_k)}{f(z)}\right) + \cdots + m\left(r, \frac{b_0(z)f(q_0z+c_0)}{f(z)}\right) + S_q(r, f) \\ &= S_q(r, f). \end{aligned}$$

(ii) *By the above result* (i) *we have*

$$m\left(r, \frac{L_q(z, f-a)}{f-a}\right) = S_q(r, f) \tag{2.1}$$

*Since $b_0(z) + \cdots + b_k(z) \equiv 0$, we get $L_q(z, a) = 0$.*
*Hence*

$$L_q(z, f-a) = L_q(z, f) - L_q(z, a) = L_q(z, f). \tag{2.2}$$

*By (2.1) and (2.2), we obtain*

$$m\left(r, \frac{L_q(z,f)}{f-a}\right) = m\left(r, \frac{L_q(z,f-a)}{f-a}\right) = S_q(r, f).$$

LEMMA 2.3. *Let $f(z)$ be a meromorphic function of zero order and $L_q(z, f) \not\equiv 0$ is of the form (1.1) such that $b_0(z) + \cdots + b_k(z) \equiv 0$. Let $l \geq 2$ and $a_1, \cdots, a_l$ be distinct complex constants. Then*

$$m(r, f) + \sum_{j=1}^{l} m\left(r, \frac{1}{f-a_j}\right) \leq 2T(r, f) - N^*(r, f) + S_q(r, f).$$

*where,*

$$N^*(r,f) = 2N(r,f) - N(r, L_q(z,f)) + N\left(r, \frac{1}{L_q(z,f)}\right)$$

*on a set of logarithmic density 1.*

Proof. By denoting

$$P(f) = \prod_{j=1}^{l}(f - a_j)$$

then we have,

$$\frac{1}{P(f)} = \sum_{j=1}^{l} \frac{\alpha_j}{f - a_j}$$

where $\alpha_j \in S_q(r,f)$. Using this, we have

$$\begin{aligned} m\left(r, \frac{1}{P(f)}\right) &\leq \sum_{j=1}^{l} m\left(r, \frac{\alpha_j}{f - a_j}\right) + S_q(r,f). \\ &\leq \sum_{j=1}^{l} m\left(r, \frac{1}{f - a_j}\right) + S_q(r,f). \end{aligned} \tag{2.3}$$

Hence, by (*ii*) of Lemma 2.2, we obtain

$$\begin{aligned} m\left(r, \frac{L_q(z,f)}{P(f)}\right) &\leq \sum_{j=1}^{l} m\left(r, \frac{L_q(z,f)}{f - a_j}\right) + S_q(r,f) \\ &= S_q(r,f) \end{aligned}$$

Using this, we obtain

$$\begin{aligned} m\left(r, \frac{1}{P(f)}\right) &= m\left(r, \frac{L_q(z,f)}{P(f)} \frac{1}{L_q(z,f)}\right) \\ &\leq m\left(r, \frac{L_q(z,f)}{P(f)}\right) + m\left(r, \frac{1}{L_q(z,f)}\right) + S_q(r,f) \\ &\leq m\left(r, \frac{1}{L_q(z,f)}\right) + S_q(r,f). \end{aligned} \tag{2.4}$$

By combining the first main theorem, (2.4) and Valiron-Mohon'ko identity, we get

$$\begin{aligned} T(r, L_q(z,f)) &= T\left(r, \frac{1}{L_q(z,f)}\right) + S_q(r,f) \\ &= m\left(r, \frac{1}{L_q(z,f)}\right) + N\left(r, \frac{1}{L_q(z,f)}\right) + S_q(r,f). \end{aligned} \tag{2.5}$$

Using (2.3), (2.4) and (2.5), we can write

$$T(r, L_q(z, f)) \geq \sum_{j=1}^{l} m\left(r, \frac{1}{f - a_j}\right) + N\left(r, \frac{1}{L_q(z, f)}\right) + S_q(r, f). \tag{2.6}$$

Thus, by ($i$) of Lemma 2.2, (2.6) reduces to

$$\begin{aligned} m(r, f) + \sum_{j=1}^{l} m\left(r, \frac{1}{f - a_j}\right) &\leq T(r, f) + m(r, L_q(z, f)) + N(r, L_q(z, f)) \\ &\quad -N\left(r, \frac{1}{L_q(z, f)}\right) - N(r, f) + S_q(r, f) \\ &\leq T(r, f) + m(r, f) + N(r, L_q(z, f)) \\ &\quad -N\left(r, \frac{1}{L_q(z, f)}\right) - N(r, f) + S_q(r, f) \\ &= 2T(r, f) + N(r, L_q(z, f)) - N\left(r, \frac{1}{L_q(z, f)}\right) \\ &\quad -2N(r, f) + S_q(r, f) \end{aligned}$$

Thus, we arrive at the conclusion of Lemma 2.3. □

LEMMA 2.4 ([24], [26]). *Let $f(z)$ be a transcendental meromorphic function of zero order and if $q, c$ are two non-zero complex constants, then we have*

$$T(r, f(qz + c)) = T(r, f) + S(r, f)$$

*on a set of logarithmic density 1.*

LEMMA 2.5 ([28]). *Let $f(z)$ be a non-constant zero-order meromorphic function and $q \in \mathbb{C} \setminus \{0\}$, then*

$$T(r, f(qz)) = T(r, f(z)) + S(r, f)$$

*on a set of logarithmic density 1.*

## 3. Proof of Theorem 1.6

PROOF. By the assumption $E(0, f) \subset E(0, L_q(z, f))$, we have

$$N\left(r, \frac{1}{f}\right) \leq N\left(r, \frac{1}{L_q(z, f)}\right). \tag{3.1}$$

Inequality (3.1) together with (*i*) of Lemma 2.2, we obtain

$$\begin{aligned}
T(r,f) &= T\left(r,\frac{1}{f}\right)+O(1)\\
&= m\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f}\right)+O(1)\\
&\le m\left(r,\frac{L_q(z,f)}{f}\right)+m\left(r,\frac{1}{L_q(z,f)}\right)+N\left(r,\frac{1}{f}\right)+O(1)\\
&\le m\left(r,\frac{L_q(z,f)}{f}\right)+m\left(r,\frac{1}{L_q(z,f)}\right)+N\left(r,\frac{1}{L_q(z,f)}\right)+O(1)\\
&\le T\left(r,\frac{1}{L_q(z,f)}\right)+S_q(r,f)\\
&\le T(r,L_q(z,f))+S_q(r,f). \qquad (3.2)
\end{aligned}$$

On the other hand by Lemma 2.4, we have

$$T(r,L_q(z,f))\le (k+1)T(r,f)+S_q(r,f). \qquad (3.3)$$

Combining (3.2) and (3.3), it follows that

$$S_q(r,f)=S(r,L_q(z,f))=S(r).$$

We set

$$\frac{L_q(z,f)}{f}=h. \qquad (3.4)$$

The assumptions $E(0,f)\subset E(0,L_q(z,f))$ and $E(\infty,f)\supset E(\infty,L_q(z,f))$ imply that $h$ is an entire function. Thus, by Lemma 2.2, we get

$$T(r,h)=m(r,h)+N(r,h)=m(r,h)=m\left(r,\frac{L_q(z,f)}{f}\right)=S(r). \qquad (3.5)$$

Next, we consider the following cases for $h$.
Case 1: If $h$ is not constant.
Since $L_q(z,f)$ and $f$ share 1 CM, we obtain from (3.4) and (3.5), that

$$N\left(r,\frac{1}{f-1}\right)\le N\left(r,\frac{1}{h-1}\right)\le S(r). \qquad (3.6)$$

By the assumptions $E(0,f)\subset E(0,L_q(z,f))$ and $E(\infty,f)\supset E(\infty,L_q(z,f))$, we notice that

$$N(r,L_q(z,f))-N(r,f)\le 0, \qquad N\left(r,\frac{1}{f}\right)-N\left(r,\frac{1}{L_q(z,f)}\right)\le 0. \qquad (3.7)$$

By applying Lemma 2.3, combining with (3.6) and (3.7), we have

$$\begin{aligned} T(r,f) &= N\left(r,\frac{1}{f-1}\right) + [N(r,L_q(z,f)) - N(r,f)] \\ &\quad + \left[N\left(r,\frac{1}{f}\right) - N\left(r,\frac{1}{L_q(z,f)}\right)\right] + S(r) \\ &\leq N\left(r,\frac{1}{f-1}\right) + S(r) = S(r). \end{aligned}$$

Which is contradiction.
Case 2: If $h$ is constant.
Subcase (i): Suppose $h \neq 1$, then 1 is Picard exceptional value of $f$ using the hypothesis $f$ and $L_q(z,f)$ share 1 CM, so that

$$N\left(r,\frac{1}{f-1}\right) = O(1).$$

As in the Case 1, we get a contradiction.
Subcase (ii): If $h = 1$, then the conclusion holds, that is $L_q(z,f) \equiv f$. □

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