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ON $(1,2)^*$-$\ddot{g}$ CLOSED FUNCTIONS IN INTUITIONISTIC FUZZY BITOPOLOGICAL SPACES

S. MUKESH PARKAVI ✉ and A. ARIVU CHELVAM

Abstract

In this article, we introduce a new type of functions on Intuitionistic Fuzzy Bitopological spaces called $(1,2)^*$- $\ddot{g}$ closed functions and $(1,2)^*$-i$\ddot{g}$ functions and study some characterizations of $(1,2)^*$- $\ddot{g}$ closed function in intuitionistic fuzzy bitopological spaces.

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1. Introduction

Zadeh [12] was introduced the fuzzy sets and Chang [3] was initiated the fuzzy topology. Kandil [6] introduced the concept of fuzzy bitopological spaces as a natural generalization of Chang's fuzzy topological spaces. Atanassov [2] was studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Malghan [7] have initiated generalized closed mappings in topological spaces. The concept of open and closed mappings in intuitionistic fuzzy topological spaces was discussed by Seok Jong Lee and Eun Pyo Lee [11]. Joung Kon Joen et al [5] gave various types of intuitionistic fuzzy open mapping. α-generalized closed mappings and contra α-generalized closed mappings in intuitionistic fuzzy topological spaces were introduced by Santhi and Sakthivel [10]. Recently these authors [9] introduced the concept $(1,2)^*$-Intuitionistic fuzzy $\ddot{g}$ continuous and irresolute mappings in bitopological spaces.

In this paper, we introduced the concept of intuitionistic fuzzy bitopological spaces as a generalization of fuzzy bitopological spaces. Next, we introduce the notations of $(1,2)^*$-$\ddot{g}$ closed functions and $(1,2)^*$- $i\ddot{g}$ closed functions in intuitionistic fuzzy bitopological spaces and studied their relation among them with suitable examples.

2. Preliminaries

Throughout this paper (X,τ_1, τ_2) and (Y,σ_1,σ_2) or simply X and Y denote the intuitionistic fuzzy bitopological spaces (briefly IFBTS). For a subset A of a space X, the closure, interior and complement of A are denoted by cl(A), int (A) and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1. [2] Let X be a non-empty set. An intuitionistic fuzzy set (briefly IFS) A in X is an object having the form, A = $\{< X, \mu_A(x), \gamma_A(x) > / x \in X\}$ where the functions μ_A : X → [0,1] and γ_A : X → [0,1] denote the degree of membership (i.e., $\mu_A(x)$) and the degree of non-membership (i.e., $\gamma_A(x)$) of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$. The set of all intuitionistic fuzzy sets in X is denote by IFS(X).

Definition 2.2. [4] An intuitionistic fuzzy topology (briefly IFT) on X is a family τ of IFSs in X satisfying the following axioms.

1. 0~, 1~ $\in \tau$
2. $H_1 \cap H_2 \in \tau$ for any $H_1, H_2 \in \tau$
3. $\cup H_i \in \tau$ for any family $\{H_i / i \in$ J$\} \subseteq \tau$.

In this state the pair (X,τ) is called an intuitionistic fuzzy topological space (briefly IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (briefly IFOS) in X. The complement of an intuitionistic fuzzy open set is called an intuitionistic fuzzy closed set (briefly IFCS) in X.

Definition 2.3. [1] Let τ_1 and τ_2 be two intuitionistic fuzzy topologies on a non-empty set X. The triple (X,τ_1,τ_2) is called an intuitionistic fuzzy bitopological spaces (briefly IFBTS), every member of $\tau_{1,2}$ is called $\tau_{1,2}$-intuitionistic fuzzy open sets ($\tau_{1,2}$-IFOS) and the complement of $\tau_{1,2}$-IFOS is $\tau_{1,2}$-intuitionistic fuzzy closed sets ($\tau_{1,2}$-IFCS).

Definition 2.4. Let f be a function from an IFBTS X into an IFBTS Y. Then f is said to be an

1. $(1,2)^*$-intuitionistic fuzzy closed function ($(1,2)^*$-IF closed function in short) if $f(A)$ is an $\tau_{1,2}$-IFCS in (Y,σ_1,σ_2) for every $\tau_{1,2}$-IFCS A of (X,τ_1,τ_2).
2. $(1,2)^*$-intuitionistic fuzzy regular closed function ($(1,2)^*$-IFR closed function in short) if $f(A)$ is an $(1,2)^*$-IFRCS in (Y,σ_1,σ_2) for every $\tau_{1,2}$-IFCS A of (X,τ_1,τ_2).
3. $(1,2)^*$-intuitionistic fuzzy generalized closed function ($(1,2)^*$-IFG closed function in short) if $f(A)$ is an $(1,2)^*$-IFGCS in (Y,σ_1,σ_2) for every $\tau_{1,2}$-IFCS A of (X,τ_1,τ_2).

3. $(1,2)^*$-$\ddot{g}$ Intuitionistic Fuzzy Closed Function

Definition 3.1. A function f: (X,τ_1,τ_2)→(Y,σ_1,σ_2) is called an $(1,2)^*$-intuitionistic fuzzy $\ddot{g}$ closed ($(1,2)^*$-IF$\ddot{g}$ closed, in short) if $f(A)$ is an$(1,2)^*$-IF$\ddot{g}$CS in (Y,σ_1,σ_2) for every $\tau_{1,2}$-IFCS A in (X,τ_1,τ_2). The complement of $(1,2)^*$-intuitionistic fuzzy $\ddot{g}$ closed function is an $(1,2)^*$-intuitionistic fuzzy $\ddot{g}$ open function.

Theorem 3.2. *Every $\tau_{1,2}$-IF closed function is an $(1,2)^*$-IF$\ddot{g}$ closed function, but not conversely.*

Proof. Let f: (X,τ_1,τ_2)→(Y,σ_1,σ_2) be an $\tau_{1,2}$-IF closed function. Let A be an $\tau_{1,2}$-IFCS in X. Since f is an $\tau_{1,2}$-IF closed function, $f(A)$ is an $\sigma_{1,2}$-IFCS in Y. Since every $\tau_{1,2}$-IFCS is an $(1,2)^*$-IF$\ddot{g}$CS [8], $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$CS in Y. Hence f is an $(1,2)^*$-IF$\ddot{g}$ closed function.

□

EXAMPLE 3.3. *Let X = {a, b} and Y = {u, v}. Let* H_1 *= <x, (0.5,0.6), (0.1,0.2)>,* H_2 *= <x, (0.5,0.4), (0.3,0.2)> and* H_3 *= <y, (0.5,0.5), (0.2,0.3)>,* H_4 *= <y, (0.4,0.5), (0.4,0.3)>. Then* $\tau_1 = \{0\sim, H_1, 1\sim\}$, $\tau_2 = \{0\sim, H_2, 1\sim\}$ *and* $\sigma_1 = \{0\sim, H_3, 1\sim\}$, $\sigma_2 = \{0\sim, H_4, 1\sim\}$ *are IFBTS on X and Y respectively. Define f:* $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *by f(a) = u, f(b) = v. Then for* $\tau_{1,2}$*-IFCS A = <x, (0.1,0.2), (0.5,0.6)> of* (X,τ_1,τ_2)*,* $f(A)$ *is not an* $\sigma_{1,2}$*-IFCS in* (Y,σ_1,σ_2)*. Therefore, f is not an* $(1,2)^*$*-IF closed function. but f is an*$(1,2)^*$*-IF*$\ddot{g}$ *closed function.*

THEOREM 3.4. *Every* $(1,2)^*$*-IF*$\ddot{g}$ *closed function is an* $(1,2)^*$*-IFG closed function, but not conversely.*

PROOF. Let $f{:}(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an $(1,2)^*$-IF$\ddot{g}$ closed function. Let A be an $\tau_{1,2}$-IFCS in X. Since f is an $(1,2)^*$-IF$\ddot{g}$ closed function, $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$CS in Y. Since every $(1,2)^*$-IF$\ddot{g}$CS is an $(1,2)^*$-IFGCS [8], $f(A)$ is an $(1,2)^*$-IFGCS in Y. Hence f is an $(1,2)^*$-IFG closed function.

□

EXAMPLE 3.5. *Let X = {a, b} and Y = {u, v}. Let* H_1*= <x, (0.1,0.3), (0.8,0.7)>,* H_2 *= <x, (0.1,0.2), (0.8,0.8)> and* H_3 *= <y, (0.8,0.8), (0.2,0.1)>,* H_4 *= <y, (0.6,0.4), (0.3,0.3)>,* H_5 *= <y, (0.5,0.3), (0.3,0.4)>. Then* $\tau_1 = \{0\sim, H_1, 1\sim\}$, $\tau_2 = \{0\sim, H_2, 1\sim\}$ *and* $\sigma_1 = \{0\sim, H_3, H_4, 1\sim\}$, $\sigma_2 = \{0\sim, H_5, 1\sim\}$ *are IFBTS on X and Y respectively. Define f:* $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *by f(a) = u, f(b) = v. Then for* $\tau_{1,2}$*-IFCS A = <x, (0.8,0.7), (0.1,0.3)> of* (X,τ_1,τ_2)*,* $f(A)$ *is not an* $(1,2)^*$*-IF*$\ddot{g}$*CS in* (Y,σ_1,σ_2)*. Therefore, f is not an* $(1,2)^*$*-IF*$\ddot{g}$ *closed function. but f is an* $(1,2)^*$*-IFG closed function.*

THEOREM 3.6. *A function f:* $(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is an* $(1,2)^*$*-IF*$\ddot{g}$ *closed if and only if f(A) is an* $(1,2)^*$*-IF*$\ddot{g}$*OS in* (Y,σ_1,σ_2) *for every* $\tau_{1,2}$*-IFOS in X.*

PROOF. Let A be an $\tau_{1,2}$-IFOS in X. This implies A^c is an $\tau_{1,2}$-IFCS in X. Since f is an $(1,2)^*$-IF$\ddot{g}$ closed, $f(A^c)$ is an $(1,2)^*$-IF$\ddot{g}$CS in Y. Since $f(A^c) = (f(A))^c$, $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$OS in Y. Hence $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$OS in Y for every $(1,2)^*$-IFOS in X.

Let A be an $\tau_{1,2}$-IFCS in X. Then A^c is an $\tau_{1,2}$-IFOS on X. By hypothesis, $f(A^c)$ is an $(1,2)^*$-IF$\ddot{g}$OS in Y. Since $f(A^c) = (f(A))^c$, $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$CS in Y. Hence f is an $(1,2)^*$-IF$\ddot{g}$ closed function.

□

REMARK 3.7. *The composition of two* $(1,2)^*$*-IF*$\ddot{g}$ *closed functions need not be an* $(1,2)^*$*-IF*$\ddot{g}$ *closed function.*

EXAMPLE 3.8. *Let X = {a, b}, Y = {c, d} and Z = {u, v} Let* H_1*= <x, (0.7,0.8), (0.3,0.2)>,* H_2 *= <x, (0.7,0.6), (0.3,0.3)> and* H_3 *= <y, (0,0), (1,1)>,* H_4 *= <z, (0.4,0.4), (0.6,0.6)>,* H_5 *= <z, (0.3,0.4), (0.6,0.5)> Then* $\tau_1 = \{0\sim, H_1, 1\sim\}$, $\tau_2 = \{0\sim, H_2, 1\sim\}$ *and* $\sigma_1 = \{0\sim, H_3, 1\sim\}$, $\sigma_2 = \{0\sim, 1\sim\}$ *and* $\delta_1 = \{0\sim, H_4, 1\sim\}$, $\delta_2 = \{0\sim, H_5, 1\sim\}$ *are IFBTS on X, Y and Z respectively. Define f:*$(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *by f(a) = c, f(b) =*

d and define g:(Y,σ_1,σ_2)→(Z,δ_1,δ_2) by g(c) = u, g(d) = v. Then f and g are $(1,2)^*$*-IF$\ddot{g}$ closed functions. consider an IFS A = <z, (0.3,0.2), (0.7,0.8)>. Then A is an $\delta_{1,2}$-IFCS in (Z,δ_1,δ_2). But (f ∘ g) (A) is not an* $(1,2)^*$*-IF$\ddot{g}$ closed set in (X,τ_1,τ_2). Hence the composition of two* $(1,2)^*$*-IF$\ddot{g}$ closed functions need not be an* $(1,2)^*$*-IF$\ddot{g}$ closed function.*

4. $(1,2)^*$-IF i-$\ddot{g}$ Closed Functions

Definition 4.1. A function f: (X,τ_1,τ_2)→(Y,σ_1,σ_2) is called an $(1,2)^*$-intuitionistic fuzzy irresolute $\ddot{g}$ closed (($(1,2)^*$-IF i-$\ddot{g}$ closed, in short) if $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$CS in (Y,σ_1,σ_2) for every $(1,2)^*$-IF$\ddot{g}$CS A in (X,τ_1,τ_2).

Theorem 4.2. *Let f: (X,τ_1,τ_2)→(Y,σ_1,σ_2) be an* $(1,2)^*$*-IF i-$\ddot{g}$ closed function. Then f is an* $(1,2)^*$*-IF$\ddot{g}$ closed function but not conversely.*

Proof. Let f be an $(1,2)^*$-IF i-$\ddot{g}$ closed function. Let A be any $\tau_{1,2}$-IFCS in X. Since every $\tau_{1,2}$-IFCS is an $(1,2)^*$-IF$\ddot{g}$CS [8] and A is an $(1,2)^*$-IF$\ddot{g}$CS in X. By hypothesis, $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$CS in X. Hence f is an$(1,2)^*$-IF$\ddot{g}$ closed function. □

example 4.3. *Let X = {a, b} and Y = {u, v}. Let H_1= <x, (0.7,0.6), (0.3,0.4)>, H_2 = <y, (0.3,0.4), (0.7,0.6)> and H_3 = <y, (0.2,0.4), (0.8,0.6)>. Then τ_1= {0∼, H_1,1∼}, τ_2 = {0∼,1∼} and σ_1 = {0∼, H_2, 1∼}, σ_2 = {0∼,H_3,1∼} are IFBTS on X and Y respectively. Define f: (X,τ_1,τ_2)→(Y,σ_1,σ_2) by f(a) = 1-u, f(b) = 1-v. Then f is an* $(1,2)^*$*-IF$\ddot{g}$ closed function. But f is not an* $(1,2)^*$*-IF i-$\ddot{g}$closed function. Since the A = <x, (0.2,0.3), (0.8,0.7)> is an* $(1,2)^*$*-IF$\ddot{g}$CS in (X,τ_1,τ_2), but* f(A) *= <y, (0.8,0.7), (0.2,0.3)> is not an* $(1,2)^*$*-IF$\ddot{g}$CS in (Y,σ_1,σ_2). Hence f is an* $(1,2)^*$*-IF$\ddot{g}$closed function but f is not an*$(1,2)^*$*-IF i-$\ddot{g}$ closed function.*

Theorem 4.4. *Let f: (X,τ_1,τ_2)→(Y,σ_1,σ_2) and g: (Y,σ_1,σ_2)→(Z,δ_1,δ_2) be an* $(1,2)^*$*-IF i-$\ddot{g}$ closed function. Then $g \circ f$: (X,τ_1,τ_2)→(Z,δ_1,δ_2) is an*$(1,2)^*$*-IF i-$\ddot{g}$ closed function.*

Proof. Let A be an $(1,2)^*$-IF$\ddot{g}$CS in X. then $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$CS in Y. Since g is an $(1,2)^*$-IF i-$\ddot{g}$ closed function, g ($f(A)$) is an $(1,2)^*$-IF$\ddot{g}$CS in Z. Hence $g \circ f$ is an $(1,2)^*$-IF i-$\ddot{g}$ closed function.

□

Theorem 4.5. *Let f: (X,τ_1,τ_2)→(Y,σ_1,σ_2) be an* $(1,2)^*$*-IF$\ddot{g}$ closed function and g: (Y,σ_1,σ_2)→(Z,δ_1,δ_2) be an* $(1,2)^*$*-IF i-$\ddot{g}$ closed function. Then $g \circ f$: (X,τ_1,τ_2)→(Z,δ_1,δ_2) is an* $(1,2)^*$*-IF$\ddot{g}$ closed function.*

Proof. Let A be an $\tau_{1,2}$-IFCS in X. Since f is an $(1,2)^*$-IF$\ddot{g}$ closed function, $f(A)$ is an $(1,2)^*$-IF$\ddot{g}$CS in Y. Since g is an $(1,2)^*$-IF i-$\ddot{g}$ closed function, g ($f(A)$) is an $(1,2)^*$-IF$\ddot{g}$CS in Z. Hence $g \circ f$ is an $(1,2)^*$-IF$\ddot{g}$ closed function.

□

5. Conclusions

Recently, we introduced $(1,2)^{*}$-IF$\ddot{g}$ closed sets in intuitionistic fuzzy bitopological spaces. Using the above concepts we learned a new type of $(1,2)^{*}$-IF$\ddot{g}$ closed functions and its various properties. And also we developed $(1,2)^{*}$-IF i-$\ddot{g}$closed functions and its results.

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S. Mukesh Parkavi, Full-Time Research Scholar, PG and Research Department of Mathematics, Mannar Thirumalai Naicker College (Autonomous), Affiliated to Madurai Kamaraj University, Madurai , India
e-mail: mukeshparkavi98@gmail.com

A. Arivu Chelvam, Associate Professor, PG and Research Department of Mathematics, Mannar Thirumalai Naicker College (Autonomous), Affiliated to Madurai Kamaraj University, Madurai , India
e-mail: arivumtnc@gmail.com