GANITA,Vol.74(1), 2024, 279-288

MHD CONVECTIVE FLOW OF CHEMICALLY REACTING VISCOELASTIC FLUID THROUGH AN INFINITE INCLINED PLATE

L. RAMA MOHAN REDDY ✉, P. VEERA SANJEEVA KUMAR, K. MOHNA BABU and B.NAGARAJA NAIK

Abstract

A theoretical study of the impacts of thermal radiation, heat generation, radiation absorption, and chemical reaction on MHD convective flow of viscoelastic fluid was conducted in an unstable situation. The flow along the infinite inclined vertical plate embedded in the porous medium is considered. The method of perturbation is used to obtain an exact solution in terms of velocity, temperature, and concentration of the governing equations. The solutions are obtained, and the graphical interpretation done by using MATLAB software. The effect of skin friction, Nusselt number, and Sherwood number, their gradients are analyzed and also numerical values are tabulated. It is observed that by changing the inclination angle from 0° to 90°, the amplitude of the flow increases.

Keywords and phrases: Thermal radiation, heat generation, radiation absorption inclined vertical plate and perturbation method.

1. Introduction

Viscoelastic fluid model is one of the subclass of rate type fluids which has gained broad attractions among the researchers in last 15 years. The fluid is known as viscoelastic fluid for both viscosity and elasticity properties. The key benefit of using this type of fluid is that it can predict the relaxation of stress while other fluids of the differential type cannot predict such results. The dynamic viscoelastic fluids have an effective technique for tumbling turbulent skin friction in major-scale flows compared to Newtonian fluids. Macroscopic properties that disregard our intuition are followed by non-Newtonian fluids.

The movement of viscoelastic fluid on a fluctuating surface was examined by Hayat et al. (2004). Kumeresan and Vijay Kumar (2017) offered an exact solution for the flow of MHD viscoelastic fluid in unstable situations. This flow surrounded by porous surfaces was investigated by Nayak et al. (2013) along with the presence of chemical reactions. The MHD free convective flow of a viscous fluid through porous medium was investigated by Singh et al. (2005). The rotation flow of viscoelastic fluid past an infinite vertical oscillating porous plate with a chemical reaction was discussed by Veera Krishna and Gangadhar Reddy (2016). Chowdary and Islam (2000) developed and recorded viscoelastic fluid past a free convection model of an immeasurable plate. The effect of heat and mass transfer on this flow was investigated by Chowdary and Kumar Das (2014, 2009) under the simultaneous occurrence of chemical reaction and radiation. Krishna et al. (2019) discussed heat and mass transfer over an infinite non-conducting vertical flat porous plate on MHD free convective flow. Chandra Reddy et al. (2015, 2018) investigated the effects of thermal and solute buoyancy on flow under various suction and parameter varieties. The unstable MHD convective flow of second grade fluid through a porous medium in a rotating parallel plate channel with a temperature-dependent source was investigated by Veera Krishna et al. (2016).

The radiative flow over an inclined plate under concurrent heat and mass transfer was specifically discussed by Bhuvaneswari et al. (2010). The numerical study of the flow properties of Jeffery nanofluids past a moving plate in the conducting area was investigated by Baddela Hari Babu et al. (2020). Srinivasa Raju et al. (2017, 2016) used the finite element approach to analyze the existence of viscous dissipative flows from Casson and then to incorporate parameters of cross diffusion. Mass diffusion and changeable temperature were considered by Rajput et al. (2017) and this form of flow was analyzed. The mass transfer and heat source effects on MHD flows through the inclined porous vertical surface were explored and concluded by Reddy et al. (2011). Hari Babu et al. (2020) investigated the Hall and ion-slip effects of rotating Jeffery fluid over an infinite vertical porous surface on MHD free convection flow.

All the previous studies discussed in the literature are confined to the study of viscoelastic fluids flow past on vertical porous plates only. Also the above studies are considered only Newtonian fluids. Nagaraju et al. (2019) noticed the limitations in the above studies and focused their studies on Non- Newtonian fluids flow past on inclined plates. The present work is an extension of studies on Non- Newtonian fluids flow past on inclined plates. In this investigation considering the angle of inclination is the main parameter and studied the influences of the Dufour effect, absorption of radiation of heat generation on inclined plate.

2. Formulation of the problem

In unsteady situations, the flow of viscoelastic fluid past an inclined upright porous plate in the conducting area is considered. The problem involves the presence of a heat source, chemical reaction, and thermal diffusion effect.

The axis of x* is taken as the flow path along the vertical plate and is normal to the axis of y*. In the direction of the y* axis, a magnetic field of equal ability is applied in transverse mode. The flow medium and plate are first maintained at temperature T^*_∞ and at all points of the fluid at the same concentration C^* levels. An impulsive motion with velocity $u = u_0$ is given to the plate when time elapses and the temperature and concentration T_w^* C_w^* are respectively preserved.

$$\frac{\partial u^*}{\partial t^*} = \nu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{K_0}{\rho}\frac{\partial^3 u^*}{\partial y^{*2}\partial t^*} - \frac{\nu}{K_p^*}u^* - \frac{\sigma B_0^2 u^*}{\rho} + g\sin\phi\beta(T^* - T_\infty^*) + g\sin\phi\beta(C^* - C_\infty^*) \quad (2.1)$$

$$\rho Cp\frac{\partial T^*}{\partial t^*} = k\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q_r}{\partial y^*} + Q^*(T^* - T_\infty^*) + Q_1(C^* - C_\infty^*) \quad (2.2)$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_r(C^* - C_\infty^*) \quad (2.3)$$

Also the following boundary conditions are considered

$t^* \leq 0 : u^* = 0, T^* = T_\infty^*, C^* = C_\infty^*$ for all y^*

$t^* > 0 : u^* = u_0, T^* = T_w^*, C^* = C_w^*$ at $y^* = 0$

$u^* = 0, T^* \to T_\infty^*, C^* \to C_\infty^* \quad as \quad y^* \to \infty$

For the event of an optically thin gray gas, the local radiant is expressed by

$$\frac{\partial q_r}{\partial y^*} = -4a^*\sigma(T_\infty^* - T^{*^4}) \quad (2.4)$$

Here $A = \frac{u_0^2}{\nu}$, It is assumed that the temperature difference within the flow are sufficiently small and that T^{*^4} may be expressed as a linear function of the temperature. This is obtained by expanding$T*^4$ in a Taylor series about the higher order terms,we get

$$T^{i^4} \approx 4T_\infty^{*^4}T^i - 3T_\infty^{*^4} \quad (2.5)$$

Substituting equations (2.4) and (2.5) in equation (2.2), we get

$$\rho Cp\frac{\partial T^*}{\partial t^*} = k\frac{\partial^2 T^*}{\partial y^{*2}} + 16a^*\sigma T_\infty^{*^3}(T_\infty^* - T^*) + Q(T^* - T_\infty^*) + Q_1(C^* - C_\infty^*) \quad (2.6)$$

The following are non-dimensional quantities

$$u = \frac{u^*}{u_0}, t = \frac{t^* u_0^2}{\nu}, y = \frac{y^* u_0}{y}, \theta = \frac{T^* - T^*r}{T_w^* - T_\infty^*}, Gr = \frac{g\beta\nu(T^*w - T_\infty^*)}{u_0^3} \quad (2.7)$$

$$Gm = \frac{g\beta^*(C_w^* - C_\infty^*)}{u_0^3}, C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}, k = \frac{\nu K_r}{u_0^2}, P = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D} \quad (2.8)$$

$$M = \frac{\sigma B_0^2\nu}{\rho u_0^2}, R = \frac{16a^*\nu^2\sigma T_\infty^{*^3}}{ku_0^2}, T = \frac{K_0^2 u_0^2}{\rho\nu^2}, Q = \frac{Q^*\nu^2}{Ku_0^2}, S_0 = \frac{D_1(T_w^* - T_\infty^*)}{\nu(C_w^* - C_\infty^*)} \quad (2.9)$$

Equation (2.1),(2.6) and (2.3) leads to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - T\frac{\partial^3 u}{\partial y^2 \partial t} - Mu - \frac{u}{K} + Gr.\sin\phi.\theta + Gm.\sin\phi.C \tag{2.10}$$

$$P_r\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - R\theta + Q\theta + \chi C \tag{2.11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - K_r C \tag{2.12}$$

The non-dimensional boundary conditions are given by

$$t \leq 0; u = 0; \theta = 0; C = 0 \;\; Forall \;\; y \tag{2.13}$$

$$t > 0; u = 1; \theta = 1; C = 1 \;\; At \;\; y = 0 \tag{2.14}$$

$$u \to 0; \theta \to 0; C \to 0 \;\; As \;\; y \to \infty \tag{2.15}$$

3. Method of Solution

In order to get the exact expressions for the velocity, temperature and concentration, we assume the trail solution as follows

$$u(y,t) = u_0(y)e^{nt} \tag{3.1}$$

$$\theta(y,t) = \theta_0(y)e^{nt} \tag{3.2}$$

$$C(y,t) = C_0(y)e^{nt} \tag{3.3}$$

The corresponding boundary conditions can be written as

$$u_0 = e^{-nt}, \theta_0 = e^{-nt}, C_0 = e^{-nt} \;\; at \;\; y = 0 \tag{3.4}$$

$$u_0 \to 0, \theta_0 \to 0, C_0 \to 0 \;\; as \;\; y \to \infty \tag{3.5}$$

Analytical solutions are provided by ordinary differential equations that satisfy the boundary conditions.

$$u = (1 - A_{12} - A_{13})e^{-A_{11}y} + A_{12}e^{-\sqrt{A_2 y}} + A_{13}e^{-\sqrt{A_1 y}} \tag{3.6}$$

$$\theta = (1 - A_4)e^{-\sqrt{A_2 y}} + A_4 e^{-\sqrt{A_1 y}} \tag{3.7}$$

$$C = e^{-\sqrt{A_1 y}} \tag{3.8}$$

Skin-friction: The skin-friction at the plate which is non-dimensional form is given by

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = A_{11} - A_{11}A_{12} - A_{11}A_{13} + A_{12}\sqrt{A_2} + A_{13}\sqrt{A_1} \tag{3.9}$$

Nusselt number: The rate of heat transfer which is the gradient of temperature is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right) = (1 - A_4)\sqrt{A_2} + A_4\sqrt{A_1} \tag{3.10}$$

Sherwood number: The rate of mass transfer which is the gradient of concentration is given by

$$Sh = -\left(\frac{\partial C}{\partial y}\right) = \sqrt{A_1} \tag{3.11}$$

4. Results and Discussion

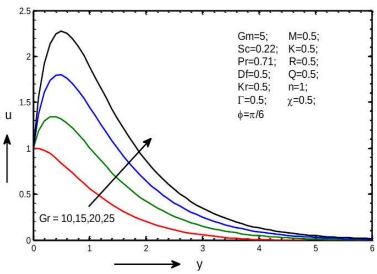
The graphs and numerical values are framed based on the exact solutions obtained from the governing equations. The MAT Lab software has used for obtaining the results. A closed analytical solution under the influence of thermal diffusion, radiation absorption and heat generation for the problem of unsteady MHD convective Non-Newtonian fluid flow. In order to get physical insight into the problem, the velocity, temperature, concentration distributions along with skin friction, Nusselt number and Sherwood number have been studied by assigning numerical values for the various parameters such number (Gr), radiation parameter (R), heat source (Q), Prandtl number (Pr), chemical reaction parameter (Kr), Porosity parameter (K), Dufour number (Df). The results are illustrated through the Figures 1–11 and Tables 1-3.

Figures 1-5 illustrate the changes of the fluid velocity under the influence of various parameters. The effect of the thermal Grashof number on velocity is presented in Figure.1. As Gr increases, so does velocity. This is due to buoyancy acting on the fluid particles because of the gravitational force that raises the velocity of the fluid. In the presence of the modified Groshof number, which also increases the fluid velocity, a similar effect is noted in Figure.2. Velocity profiles with magnetic parameter variance are shown in Figure.3.

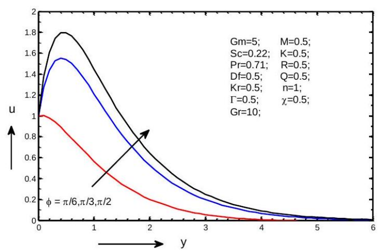
From Figure.3, it is noted that the increase in magnetic parameters reduces velocity. A magnetic force, called Lorentz force, is created in the flow field when an electrically conducting fluid moves in the presence of an applied magnetic field whose tendency is to resist the motion of the fluid. For this cause, the fluid velocity of the increasing parameter is delayed (M). The differences in velocity profiles for different values of the Permeability parameter are shown in Figure.4. It is obvious from Figure.4 that thevelocity increases as K increases. The velocity variations due to the inclination of various angles from 30 0 to 90 0 are shown in Figure.5. The growth in velocity is perfectly noted from the beginning to the end of the angles. The angular moving plate resists the motion of flow and thus growth.

The variations of the fluid temperature under the influence of various parameters are shown in Figures 6-9. Form the Figure.6, it is observed that the temperature in the viscous fluid is greatly decreased by an increase in Pr. It can be seen from Figure.6 that as Pr increases, the thickness of the thermal boundary layer decreases. The effect of a heat source on temperature is depicted in Figure.7. It is noted that as the heat source parameter increases, the temperature increases. Figure. 8 demonstrates the influence of the parameters of radiation on the distribution of temperature. It demonstrates that with increasing radiation parameter values, the temperature decreases. The temperature effect of the radiation absorption parameter is shown in Figure.9. Temperature changes are observed as the radiation absorption parameter increases

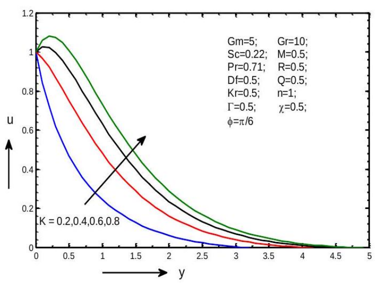
The variations of fluid concentration under the influence of various parameters are shown in Figures 10-11. The effect of the chemical reaction on concentration is depicted in Figure.10. The concentration decreases as the chemical reaction parameter increases, it is noted. The Figure. 11 demonstrates the effect of the Schmidt number on concentration. It is evident from this Figure that concentration decreases with an increase in the Schmidt number. Since the Schmidt number is a dimensionless number defined as the ratio of diffusivity of momentum and diffusivity of mass, and is used to describe fluid flows in which the process of convection of momentum and mass diffusion is simultaneous. Therefore, with an increase in the Schmidt number, the concentration boundary layer decreases.



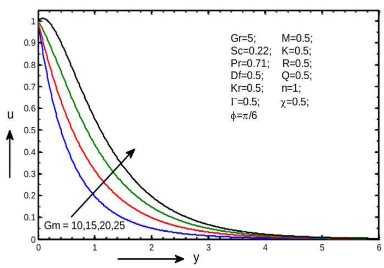
(a) Figure 1.The velocity profile for u against Gr



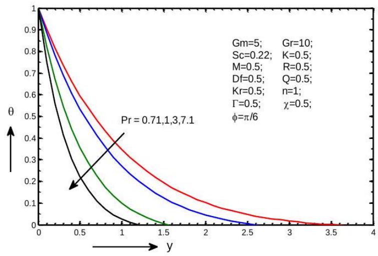
(b) Figure 2.The velocity profile for u against Gm



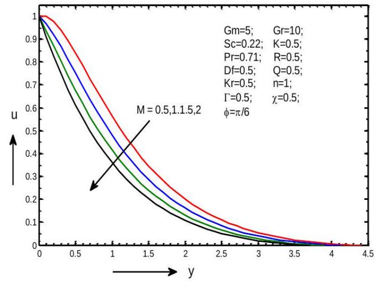
(a) Figure 3.The velocity profile for u against M



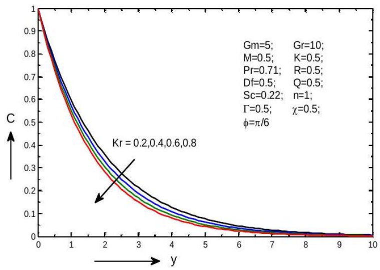
(b) Figure 4.The velocity profile for u against K



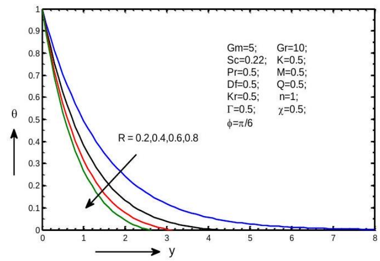
(a) Figure 5.The velocity profile for u against ϕ



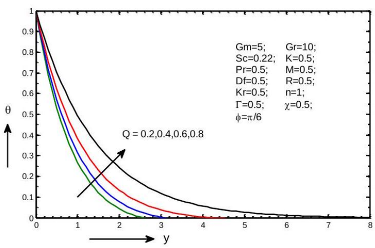
(b) Figure 6.The temperature profile for θ against Pr



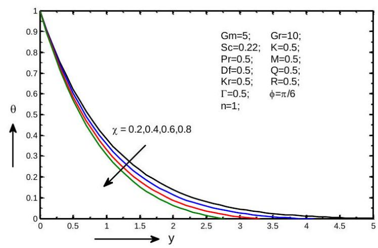
(a) Figure 7.The temperature profile for θ against Q



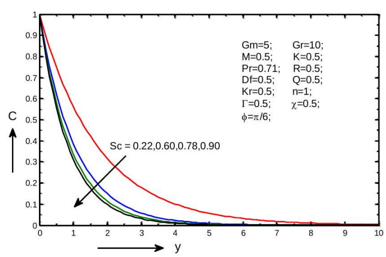
(b) Figure 8.The temperature profile for θ against R



(a) Figure 9.The temperature profile for θ against χ



(b) Figure 10.The Concentration profile for C against Kr



(a) Figure 11.The Concentration profile for C against Sc

The study of changes in skin friction is continued under the related numerical value considerations in Table 1. The velocity gradient increases as the magnetic parameter, inclination angle and Prandtl number values rise, while the overturn nature is observed with the viscoelastic parameter. As shown in Table 2, the increasing values of Pr and R and the decreasing values of Df lead to an increase in the rate of heat transfer. The concentration gradient at the foot of the plate increased as shown in Table 3 under the influence of Sc and Kr.

TABLE 1

M	α	γ	**Pr**	τ
3	$\pi/6$	0.5	0.71	-0.8566
5				-0.0539
7				0.4939
9				1.0834
	$\pi/6$			-1.9792
	$\pi/3$			0.4015
	$\pi/2$			3.5936
	$2\pi/3$			7.0126
		0.6		-1.8998
		0.6		-2.2968
		0.7		-2.8098
		0.8		-3.4023
			0.71	-1.9394
			2	-1.9526
			3	-1.5302
			7.1	-1.4012

TABLE 2

Pr	**R**	**Q**	**Df**	**Nu**
0.71	3	0.6	0.5	1.8061
1				1.9026
5				2.7353
7.1				3.0920
	3			1.8021
	4			2.0294
	5			2.2592
	6			2.3450
		0.6		1.7351
		1		1.7596
		1.4		1.5965
		1.8		1.5997
			0.5	1.7591
			1	1.5899
			1.5	1.3977
			2	-1.2254

TABLE 3

Sc	**Kr**	**Sh**
0.2200	0.5	0.6359
0.6000		0.9258
0.7800		1.0796
0.9600		1.1998
	0.5	1.1025
	1.0	1.1968
	1.5	1.2956
	2.0	1.4984

5. Conclusions

The following are the key points of this research. a. With an increase in the number of Grashof and the adjusted number of Grashof and the permeability of the porous medium and angle of inclination, the velocity increases while the presence of a magnetic parameter decreases. b. In the presence of heat source, temperature increases when in the presence of Prandtl number, radiation parameter, and parameter of absorption of radiation decreases. c. The concentration decreases with an increase in the amount of Schmidt and chemical reaction parameters.

6. Appendix:

$$A_1 = ScKr + nSc \quad A_2 = R + nPr - Q \quad A_3 = -X - DfA_1 \tag{6.1}$$

$$A_4 = \frac{A_3}{A_1 + A_2} \quad A_5 = 1 - n\gamma \quad A_6 = M + \frac{1}{K} + n \tag{6.2}$$

$$A_7 = -Gr(1 - A_4)\sin\phi \quad A_8 = -GrA_4 \sin\phi \quad A_9 = -Gm \sin\phi \tag{6.3}$$

$$A_{10} = A_8 + A_9 \quad A_{11} = \sqrt{\frac{A_6}{A_5}} \tag{6.4}$$

References

[1] Baddela Hari Babu, P.S. Rao and S.V.K.Varma, *Numerical analysis of flow characteristics of Jeffery Nano fluids past a moving plate in conducting field.*, AIP Conference Proceedings 2246 (2020).

[2] Bhuvaneswari, M, Sivasankaran, S, Kim. Y.J., *Exact analysis of radiation convective flow heat and mass transfer over an inclined plate in a porous medium.*, World Appl. Sci. J. **10** (2010) 774–778.

[3] Choudhury, R. and M. Mahanta., *Free convective MHD flow of a visco- elastic fluid past an infinite vertical channel*, Int. J. Appl. Math. **23** (2009) 189-203.

[4] Chowdhury, M. K., Islam, M. N., *MHD Free Convection Flow of Visco-Elastic Fluid Past an Infinite Vertical Porous Plate*, Heat and Mass transfer **5** (2000) 439 – 447.

[5] E. Kumaresan and A .G. Vijaya Kumar, *An exact solution on unsteady MHD viscoelastic fluid flow past an infinite vertical plate in the presence of thermal radiation*, Frontiers in Heat and Mass Transfer **8** (2017) .

[6] Hari Babu,B., Srinivasa rao, P., varma, SVK, *Hall and ion-slip effects on MHD free convection flow of rotating Jeffery fluid over an infinite vertical porous surface*, Heat transfer **50(2)** (2020) 1-23.

[7] Hayat, T., Mohyuddin, M.R., Asghar, S. and Siddiqui, A.M., *The flow of a visco-elastic fluid on an oscillating plate*, Zeitschrift für Angewandte Mathematik und Mechanikr **84** (2004) 65-70.

[8] Krishna. M.V., Reddy, M.G. and Chamkha, A.J., *Heat and mass transfer on MHD free convective flow over an infinite non-conducting vertical flat porous plate*, Int. Jour. of Fluid Mech. Res. **45(5)** (2019) 1-25.

[9] Nayak, A., Dash, G.C., Panda, *MHD Flow of a Visco-Elastic Fluid along Vertical Porous Surface With Chemical Reaction*, A. Phys. Sci. **83(2)** (2023) 153-161.

[10] P. Chandra Reddy, M.C. Raju, G.S.S. Raju, *MHD natural convective heat generating/ absorbing and radiating fluid past a vertical plate embedded in porous medium–an exact solution*, Journal of the Serbian Society for Computational Mechanics **12(2)** (2018) 106-127.

[11] P. Chandra Reddy, M.C. Raju, G.S.S. Raju, *Thermal and solutal buoyancy effect on MHD boundary layer flow of a visco-elastic fluid past a porous plate with varying suction and heat source in the presence of thermal diffusion*, Journal of Applied and Computational Mathematics **4(5)** (2015) 1-7.

[12] R. Choudhury and S. Kumar Das, *Visco-Elastic MHD Free Convective Flow through Porous Media in Presence of Radiation and Chemical Reaction with Heat and Mass Transfer*, Journal of Applied Fluid Mechanics **7(4))** (2014) 603-609.

[13] R. Srinivasa Raju, G. Jithender Reddy, G. Anitha, *MHD Casson viscous dissipative fluid flow past a vertically inclined plate in presence of heat and mass transfer*, A finite element technique, Frontiers in Heat and Mass Transfer **8** (2017)

[14] Reddy, M. G. and N. B. Reddy, *Mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium*, Journal of Applied Fluid Mechanics **4(2)** (2011) 7-11.

[15] Singh, P. and Gupta, C.B., *MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer*, Indian Journal of Theoretical Physics **53** (2005) 111-120.

[16] Srinivasa Raju, R., Mahesh Reddy, B., Rashidi, M.M., and Gorla, R.S.R., *Application of finite element method to unsteady MHD free convection flow past a vertically inclined porous plate including thermal diffusion and diffusion thermo effects*, J. Porous Media **19** (2016) 701-722.

[17] U. S. Rajput and Gaurav Kumar, *Radiation effect on MHD flow past an inclined plate with variable temperature and mass diffusion*, International Journal of Applied Science and Engineering **14(3)** (2017) 171-183.

[18] V. Naga Raju, K. Hemalatha, V. Srihari Babu, *MHD Viscoelastic Fluid Flow Past an Infinite Vertical Plate in the Presence of Radiation and Chemical Reaction*, International Journal of Applied Engineering Research **14(5)** (2019) 1062-1069.

[19] Veera Krishna, M. and Gangadhar Reddy, M., *MHD free convective rotating flow of Visco- elastic fluid past an infinite vertical oscillating porous plate with chemical reaction*, IOP Conference Series: Material Science and Engineering **149** (2016) 012217

[20] Veera Krishna, M. and Subba Reddy, G., *Unsteady MHD convective flow of second grade fluid through a porous medium in a rotating parallel plate channel with temperature dependent source*, IOP Conference Series: Material Science and Engineering **149** (2016) 012216.

L. Rama Mohan Reddy, Department of Mathematics, Rajiv Gandhi University of Knowledge Technologies-A.P, India

P. Veera Sanjeeva Kumar, Department of Mechanical Engineering, Annamacharya Institute of Technology & Sciences (Autonomous)
e-mail: sajneevpalagiri@gmail.com

K. Mohna Babu , Department of Mathematics, Rajiv Gandhi University of Knowledge Technologies-A.P, India

B.Nagaraja Naik, Department of Mathematics and CSE, Rajiv Gandhi University of Knowledge Technologies-A.P, India